BEAM MATCHING TO A PLASMA WAKE FIELD ACCELERATOR USING A RAMPED DENSITY PROFILE AT THE PLASMA BOUNDARY

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Abstract

An important aspect of plasma wake field accelerators (PWFA) is stable propagation of the drive beam. In the under dense plasma regime, the drive beam creates an ion channel which acts on the beam as a strong thick focusing lens. The ion channel causes the beam to undergo multiple betatron oscillations along the length of the plasma. There are several advantages if the beam size can be matched to a constant radius. First, simulations have shown that instabilities such as hosing are reduced when the beam is matched [1]. Second, synchrotron radiation losses are minimized when the beam is matched. Third, an initially matched beam will propagate with no significant change in beam size in spite of large energy loss or gain. Coupling to the plasma with a matched radius can be difficult in some cases. This paper shows how an appropriate density ramp at the plasma entrance can be useful for achieving a matched beam. Additionally, the density ramp is helpful in bringing a misaligned trailing beam onto the drive beam axis. A plasma source with boundary profiles useful for matching has been created for the E-164X PWFA experiments at SLAC.

INTRODUCTION

Tunnel ionization of lithium vapor by the self-field of the electron beam, has been successfully used in recent PWFA experiments at SLAC [2]. This greatly simplifies the problem of plasma production. Because tunnel ionization produces a fully ionized plasma, the plasma density profile is identical to the vapor density profile. In the under dense blow out regime, a high density electron beam exerts a large radial force on the plasma electrons creating a uniform ion column. Assuming rapid ionization and blow out of the plasma electrons, the resulting ion column profile will be identical to the vapor density profile.

The plasma source used for E-164X experiments was a lithium heat pipe oven [2]. The lithium vapor was confined to the hot zone by room temperature helium buffer gas. Helium has a much higher ionization potential than lithium, so helium ionization is not considered here. The axial density profile of the lithium vapor was measured by recording the temperature along the oven axis with a thermocouple probe. The lithium vapor density was calculated using the empirical vapor pressure formula in [3]. The data shows the axial density profile is uniform in the center and the vapor boundaries have a Gaussian shaped axial profile. This paper shows

that the rise in density at the plasma boundary can actually be helpful for achieving matched beam propagation.

BEAM PROPAGATION EQUATION MODEL

Beam transport in under dense PWFA's can be modeled by considering the ion channel to be an ideal lens with a linear focusing force. For a set of initial round beam parameters, the beam envelope, r, is obtained by solving Equation 1.

$$\frac{d^2r}{dz^2} + k^2r = \frac{\varepsilon^2}{r^3} \qquad (1)$$

$$k = \frac{\omega_p}{c}\frac{1}{\sqrt{2\gamma}}$$

where, ω_p is the plasma frequency, γ is the relativistic beam factor and ε is the beam emittance.

Beam Propagation in a Uniform Density Plasma

A beam focused at the boundary of a uniform plasma will focus and oscillate along z. For the E-164X experimental parameters, the 28.5 GeV beam undergoes several betatron oscillations along the plasma. Equation 1 was solved and plotted using Mathematica. Figure 1 shows the unmatched beam envelope for a uniform plasma with a step in density of $n_o = 2\times10^{17}$ cm^{-3}, $\varepsilon = 1\times10^{-9}$ m-r, and a 10 μm beam waist at the plasma entrance. The betatron wavelength ($2\pi/k$) is 2.5 cm. The beam focuses to a minimum size of .4 μm. The matching condition, dr/dz = 0, is $r^2 = \varepsilon/k$. For this case, matching requires the beam to be focused to a 2 μm waist at the plasma entrance.

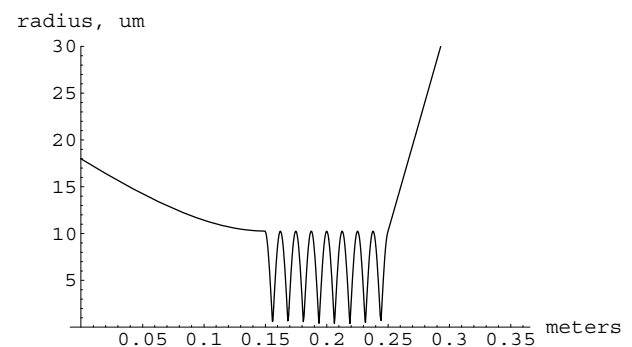

Figure 1. Plot of beam radius for $n_0 = 2\times10^{17}$ cm^{-3} and a flat density profile from z = 0.15 to 0.25 meters. The beam is focused to a 10 μm waist at the entrance, with an emittance of 1×10^{-9} m-r.

Beam Propagation for the Measured Plasma Density Profiles Used in the E-164X Experiment

For a non-uniform density profile, k is not constant, k = k(z). This can have important effects on beam propagation. As shown in Figure 2, the measured axial vapor density profiles used for E-164X are uniform in the center, but the boundaries have a Gaussian shape. When normalized to density, the measured Gaussian boundary shape was the same for all cases of interest. The boundary profile shape does not change with heater power (which is used to vary the vapor length) or helium pressure (which is used to vary the vapor density). Figure 3 shows the model profile used to calculate the beam envelope plots below.
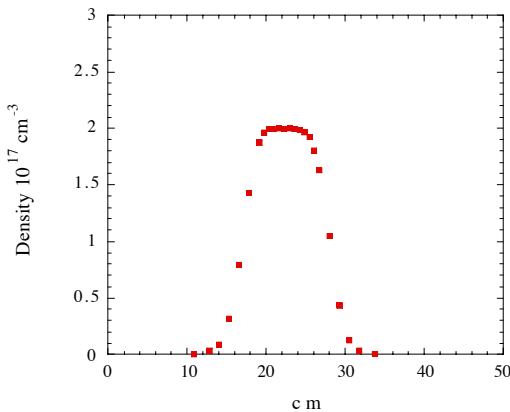
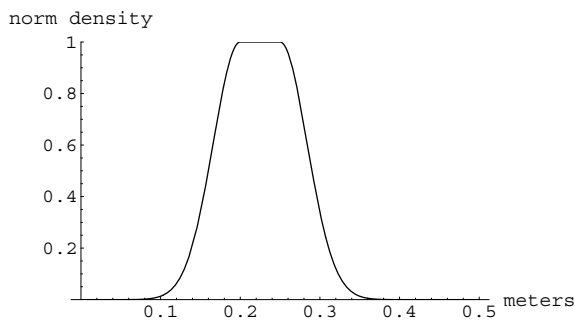

Figure 2. Measured lithium vapor density profile.

Figure 3. Normalized density profile model.

Figure 4 shows the beam envelope along z, for $n_0 = 2 \times 10^{17}$ cm^{-3}, $\varepsilon = 1 \times 10^{-9}$ m-r, and a 10 μm beam waist at z = 0.15 m. Note that as the beam propagates to higher density the beam radius gets smaller. Conservation of emittance keeps kr constant, so as k increases r decreases. Comparison shows the oscillation amplitude in Figure 4 is smaller than the amplitude in Figure 1, where the density has a step boundary.
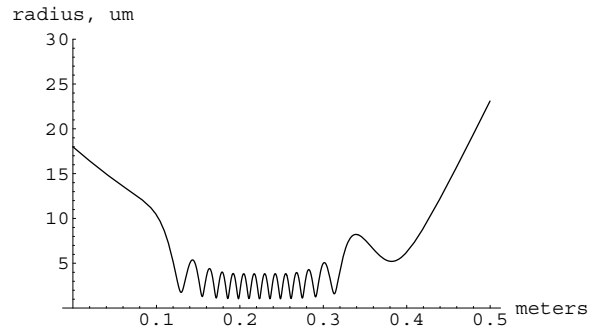

Figure 4. Plot of the beam radius for peak $n_o = 2 \times 10^{17}$ cm^{-3} using the density profile shown in Figure 3. The beam is focused in vacuum to a 10 um waist at z = .15 m.

Matched Beam Propagation for the Plasma Density Profiles Used in the E-164X Experiment

For the case of an adiabatic change in the focusing force, the density boundary can be used to focus the beam in such a way as to match the beam to the constant density portion of the plasma. The required beam parameters can be found by solving for the beam matching condition at the peak density (z = z$_{peak}$). The beam envelope can be found from Equation 1 using the initial conditions r(z$_{peak}$)^2 = ε/k(z$_{peak}$) and dr(z$_{peak}$)/dz = 0. Propagating the beam back to where $n_0 \approx 0$ and solving for r and dr/dz establishes the initial beam parameters. The initial parameters can be used to calculate the beam envelope in vacuum and set the desired beam size and waist location.

Figure 5 shows matched beam propagation using Figure 3 as the density profile model. For $n_e = 2 \times 10^{17}$ cm^{-3}, and $\varepsilon = 1 \times 10^{-9}$ m-r, a 5.8 μm waist in vacuum at z = 0.11 m is required to obtain the match. As expected the beam reaches a matched beam radius of 2 μm at the maximum plasma density.

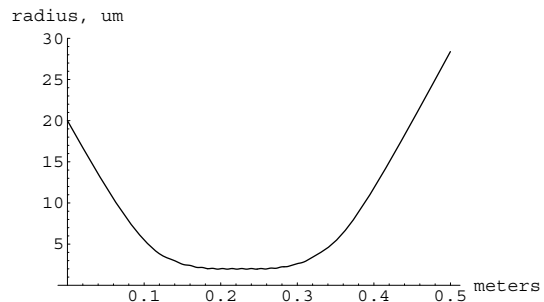Figure 5. Plot of the beam radius for matched beam propagation for peak $n_0 = 2 \times 10^{17}$ cm^{-3} using the density profile shown in Figure 3. A 5.8 um waist in vacuum at z = .11 m is required to match the beam.

Figure 6 shows that for the same beam conditions above, the beam is nearly matched for a lower density of $n_0 = 8 \times 10^{16}$ cm^{-3}. At lower density the focusing in the boundary is reduced producing the correct beam size for

near matching in this case. Similarly at higher density, the focusing in the boundary is increased and near matching is also obtained.
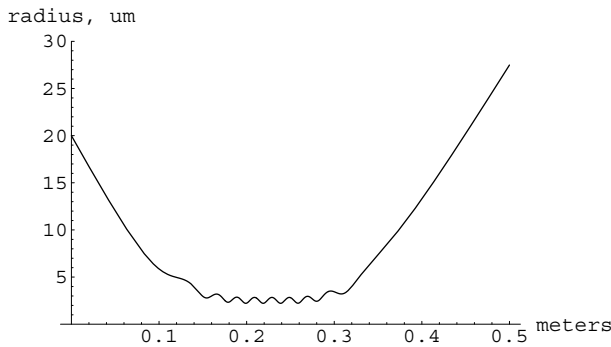

Figure 6 Plot of the beam radius for the same beam parameters as in Figure 5 for a lower peak density of $n_0 = 8 \times 10^{16}$ cm^{-3}.

Matched Beam Propagation with an Adiabatic Change in Beam Energy

The drive beam of a PWFA can loose nearly all its energy as it propagates through the plasma. This will cause k to increase along z and the geometric emittance to increase. If the beam is initially matched and the focusing force changes adiabatically, the beam will remain closely matched. An initially matched beam will propagate with no significant change to the beam radius in spite of the large energy change. The beam envelope Equation including energy loss or gain is [1],

$$\frac{d^2r}{dz^2} + \frac{1}{\gamma}\frac{d\gamma}{dz}\frac{dr}{dz} + k^2 r = \frac{\varepsilon_N^2}{\gamma^2 r^3} \qquad (2)$$

where, γ is the relativistic factor corresponding to the beam energy and $\varepsilon_N = \gamma_i\varepsilon$ is the initial normalized emittance. Note k varies along z with γ and density. For E-164X parameters the energy loss is too small to observe the effect in Eqn. (2). To illustrate the effect on beam matching including large energy loss Eqn. (2) was solved for a matched beam in a 60 cm long plasma with Gaussian boundaries and continuous beam energy loss of 40 GeV per meter. Using the same relevant parameters as Figure 5, Figure 7 demonstrates matched beam propagation of a beam with continuous energy loss.
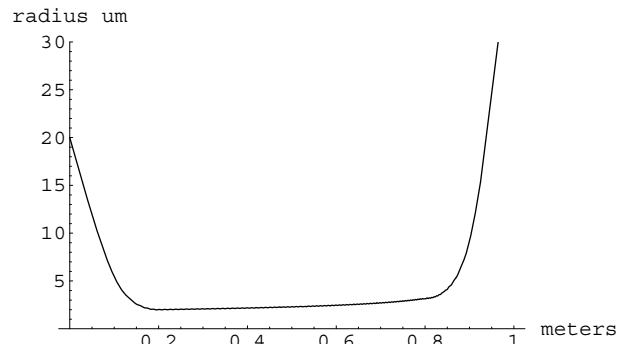

Figure 7. Plot of beam radius showing matched beam propagation in a 60 cm long, $n_0 = 2 \times 10^{17}$ cm^{-3}, plasma with Gaussian boundaries, and a continuous beam energy loss of 40 GeV per meter.

RELATED PWFA ISSUES

As shown in [1] matching is important for stability. Also matching is important to minimize synchrotron radiation losses. Synchrotron radiation loss scales as $n^2\gamma^2 r^2$ [4]. Synchrotron radiation loss becomes important in our experiments for densities above 2×10^{17} cm^{-3}. For beams with energy greater than 100 GeV the synchrotron loss would be too large for the same parameters used here. Synchrotron losses can be minimized if the beam is matched. Matched beam loss scales as, $n^{3/2}\gamma^{3/2}\varepsilon$. A matched beam with low emittance is required to avoid synchrotron losses for higher energy beams.

Keeping the trailing beam aligned to the drive beam in a PWFA could prove to be difficult. A misaligned trailing beam would normally oscillate at the initial amplitude of the offset. A density ramp at the plasma boundary provides a mechanism to reduce the oscillation amplitude, because for conserved emittance, as k increases r decreases.

References
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[2] M. J. Hogan et al., accepted for publication in Phys. Rev. Lett.
[3] A. G. Mozgovoi, et al., "The Saturated Vapor Pressure of Lithium Potassium Rubidium and Cesium", High Temperatures High Pressures 19, 425, (1987)
[4] E. Esarey et al., Phys. Rev. E, 65 056505 (2002)