

# DEVELOPING A MULTI-TIMESCALE PIC CODE FOR PLASMA ACCELERATORS

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## Abstract

An idea for advancing beam and plasma particles with different time scales in a full PIC model of plasma accelerators is proposed. Because beam particles usually respond much slower than plasma particles, large time steps can be used to update beam particles to save computation time. In this paper, we will describe how to apply this multi-timescale method in a particle-in-cell (PIC) [1] code OSIRIS [2]. Simulation results for SLAC E164 [3] experimental parameters are given and show a high degree of accuracy while gaining a factor of 4-6 in computing time. The limitations of this method are also studied. The maximum time saving is determined by driver beam energy and size of simulation box.

## INTRODUCTION

Explicit Particle-in-cell (PIC) codes give high fidelity but require considerable computation time. With the increasing beam intensity, beam energy and system complexity of particle accelerators, computing time becomes a more and more important issue in current and future accelerator research. For example, to model a 3D relativistic beam propagating through high density plasma ( $10^{16} \text{ cm}^{-3}$ ) for several meters, as in a future afterburner [4] experiment, requires for a PIC code like OSIRIS ~100,00 hours of CPU time. It is not practical to model such big problems with explicit PIC codes. Thus new mathematical algorithms, computing methods and scientific codes need to be developed to decrease simulation time. Reduced approximation codes such as QuickPIC [5] are being developed to save running time while still keeping a high degree of accuracy. QuickPIC is based on the quasi-static approximation. It can model plasma wave excitation due to an intense particle or laser beam and study beam transverse instability, but it has difficulty in handling some physics such as particle trapping. In this paper, we propose a new method to save computation time while keeping most of the functionality of OSIRIS. In this method, beam particles are advanced with two different time steps, one of which is much longer than that of plasma particles. This enables us to save computation time while obtaining correct results. This method can also be used in other particle accelerator simulation codes.

## ALGORITHM

In OSIRIS, the time step is typically set on the order of  $1/10 \omega_p^{-1}$  or even less to resolve the plasma frequency and wavelength. Relativistic beam particles usually evolve

much more slowly than plasma particles. The wavelength of beam betatron oscillation is given by  $\lambda_\beta = 2\pi\sqrt{2\gamma}c/\omega_p$  [6], which is about  $2176 c/\omega_p$  for a 30 GeV beam. This implies a possible way of using different time scales for plasma wake excitation and beam evolution. The basic idea is the following: wake fields are calculated and the beam is updated at each time step (every  $dt$ ) while plasma fills the moving simulation window. (The beam stays still relative to the simulation window.) Right after the plasma fills the window, the position and momentum of the beam is updated with a big time step  $T$  that can be several window sizes. Then fresh plasma flows in and fields are recalculated for this updated beam. This process repeats and time saving is determined by  $\frac{T}{N * dt}$  ( $N$  is number of

small steps of length  $dt$  for plasma to fill the simulation window and  $dt$  is the time step for updating plasma particles). This is illustrated in Fig. 1. This enables us to simulate problems in several days that previously required weeks.

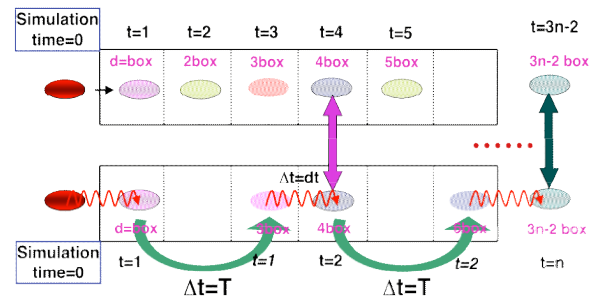


Figure 1: Illustration of time saving with multi-timescale method. Here  $T=2*N*dt$ . Top: Normal PIC algorithm; Bottom: multi-timescale beam advance

## SIMULATION RESULTS

First, we do a test run using our multi-timescale OSIRIS for SLAC E164 experiment parameters. The simulation parameters are given in Table 1. The large time step  $T$  for updating the driver beam right after the plasma flows in and fills the whole simulation box is set as  $2*N*dt$  or we say 2 window sizes.  $N$  is 588 for our simulation parameters.

The simulation results are compared to normal OSIRIS without fast-push for driver beam (See Fig. 2). Figs. 2a) and 2b) show the realspace of the beam from normal OSIRIS and multi-timescale OSIRIS. Figs. 2c) and 2d) show the comparison of average energy gain and

electrical field from both methods. All of them show good agreement. Though the beam evolves very slowly, multiple points are still needed to resolve one betatron oscillation, otherwise the fields change so much in one beam time step that the push becomes inaccurate. The error will accumulate each time we update the beam with a big step. This limits the value of beam time step  $T$  and therefore limits the time saving we can achieve with this method. In order to qualify how much we can increase the time step for advancing the driver beam, we try several cases with time step  $T$  ranging from  $N*dt$  to  $20*N*dt$ . Fig. 2 shows the realspace of the beam for  $T = dt, 2*N*dt, 4*N*dt$  and  $8*N*dt$ . Fig. 3 shows the magnitude of maximum average energy gain deviation normalized by  $Max\_E0$  ( $Max\_E0$  is the maximum average energy from

Table 1: simulation parameters for E164 experiment

Beam Energy (GeV)	30
Total particle number $N$	$2*10^{10}$
$\sigma_z$ ( $\mu\text{m}$ )	100
$\sigma_r$ ( $\mu\text{m}$ )	25
Grid Size ( $z*r$ )	$200*200$
Simulation Size ( $c/\omega_p=75\mu\text{m}$ )	$10*4$
$dt$ ( $1/\omega_p$ )	0.017
Plasma particles/cell	$2*2$
Beam particles/cell	$5*5$

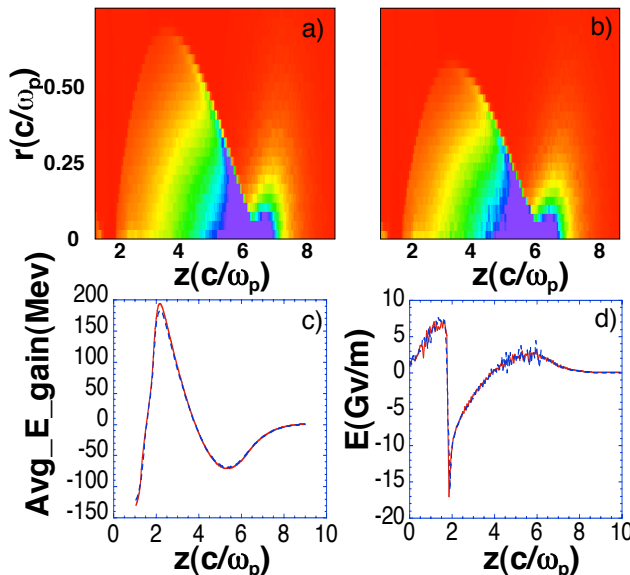


Figure 2: Comparison simulation results of multi-timescale OSIRIS and normal OSIRIS after beam propagating 34.5mm a) realspace of beam from normal OSIRIS b) realspace of beam from multi-timescale OSIRIS c) average Energy gain of beam Vs.  $z$  d) lineout of electrical field Vs.  $z$  (the solid line represents normal OSIRIS and dotted line represents multi-timescale OSIRIS)

normal OSIRIS) at different values of  $T$  (the time is normalized with window size). The total beam density deviation from that of normal OSIRIS is also shown in Fig. 3. This is obtained from the sum of the squares of the pixel by pixel difference between the images in Fig. 3.

We can see that when  $\frac{\lambda_\beta}{c*dt}$  becomes too small, there are

not enough points to resolve one betatron oscillation (as the case with time step of  $8*N*dt$  where there are only 27 points for one betatron wavelength) and leads to computational error. Since the time saving is determined by  $\frac{T}{N*dt}$ , the limitation on beam time step  $T$  decides the

maximum time saving we can achieve with multi-timescale method. Thus the maximum time saving is determined by  $\frac{\lambda_\beta}{N*c*dt*M}$  (Here  $M$  is minimum

number of points we need for resolving one betatron oscillation). For E164 parameters,  $M$  is around 50. Table 2 shows the CPU running time for different  $T$  values that give reasonable accuracy. The maximum time saving for 30 GeV driver beam is about 4-6. The saving can reach 25 for TeV beams with  $\lambda_\beta$  6 times as large as that of a 30 GeV beam.

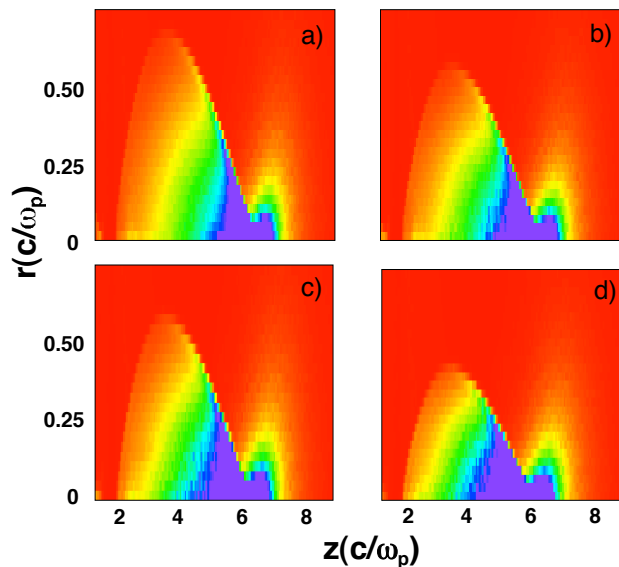


Figure 3: realspace of beam for different beam updating time steps  $T$  a)  $T= dt$  b)  $T= 2*N*dt$  c)  $4*N*dt$  and d)  $8*N*dt$ .  $dt$  is the time step for updating plasma particles and  $c*N*dt$  is the size of one simulation window.

## CONCLUSION

A multiple timescales method is developed in PIC code OSIRIS to save running time while producing reasonable physical results for plasma accelerators. The new method suggests time savings of 4-6 are possible for present beam-driven experiments and savings of 25 for TeV-class simulations. Further work is planned to test this

hypothesis over longer runs and extend the concept to laser drivers.

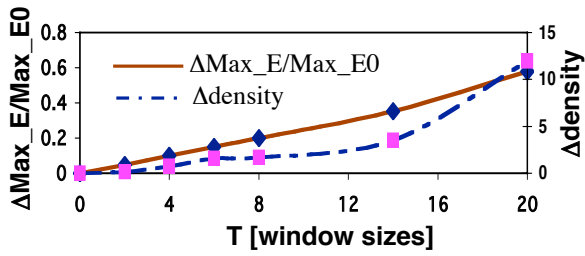


Figure 4: normalized maximum average energy gain deviation and beam density deviation for different beam advancing steps (T)

Table 2: CPU time for different T

T/window size	CPU time (s)
1	13788
2	8765
4	6467
6	3160
8	2336

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