WAKEFIELDS IN A DIELECTRIC TUBE WITH FREQUENCY DEPENDENT DIELECTRIC CONSTANT*

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Abstract

Laser driven dielectric accelerators could operate at a fundamental mode frequency where consideration must be given to the frequency dependence of the dielectric constant when calculating wakefields. Wakefields are calculated for a frequency dependence that arises from a single atomic resonance. Causality is considered, and the effects on the short range wakefields are calculated.

INTRODUCTION

One class of laser driven accelerators would have transverse dimensions comparable to the laser wavelength,[1] and a bunch charge limited to a few fC by longitudinal wakefields associated with the surrounding dielectric.[2] In addition, the effects of transverse wakefields are known to be severe for small structures.

The wakefields produced by passage of a charged particle through a dielectric tube have been studied by a number of authors [3,4,5,6] The method of analysis is to Fourier transform the currents and fields and to apply boundary conditions at the inner and outer radii of the dielectric. The results in these references are discussed for a real dielectric constant $\varepsilon_r > 1$, which is the case of frequencies well below those of the resonances of atoms forming the dielectric. However, for laser driven dielectric structures the bunch lengths are short and the corresponding frequencies are near and above the resonances. The dielectric becomes transparent at high frequency, and this could possibly reduce the wakefields substantially.

This paper extends calculations and discussions to higher frequencies taking account of the frequency dependence of the dielectric constant. We restrict ourselves to: 1) ultra-relativistic particles, 2) the limit where the outer radius $b \to \infty$ (Figure 1), 3) the azimuthal harmonics $m = 0,1$, 4) a structure with infinite length. Dimensions are scaled to the wavelength of the accelerating mode, λ. We follow the notation and equations of G. Dôme.[3] The sign convention is that trailing particles have $s > 0$, and the time dependence is $\exp(i\omega t)$.

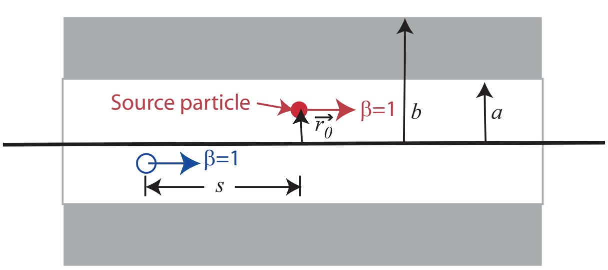

Figure 1: Dielectric tube with inner radius a and outer radius $b \to \infty$. The fields produced by the source particle located at $\vec{r}_0$ are calculated at the location of a test particle trailing by distance s.

*Work supported by Dept of Energy contract DE-AC02-76SF00515

m = 0 WAKEFIELD

The retarding voltage due to the $m = 0$ wakefield for a unit-charge source particle is[3]

$$V_0(s) = \frac{Z_0 c}{\pi\lambda^2} \frac{1}{(a/\lambda)^2} \upsilon_0(s) \tag{1}$$

where $Z_0 = 377\Omega$. The s dependence is in the factor υ_0 given by the Fourier transform

$$\upsilon_0(u) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dz \frac{\exp(izu)}{z - \dfrac{2\varepsilon_r}{\sqrt{\varepsilon_r - 1}} \dfrac{H_1^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}{H_0^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}} \tag{2}$$

where $z = \omega a/c$, $u = s/a$, and here the notation differs from that of ref. [3] to show the explicit dependence on the dielectric constant to allow frequency dependence to be introduced. It is important to note that this result was derived by applying boundary conditions at frequency ω and radius a, and it was not assumed that $\varepsilon_r > 1$ or even that it was real.

From Jackson[7] the frequency dependent dielectric constant is given by

$$\varepsilon_r = 1 + 4\pi N r_e c^2 \sum_k \frac{f_k}{\omega_k^2 - \omega^2 + i\omega\gamma_k} \tag{3}$$

where N is the electron density, and there is a sum rule relating the oscillator strengths, f_k, and the atomic number, Z. We restrict our analysis to a single resonance, which is parameterized as

$$\varepsilon_r = 1 + \frac{\alpha_1}{1 - z^2/z_1^2 + iz\gamma_1/z_1^2} \tag{4}$$

where $z_1 = \omega_1 a/c$, and γ_1 is normalized to z_1.

At low frequencies, $\omega << \omega_1$,

$$\varepsilon_r = 1 + \alpha_1 \tag{5}$$

independent of frequency. The radial wave number,

$$k_r = \frac{\omega}{c}\sqrt{\varepsilon_r - 1} \tag{6}$$

is real, and there is Cherenkov radiation. At high frequencies the dielectric behaves as a plasma with dielectric constant

$$\varepsilon_r = 1 - \frac{\alpha_1 z_1^2}{z^2}, \tag{7}$$

and the fields decay exponentially in the dielectric.

First consider low frequencies where ε_r is constant (eq. (5)). There is a pole in the complex plane at

$$z = \frac{2i(\alpha_1 + 1)}{\sqrt{\alpha_1}} \tag{8}$$

that dominates the small s behavior and can be found using the asymptotic expression for the Hankel function ratio in eq. (2), $H_1^{(2)}/H_0^{(2)} = i$. The complete expression has another pole near the positive imaginary axis, a series of poles above the negative real axis, and a cut on the negative real axis. Causality requires $\upsilon_0(u) = 0$ for particles ahead of the source ($u < 0$), and for these value of u eq. (2) can be evaluated by performing a contour integral in the lower half of the complex z-plane. There are no poles or cuts in the third and fourth quadrants, and causality is satisfied by performing the integral with z having a small negative imaginary part to avoid the cut along the negative real axis. Having made that choice eq. (2) can be evaluated for $u > 0$. The imaginary part of the integrand is an even function of u, and the real part is an odd function, so

$$\upsilon_0(u) = \frac{1}{\pi}\,\mathrm{Im}\int_0^\infty dz \frac{\exp(izu)}{z - \dfrac{2\varepsilon_r}{\sqrt{\varepsilon_r - 1}} \dfrac{H_1^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}{H_0^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}}. \tag{9}$$

This expression can be evaluated numerically because there are no cuts or poles on the positive real axis.

When $z > z_1$ for a frequency dependent ε_r (eq. (4)), $\varepsilon_r < 1$, and the Hankel functions in eq. (2) become modified Bessel functions. The sign of $\sqrt{\varepsilon_r - 1}$ is chosen so the fields decay exponentially in the dielectric. When $\gamma_1 > 0$ (as it must be for damping), two new poles appear in the upper half of the complex plane near the real axis at $|z| \gtrsim z_1$. In addition, there is a cut above the positive real axis starting at $z \sim z_1$. Many aspects of the above discussion for constant ε_r still hold. There are no poles or cuts in the lower half plane, and causality is satisfied by performing the integral with z having a small negative imaginary part. Eq. (9) holds also. In particular, there are still no cuts or poles on the positive real axis.

The results are given in Figure 2. The short distance wakefield is only weakly dependent on z_1. When $z_1 = 5$ it is ~80% of the value calculated assuming ε_r is independent of frequency, and this 20% reduction occurs only when the beam bunch is much shorter than $\sim a/10$ or so. Furthermore, z_1 cannot be substantially lower and have the structure remain a dielectric for the accelerating mode. For example, $z = 2\pi a/\lambda = 4.27$ for the accelerating mode in reference [1], and $z_1 = 5$ would correspond to an atomic resonance frequency that is only slightly above that.

To understand the result, look at the longitudinal impedance per unit length given by

$$Z_L(\omega) = \frac{Z_0}{\pi a} A(\omega a/c) \tag{10}$$

where

$$A(z) = \frac{1}{i\left(z - \dfrac{2\varepsilon_r}{\sqrt{\varepsilon_r - 1}} \dfrac{H_1^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}{H_0^{(2)}\left(z\sqrt{\varepsilon_r - 1}\right)}\right)}. \tag{11}$$

The wakefield at $s \gtrsim 0$ is determined by the real part of $A(z)$, which is shown in Figure 3. Several features can be seen. First, υ_0, which is proportional to the integral of the impedance, is dominated by the low frequency peak at $z \approx 1$ that is unaffected by the resonance. Second, for $z > z_1$ the dielectric behaves as a plasma, and plasma waves are excited by the beam. Molecules in the dielectric are unpolarized before the beam passage, and they are polarized by the beam fields initiating plasma oscillations. Although these plasma waves do not propagate as far-field Cherenkov radiation, they do contribute to the impedance and wakefield.

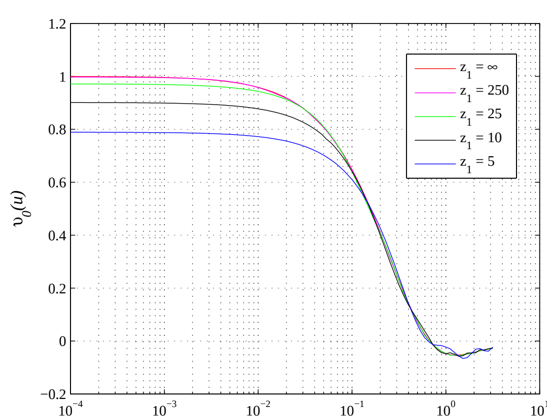

Figure 2: The s dependent factor $\upsilon_0(s)$ in eq. (1) for the $m = 0$ wakefield calculated for $\alpha_1 = 2$, $\gamma_1 = 0.001$.

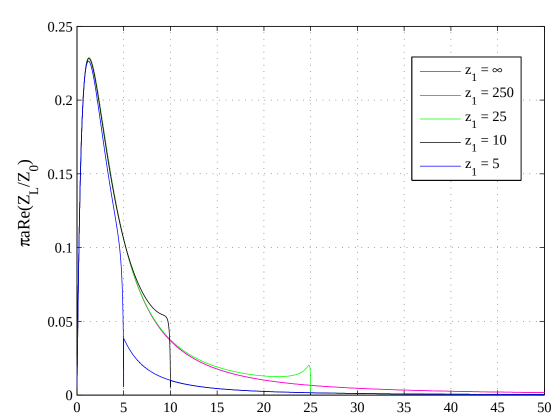

Figure 3: The real part of the longitudinal impedance given by eqs. (10) and (11).

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m = 1 WAKEFIELD

The transverse wakefield produced by a unit charge particle located at $\vec{r}_0$ with respect to the symmetry axis of the dielectric tube (in units of V/(C-m)) is [3]

$$\vec{w}_{1\perp}(s) = \frac{2cZ_H}{\lambda^2}\frac{\vec{r}_0}{a}\upsilon_1(s) \qquad (12)$$

where

$$\upsilon_1(u) = -\frac{1}{\pi}\int_{-\infty}^{\infty} dz \frac{\exp(izu)}{z^2 - \frac{2(\varepsilon_r+1)z}{\sqrt{\varepsilon_r-1}}\frac{H_2^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}{H_1^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}}. \qquad (13)$$

The same line of reasoning used for $m = 0$ can be followed to show that the transverse wakefield is causal and that υ_1 can be rewritten as

$$\upsilon_1 = -\frac{2}{\pi}\mathrm{Re}\int_0^{\infty} dz \frac{\exp(izu)}{z^2 - \frac{2(\varepsilon_r+1)z}{\sqrt{\varepsilon_r-1}}\frac{H_2^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}{H_1^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}} \qquad (14)$$

This expression can be evaluated numerically to give the results in Figure 4. Again, the frequency dependence of the dielectric constant does not have a dramatic effect.

The reason can be seen by looking at the imaginary part

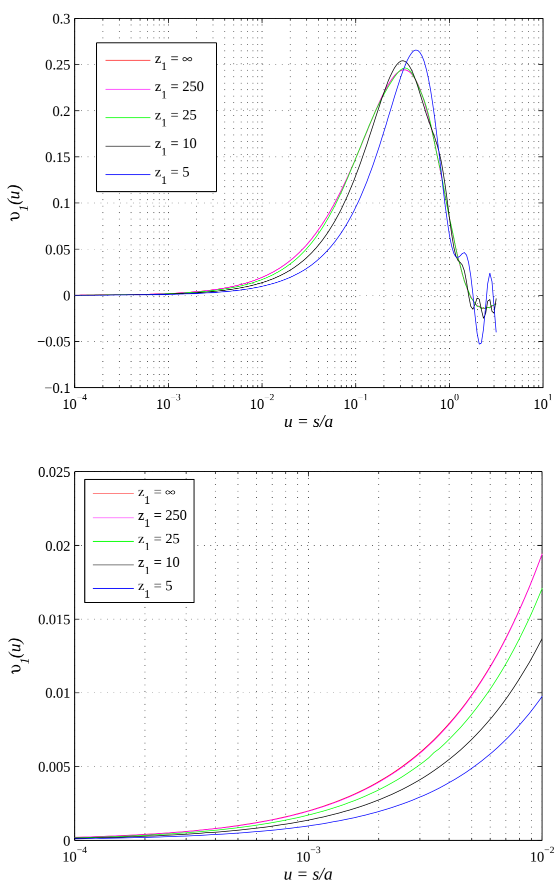

Figure 4: Factor $\upsilon_1(s)$ in eq. (12) for the transverse wakefield calculated for $\alpha_l = 2$, $\gamma_l = 0.001$.

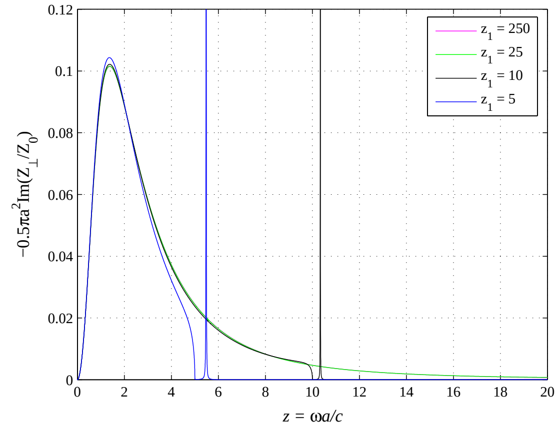

Figure 5: The imaginary part of the transverse impedance given by eqs. (15) and (16).

of the transverse impedance per unit length

$$Z_\perp(\omega) = -i\frac{2Z_0}{\pi a^2}B\left(\frac{\omega a}{c}\right) \qquad (15)$$

where

$$B(z) = \frac{1}{z^2 - \frac{2(\varepsilon_r+1)z}{\sqrt{\varepsilon_r-1}}\frac{H_2^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}{H_1^{(2)}\left(z\sqrt{\varepsilon_r-1}\right)}}. \qquad (16)$$

The integral υ_1 is dominated by the low frequency peak at $z \approx 1.5$, and this peak is only weakly dependent on z_1. Therefore, taking account of the frequency dependence of ε_r does not modify the transverse wakefield substantially.

SUMMARY & CONCLUSIONS

The frequency dependence of the dielectric constant has been taken into account using a single resonance model for the dielectric. The wakefields are not affected significantly because they are dominated by low frequency contributions. Charge limits and transverse dynamics can be calculated to good approximation using a frequency independent dielectric constant.

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