RESONANT EXCITATION OF SELECTED MODES BY A TRAIN OF ELECTRON BUNCHES IN A RECTANGULAR DIELECTRIC WAKEFIELD ACCELERATOR*

I.N.Onishchenko#, N.I.Onishchenko, G.V.Sotnikov, NSC KIPT, Kharkov, Ukraine
T.C.Marshall, Columbia University, New York City, U.S.A.

Abstract

By analysis and simulation, the effect of wakefield superposition to high amplitude is demonstrated for an arbitrary rectangular geometry which is more realizable in experiment compared with the planar slab geometry. Obtaining an intense peaked wake field in a rectangular dielectric waveguide involves the resonant excitation of only the equally spaced modes within the entire set of waveguide eigen modes which occurs when there is a coincidence between the repetition frequency of the bunch sequence and the frequency difference between selected modes.

INTRODUCTION

Recently several papers have appeared, which are devoted to acceleration of electrons by a wakefield in a dielectric filled waveguide [1-3]. Increased interest in such wakefields for accelerators is connected with the fact that short charged bunches excite simultaneously a great many radial harmonics of the dielectric waveguide which causes compression of the wakefield in the longitudinal direction and to formation of narrow peaks of axial electric field having intensity much greater than the amplitude of one radial harmonic [4].

If one uses [4,5] a uniformly-spaced sequence of bunches with rather small charge, the fields from these bunches would sum coherently, and would provide an intense accelerating field. Experimentally it is easier to create such sequence, rather than use a single bunch with the aggregate charge. Nevertheless, this method faces the following difficulties.

First, to obtain effective coherent summation of multimode fields it is necessary to provide an equal spacing of the resonance frequencies of the waveguide [4]. However in the cylindrical waveguide with partial dielectric filling, this requirement is not fulfilled.

Second, the pattern of field excitation in the dielectric waveguide with finite length qualitatively differs from the idealized model of an infinite waveguide. A good approximation for describing the waveguide of finite length without reflections at the output is the semi-infinite waveguide. The analytical solution of the wake problem in the semi-infinite waveguide has shown that when the driving bunches excite only traveling forward waves, a cancellation of the wake oscillations occurs which travels forward with the wave group velocity [6]. Therefore in a waveguide without reflections, the train of wake fields from a sequence of a large number of bunches will include wake fields from only a part of this sequence.

With the purpose to avoid some of the aforementioned difficulties in wakefield experiments at Brookhaven [1] and NSC KIPT, the use of a rectangular dielectric waveguide is planed for the future. Analysis and simulation of wakefield excitation in a dielectric waveguide of arbitrary two-dimensional transversal geometry by a regular sequence of electron bunches have been performed and are presented. For theoretical support of mentioned new experiments we are also carrying out investigation of the effects of finite structure length with reflection (resonator concept) on the excitation of wakefield that will be reported separately elsewhere.

PROBLEM STATEMENT

We consider a rectangular metal waveguide with width b ($0 \le x \le b$) and height d ($0 \le y \le d$). The waveguide is filled by homogeneous dielectric with permittivity ε. In the longitudinal direction the waveguide occupies the region $0 \le z < \infty$. From the end $z = 0$ it is short-circuited by a metal wall. We suppose that a monoenergetic point electron bunch moves with constant velocity v_0 along the axis of waveguide to the end face of waveguide. The distribution of charge density and current density of this bunch is:

$$\rho = Q_b \delta(x - x_0)\delta(y - y_0)\delta(t - t_0 - z/v_0)/v_L, \quad j_z = v_0\rho, \tag{1}$$

where Q_b is the bunch charge, t_0 is the time of bunch arrival at the waveguide, x_0, y_0 are transverse coordinates of the bunch.

Having solved a wave equation with boundary conditions on the metal walls of the waveguide we obtain the longitudinal electric field as the sum of Cherenkov radiation E_z^{cher} and the transition radiation E_z^{trans} [7]:

$$E_z(t,x,y,z,t_0,x_0,y_0) = E_z^{cher}(t,x,y,z,t_0,x_0,y_0) + E_z^{trans}(t,x,y,z,t_0,x_0,y_0), \tag{2}$$

$$E_z^{cher}(t,x,y,z,t_0,x_0,y_0) = -\frac{16\pi Q_b}{bd\varepsilon}\times \sum_{k,\,l} \sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times \cos[\omega_{kl}(t - t_0 - z/v_0)]\times \left[\Theta(t - t_0 - z/v_0) - \Theta(t - t_0 - z/v_g)\right], \tag{3}$$

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#onish@kipt.kharkov.ua

$$E_z^{trans}\left(t,x,y,z,t_0,x_0,y_0\right)=-\frac{16\pi Q_b}{bd\varepsilon}\times$$
$$\sum_{k,l}\sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times$$
$$\left\{\left[\theta\left(t-t_0-z/v_{ph}\right)-\theta\left(t-t_0-z/v_{gr}\right)\right]\times\right. \quad (4)$$
$$\sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m}-r_2^{2m}\right)J_{2m}(y)+\theta\left(t-t_0-z/v_{gr}\right)\times$$
$$\left.\left[J_0(y)+\sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m}+r_2^{-2m}\right)J_{2m}(y)\right]\right\},$$
$$\omega_{kl}^2=[(\pi k/b)^2+(\pi l/d)^2]/(\varepsilon/c^2-1/v_0^2) \quad (5)$$

are the eigen frequencies of the dielectric structure, which are in resonance with electron bunch; J_m are the cylindrical functions of m-th order, $\theta(x)$ is the Heaviside function and

$$r_1=\sqrt{\frac{t-t_0-z/v_{ph}}{t-t_0+z/v_{ph}}}\cdot\sqrt{\frac{1-v_{ph}/v_0}{1+v_{ph}/v_0}},\ v_{ph}=c/\sqrt{\varepsilon},\ ,$$
$$r_2=\sqrt{\frac{t-t_0-z/v_{ph}}{t-t_0+z/v_{ph}}}\cdot\sqrt{\frac{1+v_{gr}/v_0}{1-v_{gr}/v_0}},\ v_{gr}=c^2/v_0\varepsilon\ .$$

The Cherenkov wakefield (3) taking into account the "cancellation wave" is nonzero at $(t-t_0)v_{gr}\le z<(t-t_0)v_0$. Within this zone the envelope of the Cherenkov signal from a single bunch is constant. The quantity v_{gr} is the group velocity of the electromagnetic wave. The plane $z^{gr}=(t-t_0)v_{gr}$ is the rear edge of the wake field. This edge follows the bunch with the group velocity.

The field of the transition radiation (4) exists in the region $0\le z<(t-t_0)v_{ph}$. The quantity v_{ph} is the greatest velocity of the electromagnetic signal propagation in the dielectric waveguide; this velocity is the fastest high-frequency propagating part of the transitional signal – the so-called "precursor".

WAKEFIELD IN THE PLANNED NSC KIPT EXPERIMENT

For a bunch of finite size or for a sequence of such bunches the expression for the wake field is obtained by integration over the transverse coordinates and the times when the electrons enter the structure. It should be noted that at using symmetric (respect to XZ and YZ planes) bunches injected along waveguide axis, only odd harmonics $k=1,3,\ldots$, $l=1,3,\ldots$ are excited.

To obtain high amplitude peaked wakefield in a rectangular waveguide we should select and excite only equally spaced modes among the many others in (5). In the planar geometry [4] all modes in the set of eigen modes are equally spaced and the superposition problem for field peaking is solved automatically. As follows from (5) in the general case of arbitrary rectangular geometry, the diagonal frequencies ω_{kk} are always equally spaced. There is a good way to excite this partial series of eigen mode by using a sequence of bunches having repetition frequency equal to the frequency difference between these selected diagonal modes.

In Fig.1-Fig.2 the results of calculations for the following parameters of the NSC KIPT wake field experiment are presented: $b=4.3cm$; $d=8.6cm$; the charge of a single bunch $Q_b=-0.32nC$; electron energy is 4 MeV; transverse sizes of a bunch- $b_0=1.0cm$, $d_0=1.0cm$; the bunch length is 1.71 cm; and $\varepsilon=2.83$.

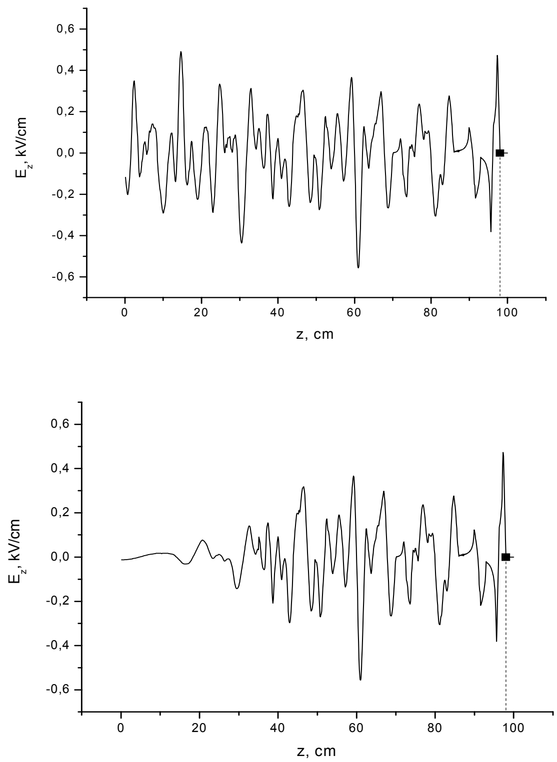

Figure 1: Wakefield excited by a single bunch in the semi-infinite dielectric waveguide having dimensions $b=4.3cm$ and $d=8.6cm$ at the time point t=3.29 ns: upper - Cerenkov field, lower - full field. The label and dash line show bunch location.

The single bunch excites many transverse harmonics (we consider 50 harmonics on x and 50 harmonics on y). As all frequencies of the excited harmonics are not divisible by the lowest eigen-frequency as an integer, the longitudinal structure of the field has an irregular character even without taking into account the transition radiation. The presence of transition radiation especially complicates the field pattern as it contains a continuous spectrum of frequencies, extending from a cut-off frequency. The wake field trailing the bunch is partly cancelled by a disturbance moving with the group velocity, and, near the input plane of the system the amplitude of the field is close to zero.

If, in the semi-infinite waveguide, the sequence of bunches is injected with repetition rate $f=2886\,MHz$,

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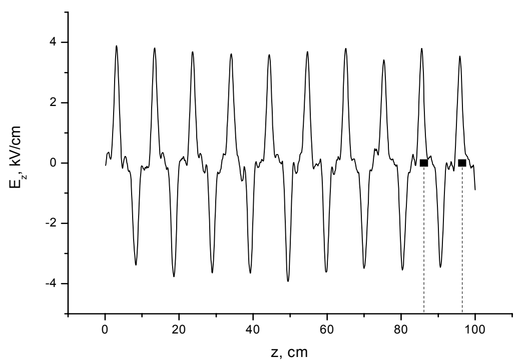
which is equal to the lowest eigen-frequency of structure and consequently is equal to the frequency difference between diagonal elements of matrix { ω_{kl} }, i.e. between frequencies ω_{kk}, the longitudinal structure of the field qualitatively changes. It becomes regular, with narrow peaks following the bunch which have the same period as the bunch train: i.e., the sequence of bunches "cuts out" from the spectrum excited by a single bunch only frequencies which are multiples of the bunch repetition rate. And, as we see from comparison of the curves in Fig. 2, the transition field destroys only an insignificant amount of the regular structure of the field. "Cancellation" of the excited oscillations moving with the group velocity reduces the maximum number of bunches which can contribute to the growth of amplitude of the field [6,7]. For the length of this system $L = 100\ cm$ this bunch quantity is 17. Additional injection of bunches does not further increase the field amplitude with this given length of structure.



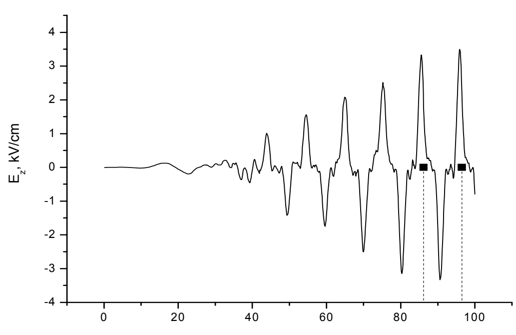


Figure 2: Wakefield excited by a sequence of 17 bunches in the semi-infinite dielectric waveguide of size $b = 4.3cm$ and $d = 8.6cm$ at the time t=8.43 ns: upper - Cherenkov field, lower - full field. Labels and dash lines show locations of the last 2 bunches of a sequence.

"QUASI-MONOPOLAR" WAKE FIELD

A sequence of bunches, injected with repetition rate equal to the lowest eigen-frequency ω_{11}, will excite the equally spaced harmonics with frequencies ω_{kk} and increase their amplitude due to coherent summation, while suppressing other non-equally spaced harmonics. For a certain relation between the transverse dimensions of the rectangular waveguide b and d, some non-diagonal elements of the matrix of eigen-frequencies ω_{kl} can also be divisible by the lowest excited eigen-frequency ω_{11} as an integer. These harmonics will survive and will be added to the harmonics with frequencies ω_{kk}.

In Fig.3 it is shown the longitudinal distribution of the wake field as excited by sequence of 5 bunches moving in a rectangular dielectric waveguide with dimensions $b = 4.88cm$; $d = 1.29b$. An exotic wake field with high amplitude peaks of one sign is excited in this case. The amplitude of the wake field of opposite sign is much lower.

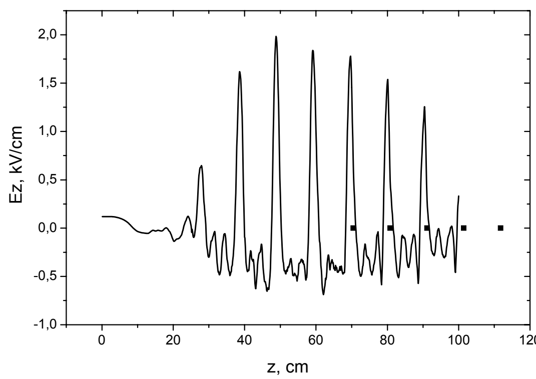


Figure 3: Wakefield excited by a sequence of 5 bunches in a semi-infinite dielectric waveguide at t=3.78 ns. Labels show locations of the last 4 bunches in the structure.

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