LEFT-HANDED METAMATERIALS STUDIES AND THEIR APPLICATION TO ACCELERATOR PHYSICS

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Abstract

Recently, there has been a growing interest in applying artificial materials, known as Left-Handed Metamaterials (LHM), to accelerator physics. These materials have both negative permittivity and permeability and therefore possess several unusual properties: the index of refraction is negative and the direction of the group velocity is antiparallel to the direction of the phase velocity (along k). These properties lead to a reverse Cherenkov effect [1,7], which has potential beam diagnostic applications, in addition to accelerator applications. Several LHM devices with different configurations are being experimentally and theoretically studied at Argonne. We are investigating the possibility of building a Cherenkov detector based on LHM and propose an experiment to observe the reverse radiation generated by an electron beam passing through a LHM. The potential advantage of a LHM detector is that the radiation in this case is emitted in the direction reversed to the direction of the beam, so it could be easier to get a clean measurement.

INTRODUCTION

When studying the interaction of an EM wave with a material it is impossible to take the response of each atom or electron into account. Instead, we rely on macroscopic parameters such as the index of refraction (n), the permittivity (ε) and the permeability (μ), to replace the details of structure in the long wavelength limit.

Most materials have both positive permeability and permittivity over a broad range of frequencies. The term for materials with both ε and μ positive is double positive, while those with both negative are called double negative materials; they are not found in nature.

There are some materials, which exhibit negative permittivity, such as a plasma below the plasma frequency, some metals at optical frequencies and silicon carbide near 10µm wavelength.

In 1968 V.Veselago [1] studied a hypothetical material, which has both ε and μ negative. He predicted several unusual properties of such materials: negative index of refraction, counter directance between group velocity and phase vector k, reverse Doppler and Cherenkov effects. But, for a long time his work was just a curious exercise.

Recently [3,4,8], it became possible to construct materials with simultaneously negative ε and μ. The interest in such metamaterials keeps growing from year to year.

MECHANISM OF NEGATIVE RESPONSE

A simple model for the dielectric properties of a material is obtained by considering the motion of a bound electron in the presence of an applied electric field. As the electric field tries to separate the electron from the positively charged nucleus, it creates an electric dipole moment. Averaging this dipole moment over the volume of the material gives rise to a macroscopic dipole moment per unit volume. A simple model for the dynamics of the displacement x of the bound electron is as follows:

$$m\ddot{x} = eE_0 e^{i\omega t} - kx - m\alpha\dot{x} \qquad (1)$$

To determine the dielectric constant we sum up the individual electric dipole moments to get the polarization, $P=Nex=\varepsilon_0\chi E$. Then the dielectric constant will be:

$$\varepsilon(\omega) = \varepsilon_0 + \frac{Ne^2/m}{(k/m) - \omega^2 + i\omega\alpha} \qquad (2)$$
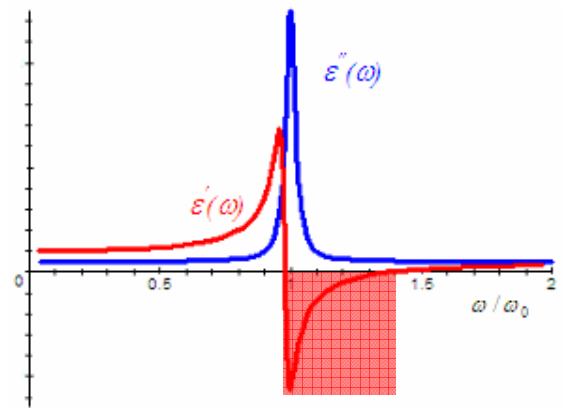

Figure 1: Real and imaginary parts of dielectric constant.

This model predicts dielectric constant to be negative in the red (figure 1) region close to resonance frequency. Similar mechanisms can provide negative permeability. The artificial material can be created have negative permittivity and permeability simultaneously in some frequency range. Such materials are sometimes called double negative.

PROPERTIES OF DOUBLE-NEGATIVE MATERIALS

Structures with negative ε and μ were termed left-handed 30 years ago by Veselago [1]. It reflects the property that the electric field $\vec{E}$, magnetic field $\vec{H}$ and the wave vector $\vec{k}$ form left-handed system instead of usual right-handed. This can be seen by writing Maxwell's equations for a plane monochromatic wave:

$$\left[\vec{k}, \vec{E}\right] = \frac{\omega}{c} \mu \vec{H} \text{ and } \left[\vec{k}, \vec{H}\right] = -\frac{\omega}{c} \varepsilon \vec{E} \qquad (3)$$

When ε and μ are simultaneously positive $\vec{E}$, $\vec{H}$ and $\vec{k}$ form right-handed set, while for double-negative materials it is left-handed. Note that the energy flow, described with Poynting vector, always forms a right-handed set with the field vectors:

$$\vec{S} \overset{def}{=} \frac{c}{4\pi} \left[\vec{E}, \vec{H}\right] \qquad (4)$$

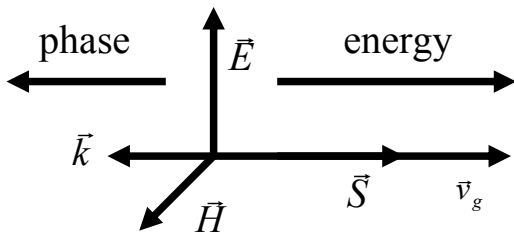In left-handed materials vectors $\vec{k}$ and $\vec{S}$ are anti-parallel.

Figure 2: Field vectors, wave vector and Poynting vector in left-handed medium

Index of refraction in LHM is also unusual. When the real parts of ε and μ are negative, the real part of index of refraction should be positive to maintain positive imaginary part (losses).

$$n = -\sqrt{\varepsilon_r \mu_r} \left[1 - \frac{i}{2} \left(\frac{\varepsilon_i}{|\varepsilon_r|} + \frac{\mu_i}{|\mu_r|} \right) \right] \qquad (5)$$

This property was experimentally observed in 2001 [3] and 2003 [4]. Some authors call left-handed materials negative (refractive) index materials because of this property.
This gives a rise to a number of physical phenomena. For instance, the Doppler effect and the Cherenkov effect are reversed.
The radiation coupling condition for Cherenkov effect is:

$$\omega = \vec{k} \cdot \vec{v} = \frac{\omega n v}{c} \cos \alpha \qquad (6)$$

Here ω is the radiated frequency, v is the particle velocity, n – material index of refraction and α is the angle between the particle trajectory and radiated photon. Therefore, when the index of refraction is negative, radiation goes backwards [2, 6].
This effect has an interesting potential for diagnostic applications [7]. Since the particle radiates backwards it may be easier to pick up a clean signal.
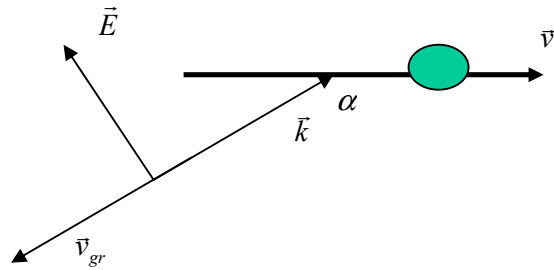

Figure 3: Cherenkov radiation in left-handed medium.

DESIGN OF LEFT-HANDED METAMATERIALS

The materials with simultaneously negative ε and μ are not found in nature. They were artificially constructed in 2000 [3]. Such man-made structures are called metamaterials. Usually it is an assembly of cell elements. As long as the size and spacing between the elements are much smaller than the electromagnetic wavelengths of interest, the incident radiation cannot distinguish the collection of elements from a homogeneous material. The idea behind making a left-handed metamaterial (LHM) is to treat electric and magnetic properties separately. Essentially, a LHM is an assembly of two kinds of cell elements. Split ring resonators (SRR) produce negative μ and a wire array (or capacitively loaded strips, CLS) produce negative ε.
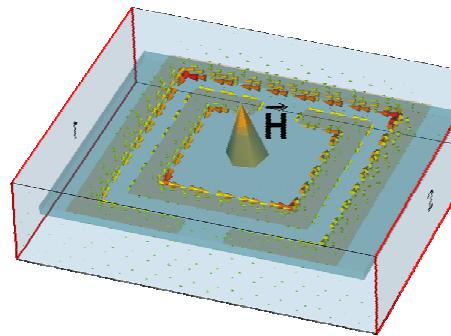
Negative permeability: Split Ring Resonators

Figure 4: Split Ring Resonator (two split conducting rings on dielectric substrate): field orientation and induced currents.

A negative response to magnetic field is produced with elements called split ring resonators [2]. SRR consists of two split coaxial conducting rings (picture 4).
When a wave propagates through a SRR artificial media, its magnetic field induces currents in the rings, creating the magnetic response (see figure 4). The dielectric response is negligible ($\varepsilon=1$). This design provides two loops of current, which form a resonant system. In terms of equivalent circuits, the SRR can be considered as an LC resonant circuit with strong inductive bond [2].

The wave propagation in the media is described with the wave equation.

$$\nabla^2\psi + \varepsilon\mu\frac{\omega^2}{c^2}\psi = 0 \qquad (7)$$

where ψ is any component of electric and magnetic field. If the product $\varepsilon\mu<0$ then physical solution is decaying exponent – wave does not propagate in such a medium. Therefore, if transmission through SRRs experiences drops ($\varepsilon\mu<0$), since there is no dielectric effect ($\varepsilon\approx1$) it happens due to negative μ [2,3].

Negative permittivity: Wire Array

A wire arrays are well-known high-pass filters for EM waves polarized with $\vec{E}$ parallel to the wires. It does not allow propagation in a certain frequency range because of negative ε [2,3]. Capacitively loaded strips (CLS) (figure 5), which were used in our design, act as an enhanced wire array and produce negative ε [8].

Manufacturing of metamaterial elements.

Simulations of LHM structure elements were performed with Microwave Studio [9] to define the dimensions of the elements in such a way that the left-handed transmission band will be at around 11.5 GHz. Then the SRR and CLS elements were etched on sheets of 10 mil (0.01 inch) Rogers 5880 substrate.
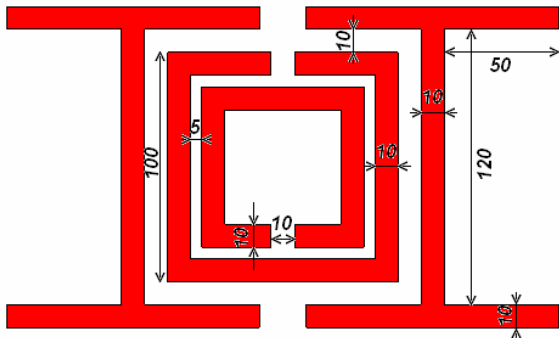

Figure 5: Typical unit cell. All sizes are in mils (0.001 inch). The size of the cell is about 2.5*mm,* which is much less than 3*cm* wavelength at 10GHz.
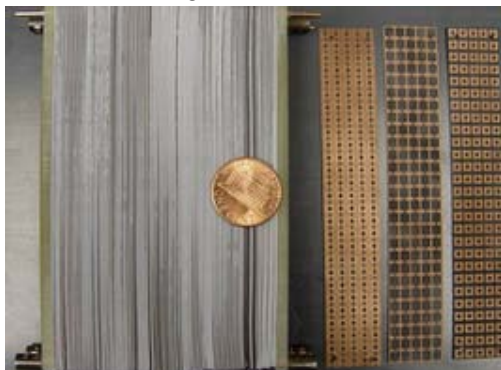

Figure 6: Pieces of printed circuit boards with SRR and CLS units on the right. To assemble metamaterial we stack the boards with Teflon spacers in between (on the left) forming a rigid structure.

The experimental studies were done using an HP 8510 network analyzer. Two X-band horn antennas were used as the source and the receiver. We measured transmission through and reflection from the structure. Results show that the left-handed transmission band is 11.6-12.2GHz.

Figure 7. Amplitude of the transmission through SRR-only, CLS-only and combined structures on a linear scale. Red zone is a left-handed transmission band, the frequency range, where SRR-only and CLS-only metamaterials do not exhibit transmission (~5%), but combined structure provides almost 50% power transmission – signature of left-handed metamaterial.

CONCLUSION

The construction of left-handed metamaterial has been achieved. A procedure, similar to the one in [8] was used to retrieve index of refraction in the LH transmission band. Our results show, that the values up to –2.2 were achieved, which is sufficient to satisfy Cherenkov radiation condition (7) [7]. We are in process of designing the experiment to study beam interaction with the LHMs.

AKNOWLEDGEMENTS

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