LIMITS OF Nb_3Sn ACCELERATOR MAGNETS*

S. Caspi[#] and P. Ferracin, LBNL, Berkeley, CA 94720, USA

Abstract

Pushing accelerator magnets beyond 10 T holds a promise of future upgrades to machines like the Tevatron at Fermilab and the LHC at CERN. Exceeding the current density limits of NbTi superconductor, Nb_3Sn is at present the only practical superconductor capable of generating fields beyond 10 T. Several Nb_3Sn pilot magnets, with fields as high as 16 T, have been built and tested, paving the way for future attempts at fields approaching 20 T. High current density conductor is required to generate high fields with reduced conductor volume. However this significantly increases the Lorentz force and stress. Future designs of coils and structures will require managing stresses of several 100's of MPa and forces of 10's of MN/m. The combined engineering requirements on size and cost of accelerator magnets will involve magnet technology that diverges from the one currently used with NbTi conductor. In this paper we shall address how far the engineering of high field magnets can be pushed, and what are the issues and limitations before such magnets can be used in particle accelerators.

INTRODUCTION

The Tevatron, the first accelerator to use superconducting magnets in its main ring, was made from NbTi conductor with a current density of 1800 A/mm^2 at 5 T and 4.2 K [1]. The bore diameter was 76 mm, the two-layer coil was 16.2 mm thick, and it could reach its short-sample field of 4.8 T at 4.3 K, with a stored energy of 98 kJ/m ("short-sample" is defined as the current vs field limit in a short superconducting wire at a given temperature). If one were to replace the Tevatron cable with identical size Nb_3Sn superconducting strands, that same magnet could reach 11.9 T at 4.3 K. The stored energy would rise to 674 kJ/m, and the original 57 MPa of coil stress, produced by the accumulation of Lorentz forces (Lorentz stress), would increase to an unacceptable level of 294 MPa (we assume a Nb_3Sn wire capable of carrying 3000 A/mm^2 in the superconductor at 12 T, 4.2 K).

When the LHC pushed NbTi conductor closer to its high-field limit, the 56 mm bore dipoles reach 9.7 T at 1.9 K, and a stored energy of 334 kJ/m (per bore) [2]. Replacing the cable in that magnet with identical size Nb_3Sn conductor (31.3 mm of overall coil thickness) would raise the field to 15.2 T, the stored energy to 900 kJ/m, and the Lorentz stress from 88 MPa to 220 MPa.

This comparison between NbTi and Nb_3Sn conductors points out both the promise and the challenge for high field magnets. Accelerator magnets, by the shear fact that a large number of them will be needed for any accelerator, can not be treated as a "one-of-a-kind" magnet. Their cost must be at a minimum and their reliability high. To accomplish that, the engineering of Nb_3Sn magnets will have to exceed all previous superconducting magnet technology. Pushing the limits on high field Nb_3Sn magnets in a way suitable for particle accelerators requires the best superconductor, a reasonably small size magnet, and a compact structure. The coil must be protected against a ten-fold increase in stored energy, and, last but not least, the conductor change in strain must be kept at a minimum. How we meet the challenge and how far we can push such magnets is the focus of this paper. We shall address the relations between field, coil size, bore diameter, stress, stored energy, and point out areas where we presently meet the challenge and areas that will require further R&D.

BRIEF Nb_3Sn MAGNET HISTORY

Nb_3Sn dipole magnets have a relatively short history. In comparison to the thousands of NbTi magnets built in the past 40 years only several dozens have been built with Nb_3Sn conductor. Among them seven record-breaking dipoles (Fig. 1) have pushed the magnetic field from 4.8 T in 1978 (BNL dipole magnet [3]) to 16 T in 2003 (LBNL HD1 [4]). Other record-breaking magnets included LBNL D10 (1984 [5]), CERN-ELIN dipole magnet (1989 [6]), University of Twente MSUT (1995 [7]), LBNL D20 (1997 [8]), and LBNL RD3b (2001 [9]). At the present, R&D on Nb_3Sn magnets is being conducted at BNL [10], FNAL [11], LBNL [12], Texas A&M University [13], CEA Saclay [14], and the University of Twente [15].

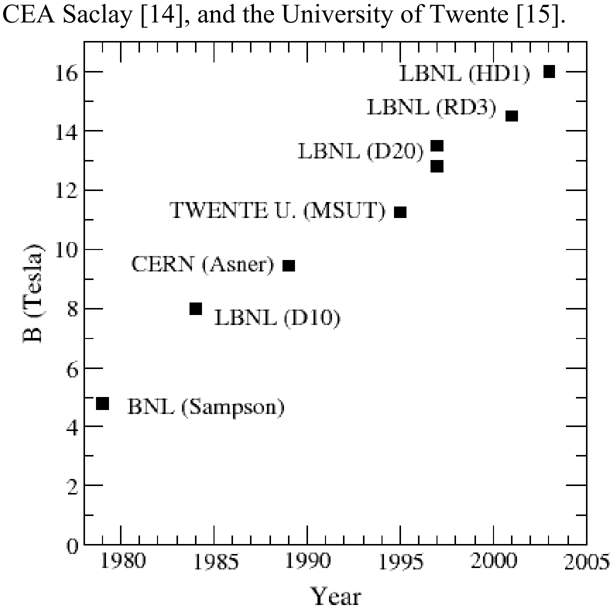

Figure 1: World record-breaking Nb_3Sn dipole magnets.

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[#]S_CASPI@lbl.gov

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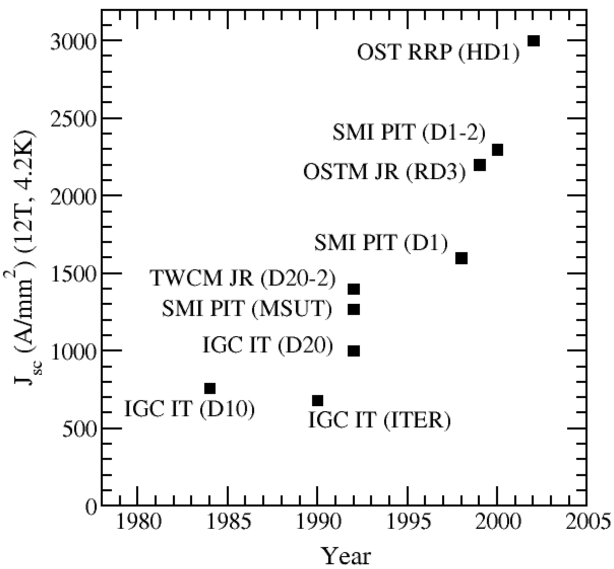

Figure 2: Improvement in Nb_3Sn current density.

The increase in field went hand in hand with progressively higher current carrying capacity of the Nb_3Sn conductor (Fig. 2). The current density of Nb_3Sn has increased almost ten fold from the early 80's to the present time [16].

DIPOLE MODEL

Model Description

To study the limits of superconducting dipole magnets, we formulated a simple but realistic model based on several assumptions: 1) the bore is round and the coil is a thick cylinder (Fig 3); 2) the engineering current density is equal to $J_e cos\theta$ and the field is therefore a pure dipole; 3) the field magnitude in the bore and along the conductor inner surface is identical; 4) the short-sample current density in the superconductor J_{ss} and the engineering current density $J_{e\text{-}ss}$ (obtained by averaging J_{ss} over copper, insulation, and voids) are a function of field B and temperature T; 5) the coil is not graded; 6) there is no ferromagnetic material nearby.

A simple relation exists between the central dipole field B_0, the engineering current density J_e, and the coil thickness w [17]:

$$B_0 = \frac{\mu_0 J_e}{2} w .$$

If we extend the engineering current density J_e up to the short-sample current density limit $J_{e\text{-}ss}$, the field will reach its corresponding short-sample value B_{ss}. By applying Summer's empirical short-sample relation [18] to the above equation, we obtain

$$w_{ss}(B_{ss},T) = \frac{2B_{ss}}{\mu_0 J_{e-ss}(B_{ss},T)} ,$$

where the coil thickness is expressed as a function of short-sample field and temperature only. We note and emphasize that the relation is <u>independent</u> of bore diameter.

We conducted our parametric analysis assuming the best commercially available superconductor (Nb_3Sn with 3000 A/mm^2 at 12 T and 4.2 K) and a Rutherford cable with 50 % non-copper, a 12 % void fraction and an 11 % insulation fraction (Fig. 4).

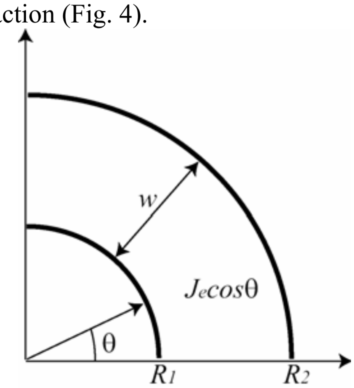

Figure 3: A pure dipole with a $J_e cos\theta$ current density.

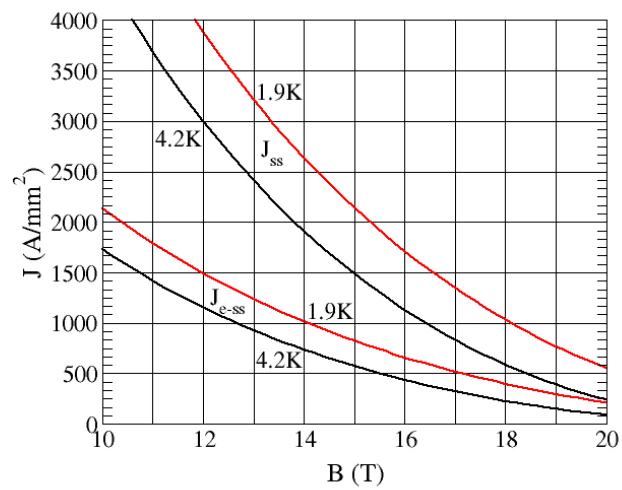

Figure 4: Short-sample current density J_{ss} and the corresponding engineering value $J_{e\text{-}ss}$ of Nb_3Sn superconductor.

PARAMETRIC DEPENDENCIES

Coil Thickness

The coil thickness w_{ss} at short-sample is plotted in Fig. 5 for two different operating temperatures. We note that, at 1.9 K, a 7 mm thick coil is sufficient to generate a 10 T field, and a 100 mm thick coil will be needed for a 19 T field. Operating at 1.9 K requires significantly less conductor than at 4.2 K, an advantage that becomes even more significant as the field approaches 20 T.

It is worth noting that, for the same current density and coil thickness w, a solenoid will generate a field that is twice that of a dipole ($B_0 = \mu_0 J_e w$). Solenoids are therefore inherently more compact, use less conductor and have already achieved fields in excess of 20 T [21]. We therefore expect high field solenoids to represent an upper boundary for high field dipoles.

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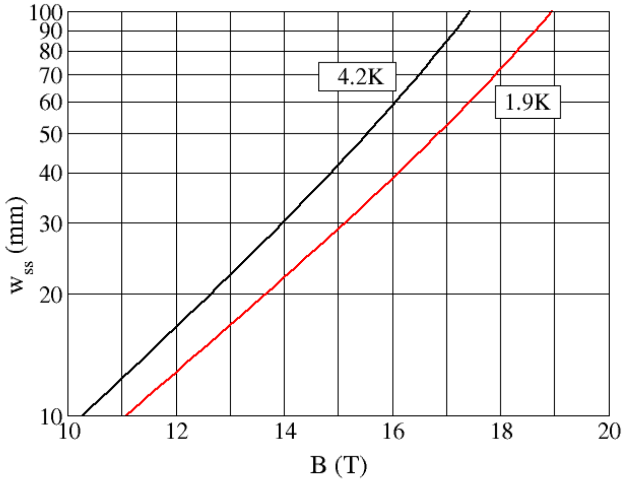

Figure 5: Coil thickness of Nb_3Sn dipole magnets at short-sample (T = 4.2 K and 1.9 K).

Bore Diameter

Since the field depends on the coil thickness and not on the bore diameter, we may claim that a dipole with a zero bore diameter has the same field as a coil with any bore diameter, as long as the coil thickness is constant. This gives us the opportunity to separate the cost of the field from the cost of the bore.

The cost of the coil is proportional to its area, which is given by $\pi w^2 + 2\pi w R_1$ (where w is the coil thickness, and R_1 is the bore radius). We associate the first term with the area of a no bore coil, and the second term with an additional area representing the contribution of a bore. Accordingly, the cost of the field is proportional to w^2, but the additional cost of the bore is linearly proportional to the bore diameter.

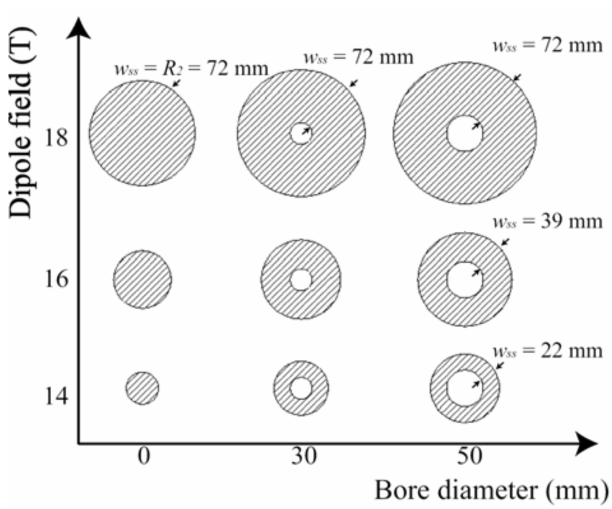

Figure 6: Coil thickness and bore diameter of various high field dipoles at short-sample (T = 1.9 K).

This dependence of the coil area (i.e. cost) on field and bore diameter is shown in Fig. 6, where we compare three different bore diameters at three different field levels. For each field the coil thickness remains the same. The proportionality between the various coil geometries has been maintained to illustrate the relative increase in coil area with increasing bore diameter. We notice that the coil area with zero bore increases by a factor of 10 as the field increases from 14 T to 18 T. In comparison, at high fields (for example at 18 T), where the current density drops (see Fig. 4), doubling the bore diameter from 25 mm to 50 mm increases the amount of conductor by only 26 %. We conclude that at very high fields, where the coil thickness approaches 100 mm, the effect of the bore diameter on the overall cost of the conductor is minor (the impact of coil grading will be addressed in the following section).

Lorentz Stress

The azimuthal Lorentz stress in a 2D $\cos\theta$ dipole is the integrated azimuthal Lorentz force with respect to θ (no shear) [20]. Expressed as

$$\sigma_\theta = \frac{\mu_0 J_e^2}{2}\frac{r}{2}\left(R_2 - \frac{R_1^3}{3r^2} - \frac{2}{3}r\right)\cos^2\theta ,$$

the stress exhibits a maximum along the coil mid-plane ($\theta = 0$), at a radius nearly two thirds of the coil thickness $r = R_{max} \sim 2/3\ w$. The above expression of stress can be rewritten as a function of field and coil thickness w,

$$\sigma_{\theta-\max} = \frac{B^2}{2\mu_0}\left(\frac{R_1}{w}\right)^2\left(\frac{R_1}{R_{\max}}\right)\left[\left(\frac{R_{\max}}{R_1}\right)^2\left(1+\frac{w}{R_1}\right)-1\right]$$

where R_{max} is calculated by setting $\partial\sigma_\theta/\partial r = 0$ and solving the cubic relation in r.

If we apply the short-sample field, coil thickness w_{ss}, and a given bore diameter to the above equation, we arrive at the maximum short-sample stress $\sigma_{\theta\text{-}max\text{-}ss}$ (Fig. 7).

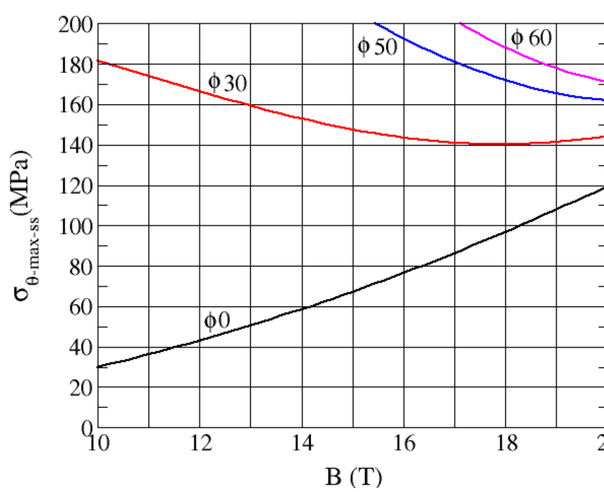

Figure 7: Maximum azimuthal Lorentz stress at short-sample (T = 1.9 K).

The zero bore solution is a monotonic increasing function of the field and is at the minimum for any bore diameter at that field. Surprisingly, for certain bore diameters and field ranges, the maximum stress decreases as the short-sample field increases. That can be explained as follows: as the field increases, so do coil thickness w_{ss} and Lorentz force; however, it is possible that the rate of

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increased coil thickness is greater than the corresponding Lorentz force, thereby reducing the stress.

We also note that at low fields (~ 10 T) the conductor is very efficient and very thin, resulting in high stress. At high fields the coil thickness dominates and the stress asymptotically approaches that for a zero bore solution, equal to $\sigma_{\theta\text{-}max\text{-}ss} = (3B_{ss}^2)/(8\mu_0)$.

Stored Energy

The stored energy E of a dipole increases quadratically with field B, bore radius R_1, and coil thickness w:

$$E = \frac{\pi B^2}{3\mu_0}\frac{w^2}{2}\left[1 + 6\left(\frac{R_1}{w}\right)^2 + 4\left(\frac{R_1}{w}\right)\right].$$

Figure 8 is a log plot of the short-sample stored energy E_{ss} for a number of different bore diameters, including a zero bore.

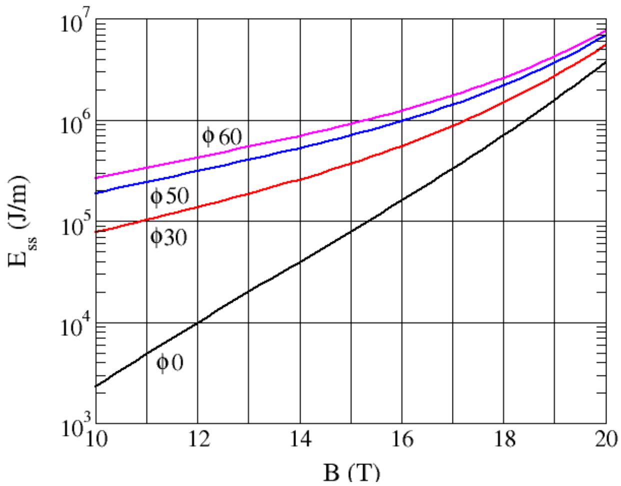

Figure 8: Short-sample stored energy in dipoles at $T = 1.9$ K for various bore diameters.

As for the stress, we can associate the energy of a zero bore diameter with the term outside the square brackets in the formula above, and the terms within the bracket as an additional bore contribution. At high fields (above 18 T) the major contributors to the stored energy are the field B and coil thickness w. The contribution by the bore is only a ratio between the radius R_1 and coil thickness w. In fact, when the coil thickness w increases, the additional contribution of bore diameter to the stored energy becomes less effective and the stored energy asymptotically approaches that of a zero bore.

If we focus on the curves representing a bore diameter closer to the LHC dipole (50-60 mm), we notice that at 15 T the stored energy is close to 1 MJ/m (a three-fold increase with respect to the actual 10 T NbTi dipole), reaching 7 MJ/m at 20 T.

GRADED COILS

Grading the coil can effectively reduce overall size while the field remains the same. Grading takes advantage of the drop in field within the coil in order to raise the current density in several discrete outer layers, thus matching that drop. That way the superconductor can reach its short-sample simultaneously throughout the cross-section, thereby reducing the overall size.

Grading significantly impacts the coil thickness at high field: for example, above 18 T, grading reduces the coil thickness by about 25 % (Fig. 9).

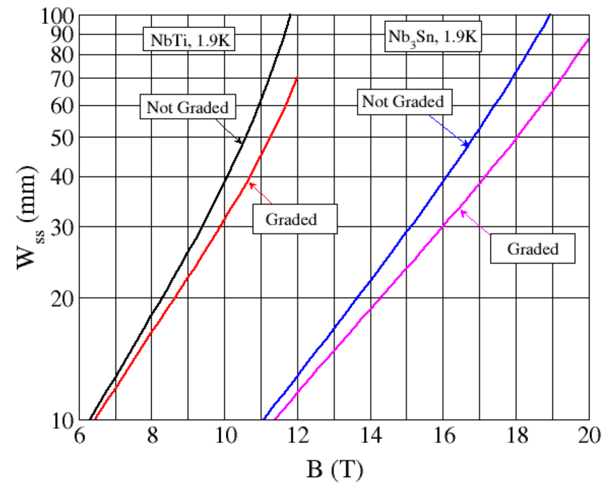

Figure 9: Variation in coil thickness between graded and un-graded coils at short-sample ($T = 1.9$ K).

It must be pointed out that grading reduces the coil thickness at the expense of an increase in conductor stress. An 18 T (at 1.9 K) dipole will reduce its coil thickness from 72 mm to 50 mm with grading (Fig. 9), but, in a 56 mm bore diameter coil, the maximum stress will rise from 170 MPa to an unacceptable level of 400 MPa (Fig. 10). Since reducing the coil thickness has a major impact on reducing the magnet cost, we are left with an important R&D issue on how to bring down the stresses. Reducing the stress through stress management and the possible introduction of various force intercepts (as proposed in [21]) will have to be weighed against the reduction in the overall engineering current density, which, as a consequence, forces an increase in coil thickness once again.

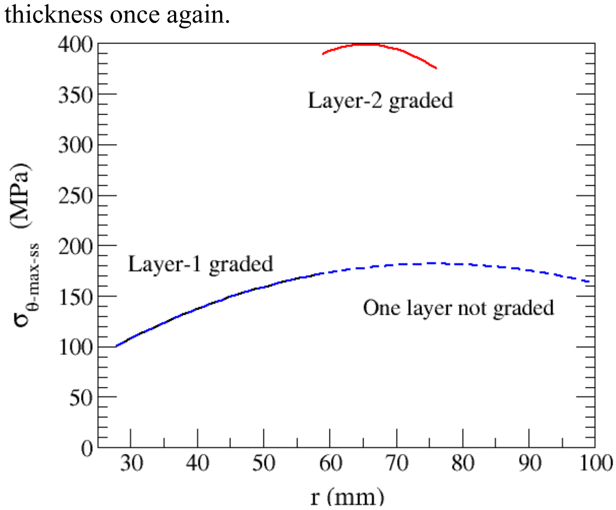

Figure 10: Grading an 18 T ($T = 1.9$ K) dipole with 56 mm bore diameter reduces the coil thickness but raises the stress.

DISCUSSION AND CONCLUSIONS

The engineering challenge of trying to balance high fields against magnet size, stress, and stored energy can actually be met today for "one-of-a-kind" dipole. The cost however, hardly acceptable even for "one-of-a-kind" magnet, is unacceptable for a main ring accelerator. Solving the engineering challenge and keeping the cost down will require new designs where each magnet component is pushed to its limits.

In order to address the issues related to accelerator magnets operating at the limit of Nb_3Sn superconductor, we have used a simple analytical model. The analytical model, whose validity was checked against Finite Element Models (FEM) of several real magnet geometries (see Table I), was applied to dipoles in the range of 10 – 20 T.

Table 1: Comparison between FEM and analytical model.

	Tevatron dipole		LHC Dipole		D20	
	FEM	*Anal.*	*FEM*	*Anal.*	*FEM*	*Anal.*
w_{input} (mm)	16.2	16.2	31.3	31.3	54.9	54.9
B_{0-ss} (T)	4.8	4.7	9.7	9.8	13.5	12.9
$\sigma_{\theta\text{-}max}$ (MPa)	57	46	88	103	150	160
E (kJ/m)	98	106	334	384	1344	1100

The following conclusions, regarding minimum coil size, coil stress, and magnet stored energy, can be drawn:

- Pushing dipole fields to 20 T would require a coil that is 150 mm thick. With a 50 mm bore, the Lorentz stress would be 160 MPa and the stored energy 6.9 MJ/m. This should be considered as an upper limit for Nb_3Sn dipole magnets.
- Decreasing the field to 18 T would require a coil that is 72 mm thick. With a 50 mm bore, the Lorentz stress would be 170 MPa and stored energy 2.2 MJ/m.
- Today's practical limit for an accelerator dipole is around 16 T ($w_{s\ s}$= 38 mm, $\sigma_{\theta\text{-}max}$ = 190 MPa, E = 1 MJ/m).
- At very high fields, the effect of the bore diameter on the overall amount of conductor, the peak stress, and the stored energy is minor.
- In a recent test of the 1 m long dipole magnet HD1, it was shown that a Nb_3Sn coil can sustain high compressive stress in the range of 150-180 MPa and reach 16 T.
- This level of stress can be considered at the present as an indicative practical limit for Nb_3Sn coils.
- By grading the coil, we can reduce coil thickness at the expanse of higher stress. As we are currently at the coils stress limit, new designs will have to be developed that intercept the Lorentz forces. However we need to be aware that stress management reduces not just the stress, but the coil efficiency as well.
- Quench protection systems will be needed to handle an order of magnitude increase in stored energy.

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