MULTIVARIATE OPTIMIZATION OF ILC PARAMETERS*

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Abstract

We present results of multiobjective optimization of the International Linear Collider (ILC) which seeks to maximize luminosity at each given total cost of the linac (capital and operating costs of cryomodules, refrigeration and RF). Evolutionary algorithms allow quick exploration of optimal sets of parameters in a complicated system such as ILC in the presence of realistic constraints as well as investigation of various what-if scenarios in potential performance. Among the parameters we varied there were accelerating gradient and Q of the cavities (in a coupled manner following a realistic Q vs. E curve), the number of particles per bunch, the bunch length, number of bunches in the train, etc. We find an optimum which decreases (relative to TESLA TDR baseline) the total linac cost by 22%, capital cost by 25% at the same luminosity of $3 \times 10^{38}\ \mathrm{m}^{-2}\mathrm{s}^{-1}$. For this optimum the gradient is $35\,$MV/m, the final spot size is $3.6\,$nm, and the beam power is $15.9\,$MW. Changing the luminosity by $10^{38}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$ results in 10% change in the total linac cost and 4% in the capital cost. We have also explored the optimal fronts of luminosity vs. cost for several other scenarios using the same approach.

MULTIOBJECTIVE OPTIMIZATIONS

In any problem involving many decision variables, one typically seeks to find an optimum choice of such variables by minimizing/maximizing a certain merit function which corresponds in a certain way to the overall performance of the system. Various parameters that characterize the system are combined to form such merit function (or objective function) and through the optimization of this single objective one is typically able to find a single set of optimal decision variables. In reality, it is often the case that a single objective cannot adequately describe a complex system, and, instead, it becomes convinient to employ more than one objective. The optimization problem becomes a multiobjective optimization problem, in which one seeks to minimize/maximize several objectives simultaneously. A classical example of such problem is cost vs. benefit optimization in economics. One desires to know the maximum benefit he can have for any given expense in a particular economic setting. Once such a trade-off of cost vs. benefit, also called *optimal front*, is known, a knowledgeable choice can be made with regards to the right amount of money that should be spent to obtain the wanted benefit.

Multiobjective optimization problem can be defined as following:

$$\left.\begin{array}{lll} \text{maximize} & f_m(\mathbf{x}), & m = 1,\ldots,M; \\ \text{subject to} & g_j(\mathbf{x}) \geq 0, & j = 1,\ldots,J; \\ & x_i^{(\mathrm{L})} \leq x_i \leq x_i^{(\mathrm{U})}, & i = 1,\ldots,n. \end{array}\right\}$$

To compare between two possible solutions one can employ the concept of *dominance*: a solution $\mathbf{x_1}$ is said to *dominate* another solution $\mathbf{x_2}$ if solution $\mathbf{x_1}$ is no worse than $\mathbf{x_2}$ in all objectives f_m ($m = 1,\ldots,M$) and is better at least in one objective. Then, the *optimal Pareto front* that one seeks to obtain can be defined as being composed of all solutions $\mathbf{x}$ that are both *feasible* (i.e. constraints $g_j(\mathbf{x}) \geq 0$ are met for all $j = 1,\ldots,J$) and are not dominated by any other possible choice of $\mathbf{x}$ (which is also feasible).

We have used modified versions of evolutionary algorithms SPEA-II [1] and NSGA2 [2] to obtain optimal fronts in performance of the International Linear Collider. Evolutionary algorithms mimic the natural selection process in nature and the optimization proceeds by ‘improving’ a fixed size set of trial solutions $\{\mathbf{x}\}$, called *population*. The fittest individuals in the population (e.g. non-dominated ones) are primary candidates to produce ‘offspring’ trial candidates. Upon evaluation of all objectives and constraints (i.e. f_m and g_j) these individuals are either forfeited or carried over to the next generation cycle depending on whether they improve over their predecessors. For more details on evolutionary algorithms, the reader is referred to [3].

ILC MODEL

The following two functions $\mathcal{L}$ and $-C_{\mathrm{tot}}$ were used as objectives in our optimizations, corresponding to luminosity and (negative) total cost of the linac:

$$\begin{aligned} \mathcal{L} &= \frac{N^2 f}{4\pi\sigma_x\sigma_y} HD \\ C_{\mathrm{tot}} &= C_{\mathrm{cap}} + C_{\mathrm{op}} \end{aligned}$$

here HD is disruption enhancement parameter, N is number of particles per bunch, f is beam collision rate, $\sigma_{x,y}$ is horizontal or vertical beam size at the IP. C_{cap} and C_{op} are capital and operational costs of the ILC linac respectively.

All optimizations were subject to at least the following constraints (with certain additional constraints added to explore various possible scenarios as discussed later):

$$\begin{array}{rcl@{\qquad}rcl} t_{\mathrm{rf}} & \leq & 2.2\,\mathrm{ms}, & 100 \leq & R & \leq 120 \\ D_y & \leq & 25, & \Upsilon & \leq & 0.1 \\ \delta & \leq & 0.05, & n_\gamma & \leq & 2 \\ \sigma_z & \leq & \beta_y^*, & P_{\mathrm{b}} & \leq & 30\,\mathrm{MW} \end{array}$$

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here $t_{\rm rf}$ is the time that RF stays on, $R = \sigma_x/\sigma_y$ is the aspect ratio, D_y is vertical disruption parameter, Υ is beamstrahlung parameter, δ is fractional energy loss, n_γ number of γ-quanta created, β_y^* and σ_z are vertical β-function and the bunch length at the IP correspondingly, and $P_{\rm b}$ is the beam power.

Refer to [4] for a complete description of all the formulas in the model. In addition, it was assumed that the total operational life-time of the accelerator will be 5 years and the cost of electricity was assumed to be 10 ¢/kW-h. The two objectives were functions of ten decision variables with the corresponding lower and upper boundaries:

$$\begin{array}{ll} 25 \le E\,(\mathrm{MV/m}) \le 45, & 10^{10} \le N \le 2\times10^{10}, \\ 0.15 \le \sigma_z\,(\mathrm{mm}) \le 0.3, & 168 \le t_{\rm bs}\,(\mathrm{ns}) \le 674, \\ 0.1 \le f_0\,(\mathrm{Hz}) \le 10, & 1000 \le n_0 \le 3000, \\ 5 \le \epsilon_x\,(\mu\mathrm{m}) \le 20, & 30 \le \epsilon_y\,(\mathrm{nm}) \le 60, \\ 0.75 \le \beta_x^*\,(\mathrm{cm}) \le 3.0, & 0.2 \le \beta_y^*\,(\mathrm{mm}) \le 0.8. \end{array}$$

Here E is accelerating gradient, $t_{\rm bs}$ is bunch separation, f_0 is repetition rate, n_0 is number of bunches per pulse, $\epsilon_{x,y}$ are normalized emittances and β_x^* is horizontal β-function at the IP. Q and E are related in a typical Q-curve fashion: $Q = [5-10]\times10^9$ for $E = [25-45]$ MV/m range.

OPTIMIZATION RESULTS

Fig. 1 shows the results of multiobjective optimization for several possible scenarios. The curve named 'base' corresponds to the constraints specified in the previous section. '$E = 30\,$MV/m' shows the optimal front if the gradient is kept fixed to that value. Certain concerns exist that too high a value of disruption parameter may result in kink instability. In order to address this we show the curve labeled '$D_y = 15$'. Curves '$\epsilon_y = 6\times10^{-8}\,$m' and '$\sigma_y = 5\times10^{-9}\,$m' show optimal curves when either vertical emittance or the beam size at the IP is kept to somewhat higher values than the more optimistic numbers. Finally, the plot also shows the 'original point' that corresponds to the old TESLA TDR design values. Cost coefficients in [4] have been adjusted to match the TDR costs for linac, RF and refrigeration.

Fig. 2 shows the improvement one can expect in the optimal luminosity vs. cost curve if $Q = 10^{10}$ regardless of the cavity gradient.

Table 1 presents the 'snapshots' of parameters that correspond to the lowest cost for $\mathcal{L} = 3\times10^{38}\,\mathrm{m^{-2}s^{-1}}$ for five different cases: a) base, b) $D_y = 15$, c) $\epsilon_y = 6\times10^{-8}\,$m, d) $Q = 10^{10}$, f) $E = 30\,\mathrm{MV/m}$ as well as g) the original TESLA TDR point. Additional parameters encountered in the Table 1 are: wall plug power $P_{\rm AC}$, cryogenic power $P_{\rm cryo}$, loaded $Q_{\rm L}$ and number of klystrons $N_{\rm kly}$.

DISCUSSION & CONCLUSION

We have employed multiobjective genetic algorithms in optimizations of the ILC luminosity vs. cost. This approach allows quick exploration of various scenarios for the ILC.

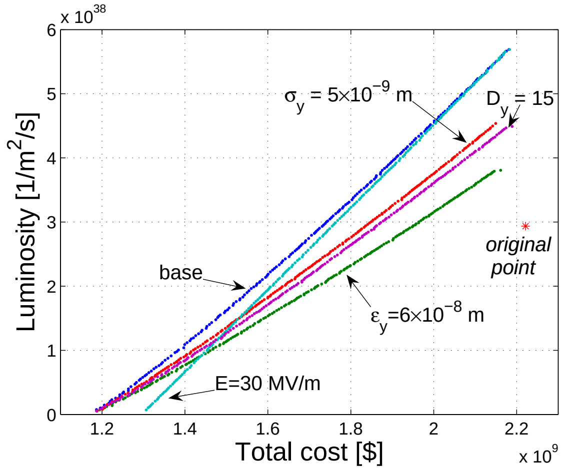

Figure 1: Optimal fronts of luminosity vs. total linac cost for the ILC.

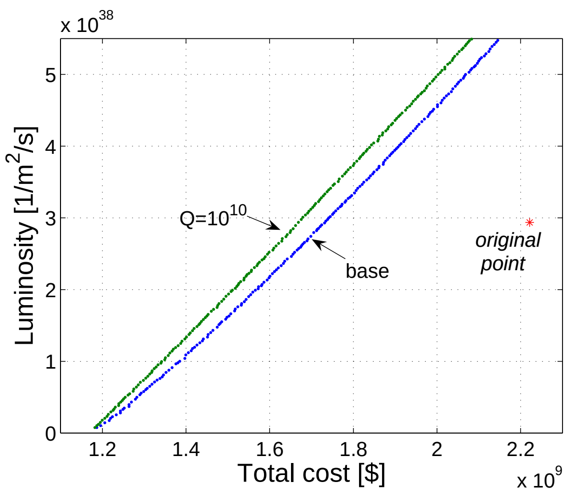

Figure 2: Optimal fronts of luminosity vs. total linac cost for the ILC.

The model can be of almost arbitrary degree of sophistication, e.g. include beam tracking to provide more accurate estimates of the objective function(s) of interest in case these functions are not ammenable to simpler analytical expressions. For example, this approach has been used in optimizing the injector design for the Energy Recovery Linac which involved beam tracking in the space-charge dominated region of the accelerator [5].

The analysis of the simple model [4] presented here allows us to make several observations. The optimum choice of parameters for $\mathcal{L} = 3\times10^{38}\,\mathrm{m^{-2}s^{-1}}$ ('base' curve of Fig. 1) decreases the total linac cost by 22% and the capital cost by 25% relative to the original specifications, which represents a substantial gain. We have examined the cost reduction of choosing $\mathcal{L} = 2\times10^{38}\,\mathrm{m^{-2}s^{-1}}$ instead of $3\times10^{38}\,\mathrm{m^{-2}s^{-1}}$. For the optimal base curve the total cost drops by 10% and the capital cost by 4%. Overall, the total cost savings relative to the original TDR point is 29% and

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Table 1: Parameters for $\mathcal{L} = 3 \times 10^{38}\,\mathrm{m^{-2}s^{-1}}$ for various scenarios.

Parameter	a)	b)	c)	d)	e)	f)	g)
E (MV/m)	34.6	32.7	34.2	39.8	30.0	35.3	25.0
N ($\times 10^{10}$)	1.75	1.74	2.00	1.75	1.64	1.48	2.00
f_0 (Hz)	3.8	4.8	4.9	3.8	4.0	4.4	5.0
σ_z (mm)	0.19	0.16	0.19	0.19	0.19	0.19	0.30
t_{bs} (μs)	0.42	0.44	0.46	0.36	0.45	0.27	0.34
ϵ_x (μm)	8.3	8.6	11.5	8.2	8.0	6.0	10.0
ϵ_y (nm)	30	30	60	30	30	30	30
β_x^* (cm)	1.1	1.4	1.0	1.1	1.0	1.1	1.5
β_y^* (mm)	0.21	0.28	0.20	0.21	0.20	0.20	0.40
$\mathcal{L}$ ($\mathrm{m^{-2}s^{-1}}$)	3×10^{38}	3×10^{38}	3×10^{38}	3×10^{38}	3×10^{38}	3×10^{38}	3×10^{38}
C_{tot} ($)	1.74×10^9	1.87×10^9	1.96×10^9	1.68×10^9	1.76×10^9	1.83×10^9	2.22×10^9
C_{cap} ($)	1.33×10^9	1.37×10^9	1.40×10^9	1.27×10^9	1.37×10^9	1.41×10^9	1.76×10^9
σ_x (μm)	0.43	0.50	0.50	0.43	0.40	0.37	0.55
σ_y (nm)	3.6	4.2	5.0	3.6	3.5	3.5	5.0
R	119	120	100	120	115	105	112
P_{b} (MW)	15.9	20.1	23.7	15.9	15.7	15.6	22.6
P_{AC} (MW)	94.7	114	129	94.0	89.8	95.4	105.6
Q	7.8×10^9	8.3×10^9	7.9×10^9	10×10^9	9.0×10^9	7.7×10^9	10×10^9
Q_{L}	5.0×10^6	5.0×10^6	4.8×10^6	5.0×10^6	5.0×10^6	4.0×10^6	2.5×10^6
N_{kly}	422	400	437	485	367	543	600
P_{cryo} (kW)	42.2	48.3	53.7	35.8	39.4	37.7	37
D_y	24.9	15	17.6	24.9	25	24.9	25
HD	1.68	1.78	1.57	1.68	1.66	1.68	1.79
δ	0.050	0.045	0.050	0.050	0.050	0.050	0.029
n_γ	1.62	1.36	1.59	1.62	1.61	1.62	1.48
Υ	0.095	0.100	0.095	0.096	0.097	0.098	0.055
t_{rf} (ms)	2.1	2.2	2.2	1.9	2.2	1.5	1.4

a) base; b) $D_y = 15$; c) $\epsilon_y = 6 \times 10^{-8}$ m; d) $Q = 10^{10}$;
e) $E = 30\,\mathrm{MV/m}$; f) $t_{\mathrm{rf}} = 1.5\,\mathrm{ms}$; g) original TDR point

the capital savings is 27%.

One of the most important parameters under discussion now is the choice of the gradient for the $0.5\,\mathrm{TeV}$ collider. Comparing the optimal base curve with the optimal $30\,\mathrm{MV/m}$ curve in Fig. 1 shows a small total cost difference for $\mathcal{L} \geq 2 \times 10^{38}\,\mathrm{m^{-2}s^{-1}}$. The choice of $30\,\mathrm{MV/m}$ over $35\,\mathrm{MV/m}$ will certainly provide a larger safety margin, considering that the best 9-cell cavities today reach about $35\,\mathrm{MV/m}$.

If Q values of 10^{10} can be maintained at high accelerating fields, then the optimum gradient moves up to $40\,\mathrm{MV/m}$ (Table 1 - column d) with a total cost savings of 24% over the TDR original point as compared to a total cost savings of about 21% for the $30\,\mathrm{MV/m}$ choice. Therefore the gain in choosing a risky gradient of $40\,\mathrm{MV/m}$ and Q of 10^{10} is limited to only 3%.

The optimal front $D_y = 15$ of Fig. 1 shows that it is possible to avoid the kink instablility and still get a total cost savings (16%) over the TDR original point. Similarly, the optimal front $\epsilon_y = 6 \times 10^{-8}$ m of Fig. 1 shows that it is possible to relieve some of the damping ring challenges by increasing the vertical emittance by a factor of 2 and still maintain a cost savings of 12%.

Finally, in almost all cases the optimal fronts push towards smaller bunch lengths, smaller spot sizes, and smaller beam power. This results in higher (but tolerable) beamstrahlung and higher (but tolerable) energy spreads. In the linac the optimal fronts push towards fewer klystrons, longer rf pulse lengths, larger bunch spacing, slightly lower repetition rate and higher Q_{L}. This will of course increase the challenges for bunch compressors and RF control systems in the presence of Lorentz force detuning. Some of the latter challenges would be lessened by the choice of $30\,\mathrm{MV/m}$ instead of $35\,\mathrm{MV/m}$.

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