RESISTIVE WALL WAKEFIELD IN THE LCLS UNDULATOR*

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INTRODUCTION

In the Linac Coherent Light Source (LCLS) [1], the relative energy variation induced within the bunch in the undulator region must be kept to within a few times the Pierce parameter (to $\sim 0.02\%$). If it becomes larger, part of the beam will not reach saturation. The largest contributor to energy change in the undulator region is the (longitudinal) resistive wall wakefield of the beam pipe. In this region the LCLS bunch is short (the rms length $\sigma_z = 20~\mu$m), and the shape can be described as uniform, but with "horns" of charge at the head and tail. According to earlier calculations, the induced energy variation within the LCLS bunch was acceptable. These earlier calculations, however, included only the dc conductivity of the metal and neglected so-called ac conductivity effects.

Electrical conductivity in normal metals can be described up to infrared frequencies by the Drude-Sommerfeld free-electron model of conductivity [2]. According to this model the (dc) conductivity of a metal is given by $\sigma = ne^2\tau/m$, with n the density of conduction electrons, e the charge of the electron, τ the relaxation time, and m the mass of the electron. The ac conductivity, a response to applied oscillating fields, is given by

$$\tilde{\sigma} = \frac{\sigma}{1 - ikc\tau} , \qquad (1)$$

where $k = \omega/c$, with ω the radial frequency of the fields and c the speed of light. For copper at room temperature $c\tau = 8.1~\mu$m; for a typical bunch frequency of $k \sim 1/\sigma_z = (20~\mu\text{m})^{-1}$ the second term in the denominator of Eq. 1 equals 0.4; for the actual LCLS bunch shape, with its sharp leading spike, the ac character of conductivity is even more pronounced. Thus the second term in Eq. 1 cannot be neglected.

The short-range resistive wall wake in a round beam pipe, including ac conductivity, was obtained in Ref. [3]. We here apply the results to the bunch in the LCLS undulator region. To study the effect of widening the beam pipe aperture we calculate next the ac wake between parallel resistive plates (or on the axis of a flat beam pipe). Finally, we consider another effect that may be important at high frequencies, the (room temperature) anomalous skin effect. More details of this work can be found in Ref. [4].

As LCLS undulator region parameters we will take: bunch energy $E = 14$ GeV, charge $eN = 1$ nC, and rms length $\sigma_z = 20~\mu$m; undulator beam pipe radius $a = 2.5$ mm and total length $L = 130$ m.

AC CONDUCTIVITY

For a round beam pipe of radius a and dc conductivity σ the longitudinal impedance is given by [5]

$$Z(k) = \left(\frac{2}{ca}\right)\left[\frac{\lambda}{k} - \frac{ika}{2}\right]^{-1} , \qquad (2)$$

where parameter λ is given by

$$\lambda = \sqrt{\frac{2\pi\sigma|k|}{c}}[i + \text{sign}(k)] \qquad (3)$$

(we work in Gaussian units). The wake is the inverse Fourier transform of the impedance [3],[5]:

$$W(s) = \frac{16}{a^2}\left[\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\,x^2 e^{-x^2 s/s_0}}{x^6+8}\right] \qquad (4)$$

where s is the distance a test particle is *behind* the exciting charge (positive values indicate energy loss). $W(s) = 0$ for $s < 0$. The characteristic distance $s_0 = (ca^2/2\pi\sigma)^{1/3}$. For copper $\sigma = 5.8 \times 10^{17}$ s^{-1}; with $a = 2.5$ mm, $s_0 = 8.1~\mu$m. For $s \gg s_0$ Eq. 4 becomes the familiar long-range result $W(s) = -\sqrt{c/\sigma}/(2\pi a s^{3/2})$.

The calculation of the resistive wall wake including ac conductivity follows the same procedure, but with σ in parameter λ (Eq. 3) replaced by $\tilde{\sigma}$ of Eq. 1 [3]. A new dimensionless parameter is introduced, $\Gamma = c\tau/s_0$, which for the LCLS beam pipe equals 1.0. Fig. 1 compares the dc and ac impedances; we see that the ac resonance is narrower. (Note that the dc impedance or wake is discussed in this report for comparison purposes only; the dc results are nonphysical and inconsistent with the free-electron model.)

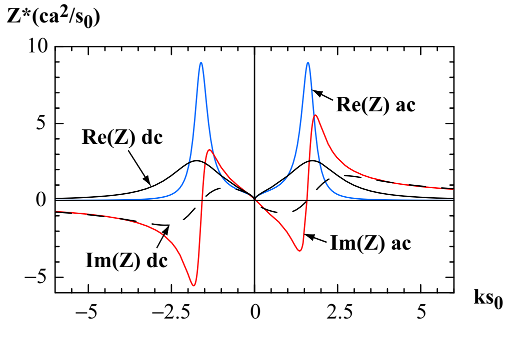

Figure 1: The impedance $Z(k)$ for the LCLS beam pipe, assuming dc and ac ($\Gamma = 1.0$) conductivities.

The wake is obtained by (numerically) inverse Fourier transforming the impedance. For $\Gamma \gtrsim 1$ (and s not too

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large) it can be approximated by a damped resonance [3]:

$$W(s) \approx \frac{4}{a^2}\, e^{-s/4c\tau} \cos\left[\sqrt{2k_p/a}\, s\right] , \qquad (5)$$

with $k_p = \sqrt{4\pi\sigma/c^2\tau}$, the plasma frequency of the metal. For a non-smooth bunch shape like in the LCLS the wake effect can be reduced (as we shall see) by increasing the damping in the wake. This is done by choosing a metal with small τ (and still reasonable σ). Such a metal is aluminum, where $\sigma = 3.8 \times 10^{17}$ s^{-1} and $c\tau = 2.4$ μm; for $a = 2.5$ mm, $s_0 = 9.3$ μm and $\Gamma = 0.26$. In Fig. 2 we plot the dc and ac wakes of Cu, and the ac wake of Al. Note that the ac wake of Al damps more rapidly than that of Cu.

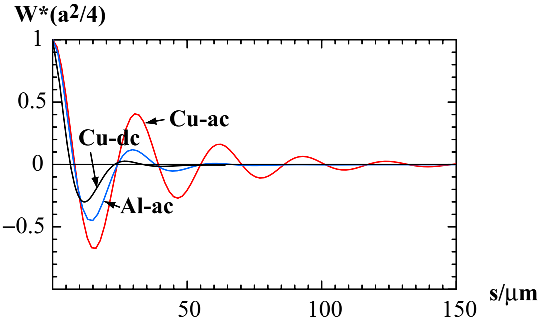

Figure 2: The dc and ac wakes for Cu, and the ac wake for Al, for the LCLS undulator beam pipe.

The energy change induced within a bunch, $\Delta E(s)$, is obtained by a convolution of the wake with the longitudinal bunch distribution, λ_z (we let $\Delta E > 0$ indicate energy gain). For LCLS parameters with numerically obtained λ_z [6], we obtain—for Cu-dc, Cu-ac, and Al-ac—$\Delta E(s)$ at the end of the undulator (see Fig. 3a; frame c gives λ_z, with head to the left). Because of uncertainties in the emittance and energy spread within the horns of the beam, it is not clear that these parts of the beam will lase. Therefore, let us define as figure of merit, δ_E, the total variation in (relative) energy change over that 30 μm length of beam that gives the minimum result. Results are given in Table I (the first row). From FEL simulations, it appears that particles within a window in $\Delta E/E$ with width $\lesssim 4\rho$ (ρ is the Pierce parameter) will lase [7]. For the LCLS $\rho = 5 \times 10^{-4}$, and the table shows that δ_E for Cu-ac is very large compared to 4ρ, for Al-ac it is better but still large. To show the dependence on a, in Table I (the third row) we give results for a doubled, and see that $\delta_E < 4\rho$ for all cases.

Table 1: Energy variation criterion, δ_E, at the end of the LCLS undulator, for different beam pipe parameters.

Pipe Shape	**Cu-dc**	**Cu-ac**	**Al-ac**
Round	0.16%	0.57%	0.34%
Flat	0.14%	0.39%	0.20%
Round, $a = 5$ mm	0.07%	0.17%	0.11%

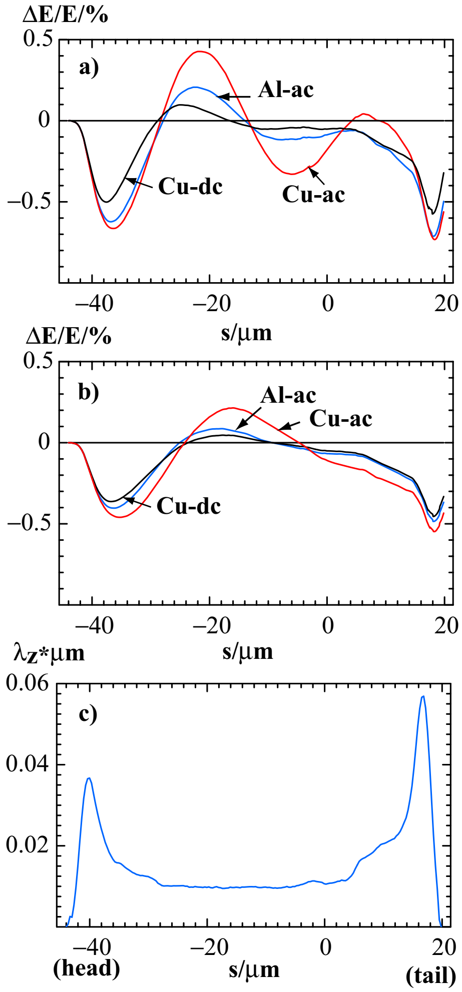

Figure 3: Energy deviation induced in the LCLS bunch in the undulator with a round beam pipe (a), and for a flat one (b); the longitudinal bunch shape is given in (c).

FLAT BEAM PIPE

To investigate the benefit of widening the beam pipe horizontally (keeping the vertical aperture fixed), we calculate the ac wake between two parallel plates. Henke and Napoly obtained the dc impedance (and wake) for this problem [8]. Their result, in our notation, is [1]

$$Z(k) = \frac{1}{c}\int_{-\infty}^{\infty} \frac{dq}{\cosh(qa)[\frac{\lambda}{k}\cosh(qa) - \frac{ik}{q}\sinh(qa)]} \qquad (6)$$

where $2a$ is now the separation of the plates and the parameter λ is the same as used above. We have simplified their equation by dropping small order terms.

We have repeated the impedance and wake calculations for a flat chamber, for both dc and ac cases. The wakefields are shown in Fig. 4 (compare with Fig. 2). The wakes at the origin are smaller by a factor $\pi^2/16$ [8], and the ac wakes damp more quickly (their Q factor is lower) and the

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[1] We believe that there is a factor $1/4\pi$ typo in their Eq. 16.

oscillation period is longer than in the round case. Fitting to a damped oscillator wake, we find that for $\Gamma \gtrsim 1$ the fitted $k \approx \sqrt{k_p/a}$ and $Q \approx 2.4 \ln(\Gamma + 1.5)$. In Fig. 3b we give $\Delta E(s)$ in the LCLS assuming a flat chamber, and see that it is reduced (compare with Fig. 3a). The criterion δ_E for these cases is given in Table I (the second row). We see that the ac results are $\sim 35\%$ better than in the round case.

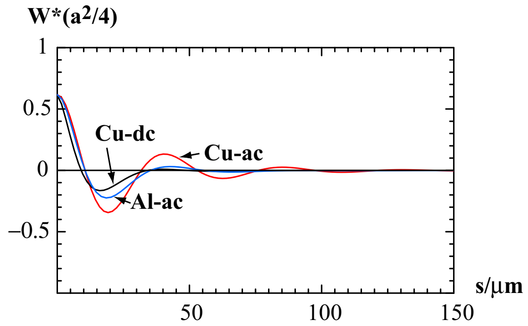

Figure 4: For a flat chamber, the ac and dc wakes for Cu, and the ac wake for Al, for the LCLS undulator beam pipe.

ANOMALOUS SKIN EFFECT

The anomalous skin effect (ASE), in theory, applies whenever the mean free path $\ell = v_F \tau$ (v_F is the Fermi velocity) becomes large compared to the classical skin depth, δ_{sk}. At $k = (20\ \mu\mathrm{m})^{-1}$ the skin depth and mean free path for Cu are both 40 nm. The LCLS bunch, however, contains much higher frequencies due to the horns, and one might expect ASE to be important. Following the theory of Reuter and Sondheimer (R-S) [9], we can obtain the impedance and wake, including ASE. We here sketch the calculation (for more details see Ref. [4]).

We define a dimensionless ASE parameter, $\Lambda = 3(\ell/\delta_{sk})^2/2kc\tau$. For Cu at room temperature $\Lambda = 3.4$. R-S give their solution in terms of the surface impedance, $Z_s = R_s + iX_s = 4\pi\mathcal{E}/c\mathcal{H}$, where $\mathcal{E}$ and $\mathcal{H}$ are the tangential electric and magnetic fields on the metal surface. Taking their expressions for R_s, X_s, the impedance for a round beam pipe is obtained by inserting

$$\lambda = \frac{4\pi|k|/c}{R_s\,\mathrm{sgn}(k) - iX_s} \tag{7}$$

into Eq. 2. The wakefield is then obtained by numerically inverse Fourier transforming Z. In Fig. 5 we plot the wake of a round, copper LCLS pipe when ASE is included in the calculation, and compare with the non-ASE result. The ASE wake is slightly weaker, but the difference is small and we see that this effect can be ignored.

RELATED WORK

Recent work on two fronts has complemented our work. (i) Verification of electrical properties of Cu and Al: Data in the literature does not seem to cover the entire range of interest to us, namely wavelengths in the range 10–100 μm. J. Tu has performed reflectivity measurements at Brookhaven on Cu and Al samples [10]. Preliminary results are in reasonably good agreement with our assumed free-electron model [11]. (ii) The relative effect on FEL performance of the different wakes considered here has been reasonably well confirmed through analytical methods [7] and detailed simulations [12].

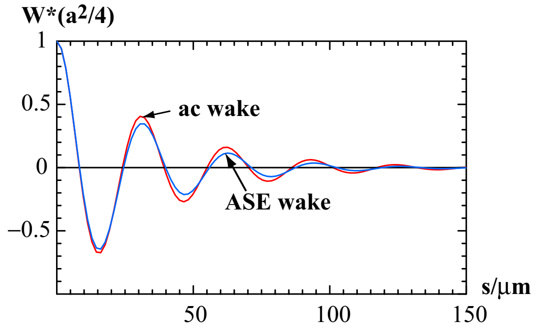

Figure 5: Wake of LCLS beam pipe (Cu, round) when including ASE (blue), or ac conductivity only (red).

CONCLUSION

We have shown that in the LCLS undulator region, because of the high frequency content in the bunch shape, the energy variation induced by the resistive wall wake is much larger when the (proper) ac conductivity calculation is performed, than for the often-used dc calculation. Using a round copper beam pipe will result in a (relative) energy variation many times the Pierce parameter, implying that a large fraction of beam will not reach saturation. Our results suggest that this situation can be improved by, instead, choosing a flat, aluminum beam pipe.

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