PRESSURE FIELD DISTRIBUTION IN A CONICAL TUBE WITH TRANSIENT AND OUTGASSING GAS SOURCES

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Abstract

This work presents and describes in detail the pressure profile in a conical tube with the unavoidable steady-state outgassing, plus a transient gas source, like, for instance, in an accelerator, when particles from the beam hit the walls. Mathematical and physical formulations are given and detailed; specific conductance, specific throughput and a detailed discussion about the boundary conditions are presented. These concepts and approach are applied to usual realistic cases, such as conical tubes, with typical laboratory dimensions.

INTRODUCTION

Several areas of applied physics deal with problems in high-vacuum and ultra high-vacuum technology that present tubular form. In many cases one finds conical tubes, which are frequently present in particle accelerators, colliders, storage rings, gravitational antennas, and some electron devices. Those systems frequently use parts that present tubular geometry with variable cross section along the tube axis. In particular, we deal with the special and important case when beam particles or beam radiation strike the walls. This case can be considered as a gas source localised in position and in time. The mathematical tools available to deal with this kind of geometry (variable cross section) are restricted to an approach that considers the vacuum system as composed by discrete parts, vacuum chamber, line pumping and vacuum pumps. This approach simplifies the treatment, but at the price of giving only, in some cases, average values for the pressure of the system. In tubular high-vacuum systems, the pressure along the axis of the tube may vary by orders of magnitude [1]. In this work we present a treatment, which solves analytically the steady-state differential equation for the case of azimuthal symmetry with variable cross section along the axis (considering the gas source due to natural outgassing); plus a numerical solution for the case of a transient gas source.

This work deals with a very common case in vacuum technology, the conic tube. From the overall dimensions and material employed, we define the specific conductance and the degassing rate per unit length of the tube. With these we are able to derive the pressure field, as well as the throughput, along the length of the tube [2-3]. We assume that high-vacuum pumps with equal pumping speeds are pumping both extremes of the tube. We analyze the steady-state and the transient modes. The differential equation is suited to deal with any transient case, if a time-dependent gas source is given. Another example is the situation where part of the vacuum system is vented, and then evacuated again [3].

In this work we determine the steady-state pressure field in a conic tube with a constant degassing rate per unit area and a gas source impulsive in time and in position.

PHYSICAL AND MATHEMATICAL MODELING

The adopted model assumes azimuthal symmetry, so that the tube surface can be defined as the revolution of a line around the axis of the tube. Other relevant quantities can be defined as well. The specific conductance at each point along the *x*-axis of the tube is defined as [1,3,4]:

$$c(x) \equiv \frac{16}{3}\left(\frac{RT}{2\pi M}\right)^{\frac{1}{2}} f^3(x)$$

where the function $f(x)$ represents the line that, by revolution around the x-axis, defines the surface of the tube. For the gas N_2 at the temperature 293 K the expression bellow will be expression (1):

$$c(x) = 96 f^3(x)\,. \tag{1}$$

The unit of $c(x)$ is $\mathrm{l.s^{-1}.cm}$. With expression (1) one can calculate the conductance of any cross section of a given tube. Analogously we can define the specific degassing rate per unit length, which is also a function of x, and can be defined as:

$$q(x) \equiv 2\pi q_0 f(x)\left[1+\left(\frac{df(x)}{dx}\right)^2\right]^{\frac{1}{2}} \tag{2}$$

where q_0 is the degassing rate per unit area of the material. The unit of $q(x)$ is $\mathrm{mbar.l.s^{-1}.cm^{-1}}$ [1]. The model assumes the molecular flow regimen, and the diffusion equation will be [1-3]:

$$c(x)\frac{\partial^2 p(x,t)}{\partial x^2}+\frac{dc(x)}{dx}\frac{\partial p(x,t)}{\partial x}=$$
$$=-q(x,t)+\nu(x)\frac{\partial p(x,t)}{\partial t}. \qquad (3)$$

The gas source can be represented by the following function:

$$q(x,t)=q_S+q_T(x,t), \qquad (4)$$

where q_S represents the outgassing, and q_T the transient. In this case we can represent the total degassing rate with the following function:

$$q(x,t)=q_S+q'\delta(x-x')\delta(t-t') \qquad (5)$$

where q' represents the amount of gas (in mbar.liters), and $\delta(x)$ is Dirac's delta function liberated at $x = x'$ in $t = t'$.

Since the differential equation is linear, the general solution will be the sum of the particular solutions for the impulsive sources, and may be written as:

$$p_G(x,t)=p_S(x)+p_T(x,t) \qquad (6)$$

where p_S represents the outgassing, and p_T the transient.

Equation (3) yields the following exact solution to the steady-state case [3-4]:

$$p_S(x)=-\frac{C_1}{2a(ax+b)^2}-\frac{bB}{196a^2(ax+b)^2}+\frac{Ab^2}{384a^3(ax+b)^2}$$
$$+\frac{B}{96a^2(ax+b)}-\frac{bA}{96a^3(ax+b)}-\frac{A\ln(ax+b)}{192a^3}+C_2.$$

The values of the constants are the following:

$$a=\frac{D-d}{2L}, \quad b=\frac{d}{2},$$
$$A=2\pi q_0\left(1+a^2\right)^{\frac{1}{2}}a, \quad B=2\pi q_0\left(1+a^2\right)^{\frac{1}{2}}b.$$

To solve this kind of equation ($a \neq 0$) one needs to define the boundary conditions to find the constants C_1 and C_2. In general, this is done specifying the pressure (Dirichlet's boundary condition) and the throughput (Neumann's boundary condition) at a particular point of the tube or two values of one of these quantities. To solve the more interesting cases in high-vacuum systems, the boundary conditions are a mixing (linear combination) of the value of pressure and the throughput (Robin's boundary condition) in particular point, in general in the place of vacuum pumps; this kind of boundary condition is very powerful. The discussion of the boundary conditions will be presented in section 3, with a detailed description of the geometry of the system.

DEFINITION OF THE GEOMETRY

We are dealing with an axis symmetric (azimuthal symmetry) tube with variable cross-section and we treat a conic tube as an example. The specific conductance changes along the axis, as well as the specific degassing rate per unit length. The gas source is considered to be due to the natural degassing of the materials of the structure, which we consider to be made of stainless steel. In this case the degassing rate per unit area is $q_S = 10^{-9}$ mbar.l.s^{-1}.cm^{-2}. This is a typical value for this material, considering common cleaning processes in high-vacuum technology. A schematic drawing of the conic tube is presented in Fig. 1.

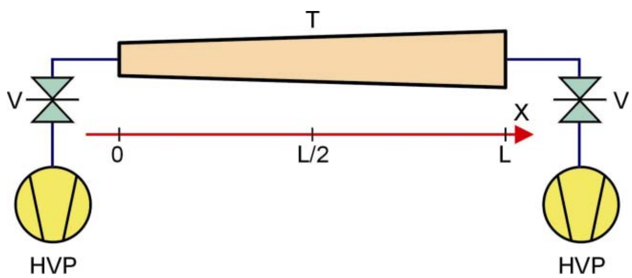

Figure 1: Schematic drawing of the system: T – conic tube; V – valve; and HVP – high-vacuum pump.

The function defining the line that, by revolution around the x-axis, defines the conic tube, is:

$$f(x)=\left(\frac{D-d}{2L}\right)x+\frac{d}{2}$$

where L is the length; d the smaller diameter; and D the larger diameter of the tube. The examples that will be given adopt $L = 400$ cm, $d = 3$ cm, and $D = 6$ cm. The total steady-state throughput generated by the walls of the tube is $Q_T = 5.66 \times 10^{-6}$ mbar.l.s^{-1}. We can write the throughput at the extremities of the tube as:

$$Q_T=Q_L+Q_R=S_L p_S(0)+S_R p_S(L)$$
$$=+c(0)\left.\frac{dp_S(x)}{dx}\right|_{x=0}-c(L)\left.\frac{dp_S(x)}{dx}\right|_{x=L},$$

where S_L and S_R are the pumping speeds at the left and right ends of the tube, respectively. In our example, we consider that $S_L = S_R = 50$ l.s^{-1}. This calculation is useful to check the consistency of the results for the pressure field along the axis of the tube, since the sum of the throughputs at the ends should give the total throughput.

The following boundary conditions are assumed [4]:

- all the gas reaching the pumps is pumped, both for the transient and the steady state solutions, so that at $x = 0$ (Robin's Boundary Condition),

$$c(0)\frac{\partial p_G(x,t)}{\partial x}\bigg|_{x=0} = S_L p_G(0,t)\,,\ \forall t \geq 0\,,$$

and at $x = L$ (Robin's Boundary Condition),

$$c(L)\frac{\partial p_G(x,t)}{\partial x}\bigg|_{x=L} = -S_R p_G(L,t)\,,\ \forall t \geq 0\,.$$

- the initial condition is

$$p_G(x,0) = p_S(x), \quad \forall x, \ \ 0 \leq x \leq L.$$

RESULTS AND DISCUSSION

The solution was obtained using both analytical and numerical procedures. The numerical solution, to solve the transient gas source, was obtained using a standard Galerkin finite element method for the spatial discretization and an implicit Euler scheme for the temporal discretization.

Once the characteristics of the system are specified and the boundary conditions adopted, we can solve equation (3) and find the steady-state and transient-state pressure fields along the tube. Figure 2 shows the total pressure field distribution along the x-axis.

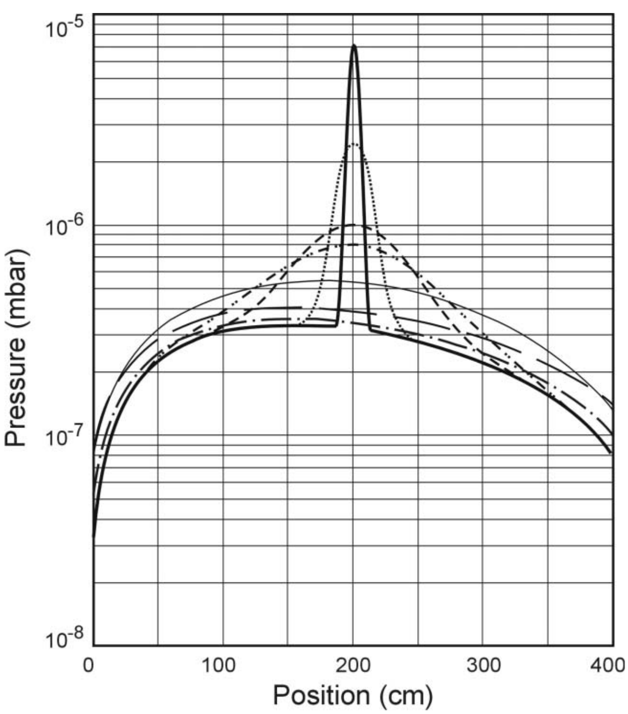

Figure 2: Pressure field along the axis of the conic tube: at $t = 10^{-4}$ s (thick solid line); $t = 10^{-3}$ s (dotted line); $t = 10^{-2}$ s (dashed line); $t = 2\text{x}10^{-2}$ s (two-dot-dashed line); $t = 10^{-1}$ s (thin solid line); $t = 10^{0}$ s (long-dashed line); $t = 10^{1}$ s (dot-dashed line).

Several effects contribute to produce an asymmetric pressure field distribution as shown in Fig. 2. The first is that the specific conductance changes along the tube. In addition one must also consider the change in throughput from the walls caused by the change in diameter of the tube. The steady-state results presented here were confirmed by a Monte Carlo calculation [6].

We can see the pressure profile along the axis of the tube at seven different times. At $t = 10^{-4}$ s one can see the gas from the transient source ($q' = 10^{-6}$ mbar.liters, $x' = 200$ cm and $t' = 0$ s) distributed in roughly ±15 cm around $x = 200$ cm. At $t = 10^{-3}$ s the pressure peak is lower, and distributed in a wider region. As time evolves, one can see the gas burst filling the whole tube and reaching the pumps. After about 10 s, the pressure is practically back at the steady-state level. One can see that the pressure varies more steeply (higher gradient in absolute value) at the left hand side of the tube, next to the end, since the specific conductance is lowest in this region. The point of maximum pressure for the steady-state condition occurs at $x = 151.6$ cm.

This kind of analysis can be very useful in the design of vacuum systems, in order to optimize the number of pumps along the vacuum line.

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