HIGH-PRECISION RESONANT CAVITY BEAM POSITION, EMITTANCE AND THIRD-MOMENT MONITORS*

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Abstract

Linear colliders and FEL facilities need fast, nondestructive beam position and profile monitors to facilitate machine tune-up, and for use with feedback control. FAR-TECH, Inc., in collaboration with SLAC, is developing a resonant cavity diagnostic to simultaneously measure the dipole, quadrupole and sextupole moments of the beam distribution. Measurements of dipole and quadrupole moments at multiple locations yield information about beam orbit and emittance. The sextupole moment can reveal information about beam asymmetry which is useful in diagnosing beam tail deflections caused by short-range dipole wakefields. In addition to the resonance enhancement of a single-cell cavity, use of a multi-cell standing-wave structure further enhances signal strength and improves the resolution of the device. An estimated resolution is better than 1 µm in rms beam size and better than 1 nm in beam position.

INTRODUCTION

Resonant cavity-based beam position monitors (BPM's) have achieved state-of-the-art beam position resolution approaching tens of nanometers. These devices measure the first moment of the beam distribution, $\langle x\rangle$ or $\langle y\rangle$ in the transverse coordinates. The same method can be used with cavities operating with a higher harmonic to measure the second-order (quadrupole), and third order (sextupole) beam moments. The second-order moment measures either $\langle xy\rangle$ or $\langle x^2-y^2\rangle$ depending on the orientation of the cavity, while the third-order moment measures $\langle y^3-3x^2y\rangle$ or $\langle x^3-3y^2x\rangle$. A technique for emittance measurement using quadrupole moment information from stripline BPM's has been suggested by Miller [1].

In a rectangular pillbox, the cavity mode for measuring the beam quadrupole moment is TM_{220}, shown in Figure 1. In the orientation shown, this cavity can measure the skew quadrupole $\langle xy\rangle$. The normal quadrupole $\langle x^2-y^2\rangle$ can be measured by rotating the cavity by 45 degrees about the beam axis. Evaluating this expression in the case of a beam with transverse sizes σ_x and σ_y, the output power will be proportional to,

$$P_{out}\propto(\langle x\rangle^2-\langle y\rangle^2+\sigma_x^2-\sigma_y^2)^2.$$

In order to extract information about the beam sizes, one needs to perform a separate dipole measurement as close as possible to the quadrupole cavity, in order to subtract the influence of $\langle x\rangle$ and $\langle y\rangle$ in the expression.

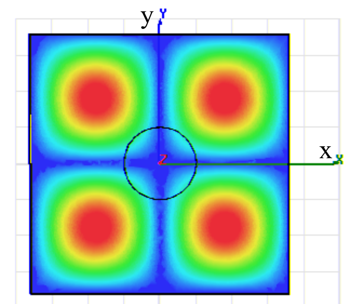

Figure 1: Field pattern in a rectangular cavity with a beam pipe. Shading indicates $|E_z|$ on the midplane, and the circle is the beam pipe.

In order to calculate the proportionality factor in the output power, the shunt impedance is evaluated for a given cavity geometry. For a rectangular pillbox with transverse lengths of $L_x=L_y\simeq3.6$ cm, a length of $L_z=0.97$ cm, and a beam pipe of 1.0 cm diameter, the resonance frequency is 11.424 GHz and $Q_{wall}=10{,}000$. The shunt impedance $R_s=V^2/P_{wall}$ where V is the maximum voltage seen by a relativistic particle crossing the cavity at a given offset and P_{wall} the power dissipated in the cavity wall. For beam offsets not too far from the beam pipe center the shunt impedance for this cavity is,

$$R_s=1.02\times10^{-10}(\langle x^2\rangle-\langle y^2\rangle)^2,$$

with x and y in meters and R_s in units of Ohms.

The cavity output power can be evaluated for two important regimes: the single bunch regime without the resonant enhancement effect, and for a train of bunches, where the cavity power is resonantly built up to reach a steady-state value. In the single-bunch case, the output power after the bunch passage is:

$$P=\frac{\omega^2q^2}{4Q_eQ_w}R_s(x,y)$$

where Q_w and Q_e are the quality factors due to wall and external coupling losses, respectively; the loaded Q, Q_L is given by $1/Q_L=1/Q_w+1/Q_e$, and q is the bunch charge. For multibunch steady-state conditions, the output power is,

$$P_{out}=(Q_L^2/Q_eQ_w)R_s(x,y)I_{beam}^2$$

where I_{beam} is the average beam current bunched at the RF frequency, and in both cases finite bunch length effects are ignored. Applying this formula to a high-current beam such as that of the NLC design, assuming $I_{beam}=0.714$ A, $\sigma_x=1$ µm, $\sigma_y\ll\sigma_x$, and critical coupling $(Q_L^2/Q_wQ_e=1/4)$, the output power will be $P_{out}=1.2\times10^{-11}$ W. A power of this level is several orders

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of magnitude above the cavity thermal noise filtered around the central frequency, and is thus considered detectable. In practice, resolution is limited by nearby parasitic modes, for example TM_{130}.

The beam third-order moment is less sensitive than the quadrupole moment, but it can give information about beam asymmetry. This mode has azimuthal mode number m=3, so that the electric field is proportional to $\cos(3\theta)$ in terms of the azimuthal angle θ. Typical shunt impedances (in Ohms) for a single cavity are:

$$R_s = 5\times10^{-19}\langle x^3 - 3y^2x\rangle^2.$$

This mode can give information about beam asymmetry. One situation where this would be valuable is to diagnose the beam tail caused by short-range transverse wakes in a linac. Since the measurement technique is non-intercepting, the signal can be used as the basis for a feedback loop. Since the cavity field is proportional to $\cos(3\theta)$, a pair of cavities can determine the tail orientation to within an additive multiple of 120 degrees. The feedback scheme can use a dithering technique to alter the beam trajectory in all three directions to determine which one causes the greatest improvement in beam quality.

Interpretation of the raw signal from a sextupole mode cavity requires knowledge of the beam position, as well as both polarizations of the quadrupole moment at the cavity location. The structure output power is then,

$$P_{out}^{1/2} \propto \langle x\rangle^3 - 3\langle x\rangle\langle y\rangle^2 - 6\langle y\rangle\langle xy\rangle + 3\langle x\rangle\langle x^2 - y^2\rangle + \langle x^3 - 3xy^2\rangle,$$

where the last term is the third order moment, and the remaining terms are derived from separate dipole and quadrupole mode measurements.

WAVEGUIDE NETWORKS

The waveguide coupling scheme for a quadrupole mode should reject the other cavity frequencies, in particular the fundamental (TM_{110}) mode. For a cavity that simultaneously resonates in both quadrupole and dipole modes, this can be done with a subtraction network schematically depicted in Figure 2. The signal in the waveguides connected to the structure is determined by the cavity magnetic field near the coupling slot. These signals can be subtracted in a set of cascading magic T's in a way that rejects the fundamental mode.

In Figure 2a, four waveguides are coupled to the peripheral B-field in each quadrant of the cavity, with a set of magic T's and a hybrid coupler directing each mode to the proper output port. In Figure 2b, two waveguides are coupled to the radial magnetic field and connected in a single magic T. This can be done by longitudinally offsetting the waveguide so it contacts the face of the cavity [3], or the waveguide can located at an iris between cavities, similar to the CLIC HOM damping scheme [4]. In either case, the TM_{110} mode is suppressed because it does not have a radial magnetic field. The scheme in Figure 2b is simpler to implement despite the fact that it only provides one of the two dipole modes, and therefore must be duplicated in order to obtain the other dipole polarization.

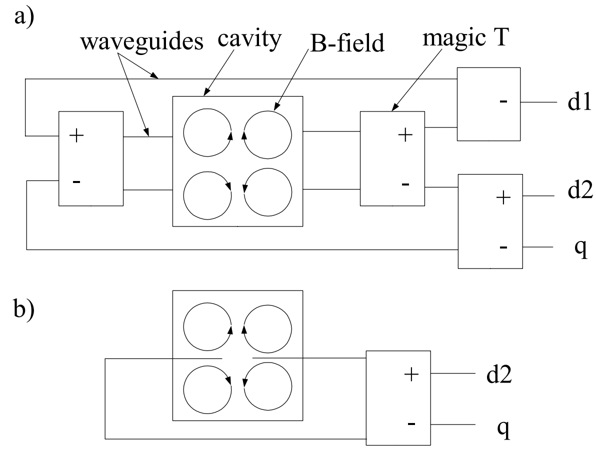

Figure 2: Schematic representation of two possible designs for a waveguide subtraction network. Waveguides are represented by lines, and d1, d2 and q signify the dipole and quadrupole output signals.

MULTI-CELL CAVITIES

The output signal can be increased by using a multi-cell standing wave design, which can result in nearly an order of magnitude more power in the case of a nine cells, as is discussed in a paper by Kim, *et al.* [2]. This cavity is designed to provide information about both dipole and quadrupole beam moments. When used for measuring information about a bunch train, both the dipole and quadrupole frequencies should be multiples of the bunch repetition frequency. Assuming NLC parameters (714 MHz bunch frequency), the dipole mode is at the 12th harmonic, and the quadrupole at the 16th harmonic. The ratio of these two modes can be adjusted with the choice of iris dimensions, and by placing radial fins which are at the electric field minima of the quadrupole mode, but at the maxima of the dipole modes. The quadrupole mode is designed to operate with π phase advance per cell, while the dipole mode uses the less efficient $3\pi/4$-mode. With a fixed cell length, the ratio between phase advance per cell should be the same as the ratio between the two frequencies, or 16/12 in this case.

FABRICATION TOLERANCES

Dipole Cell Misalignments

The section on multi-cell cavities presented a way to increase the shunt impedance of the measurement device by adding extra cells, as is done for accelerating structures. In the case of dipole and higher order modes, there are some additional aspects to consider.

A nine-cell standing-wave cavity will have nine resonances within the dipole band, ranging from phase advance per cell $\Delta\phi = 0$ to π in increments of $\pi/8$. A typical accelerating-mode structure is designed to be synchronous with only one of these frequencies, most often the π-mode. In dipole and higher order cavities, we must also consider the effect of small transverse misalignments between the central axis and the electrical center of a cell. Even with fairly tight mechanical tolerances during construction, cells can still be misaligned transversely by about 25 μm. A single bunch propagating on-center in such a structure will radiate power in each cell according to the misalignment in that cell, ~25 μm. For a beam that is "on-center", there will be very little power coupled to the synchronous mode, with the rest of the power being coupled to the remaining modes. This can be shown by performing a sum over the voltage interaction with each cell. The voltage of a particle interacting with a given cavity mode is:

$$V \propto \sum_n (x - x_n) \cos(nkL_z) \cos(\omega L_z / c),$$

where the index n refers to the cell number, L_z is the distance between cell centers, and x_n is the cell misalignment, and $k = \Delta\phi / L_z$. With no misalignments, the only significant coupling is to the synchronous mode, which usually has $c\Delta\phi = \omega L_z$. However, for misalignments much larger than the beam offset, each term in the above sum has an arbitrary sign depending on the cell misalignment. This can result in non-zero coupling for the remaining modes.

Power coupled to the unwanted modes can be suppressed in several ways. In some instances, the coupling cell is a dead cell for the mode in question, which greatly diminishes the out-coupled power. This is the case for the odd-numbered phase advances, π/8, 3π/8 through 7π/8 in a nine-cell structure coupled through the center cell.

The power in the extra modes can also be filtered out if the modes are well-separated in frequency. The filter pass-band should be wider than the cavity resonance, or the signal for the desired mode will be attenuated.

Quadrupole Cell Misalignments

The effect of quadrupole cell misalignments can be evaluated similarly to that of the dipole case. Different from the dipole case, quadrupole misalignments can also affect the power coupled to the synchronous mode. For the simple example of a two-cell cavity with misalignments $\pm\delta$ along x, the cavity voltage response will be,

$$V \propto \langle (x-\delta)^2 - y^2 \rangle + \langle (x+\delta)^2 - y^2 \rangle = 2\langle x^2 - y^2 \rangle + 2\delta^2 .$$

This response is a level-shifted version of the original signal, and the $V = 0$ curves in this case become hyperbolic.

For misalignments only along one axis, this level shift is the predominant effect, with relatively little power being coupled to modes with different $\Delta\phi$. This can be corrected by tilting the entire cavity about the other axis (the y-axis). In the general case of misalignments randomly distributed along both axes, significant power can be coupled to the remaining modes.

Quadrupole Cell Imperfections

A level-shifted voltage pattern of the type discussed above can also exist in a single cell when considering finite cell fabrication errors. We have performed HFSS[5] simulations of the case of two areas of the cavity wall near the iris are raised by some amount. For 10 micron high, 3 mm diameter imperfections near two of the electric field maxima (180 degrees apart), the point of closest approach of the $V = 0$ curve to the axis is 180 microns.

FIVE-CELL DESIGN

In order to relax the fabrication tolerances, our latest design is of a cavity featuring only five cells. A 5-cell structure has a π-mode as well as a 3π/4 mode, like the 9-cell structure, but no 7π/8 mode. By selecting a 5-cell design and increasing the cell-to-cell coupling, the π mode to nearest neighbor separation has been increased from 4 MHz in the 9-cell design to 40 MHz in the 5-cell, which facilitates the filter design. The fewer number of cells also eases alignment considerations, since in a 5-cell object only three cells need to be independently aligned to the centerline defined by the remaining two cells.

We have also found a means of optimizing the cell geometry which results in an improved shunt impedance, to be discussed in a future publication. With this refinement, the use of a smaller number of cells does not sacrifice the shunt impedance, and in fact improves it by 40%. This type of optimization can also be applied to sextupole mode cavities, with shunt impedances improved by as much as a factor of 50.

CONCLUSION

We have studied multi-cell resonant cavities for measuring the beam dipole, quadrupole and sextupole moments. In the case of fabrication errors, power can be coupled to unwanted modes with different phase advance per cell, and the quadrupole signal can be level-shifted. Fabrication tolerances can be relaxed by reducing the number of cells to five, or even three.

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