DESIGN OF A MAGNET SYSTEM FOR A MUON COOLING RING

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Abstract

A hydrogen gas filled storage ring appears to be a promising approach to reducing the emittance of a muon beam for use in a neutrino factory or a muon collider. A small muon cooling ring is being studied to test the feasibility of cooling by this method. This paper describes the magnet system to circulate the muons. The magnet design is optimized to produce a large dynamic aperture to contain the muon beam with minimum losses. Muons are tracked through the field to verify the design.

INTRODUCTION

Both muon colliders and neutrino factories use storage rings, which circulate muons. In order to achieve the required emittance the muon beams need to be cooled. Our group has been studying a small storage ring for the purpose of cooling a muon beam using ionization cooling [1]. The cooling ring consists of four cells, each with a dipole that bends the reference beam by 90°. These dipole magnets have large vertical apertures in order to minimize the losses of the large emittance beam. The pole edges are rotated with respect to the incident closed orbit path in order to provide horizontal focusing. The ring aperture is filled with pressurized hydrogen gas to act as the absorber for the ionization cooling. RF cavities are placed in the free space between adjacent magnets to replace the energy lost in the hydrogen. Table 1 shows the parameters that describe this ring. The cooling aspects of this ring will be discussed in another paper submitted to this conference [2]. The accelerator lattice for the cooling ring was obtained using the SYNCH [3] lattice design program, which assumes a hardedge description of the magnetic fields. This paper will describe the design of the magnet system with realistic fields to be used in this ring. The fields generated by the magnets are used to track muons in the ring. Because of the small size of the ring, the local variation of the field along the ring is important. The tracking of muons is made using the ICOOL simulation code [4], where the field is supplied to ICOOL as a Fourier decomposition of the field harmonics along the closed orbit path. The simulation is used to establish the dynamic aperture of the cooling ring.

MAGNET DESIGN

The dipole magnet in each cell should bend a 172.12 MeV/c reference muon on the closed orbit by 90°. The magnet pole faces are rotated with respect to the reference beam by 22.5° in order to provide edge focusing in the horizontal plane. Fig 1 shows a diagram of the mid plane geometry of one cell of the ring. It is desirable to have a horizontal aperture of ±20 cm about the closed orbit. The field should be as uniform as possible in that region. The vertical aperture needs to be ±10 cm in order to contain the full beam. The dipole field is chosen to be 1.8 Tesla so that the iron yoke is not significantly saturated. The coils are placed around the poles. Fig 2 shows a vertical cross section through the symmetry plane of the magnet. The horizontal dimensions shown in the diagram are measured from the center of the ring.

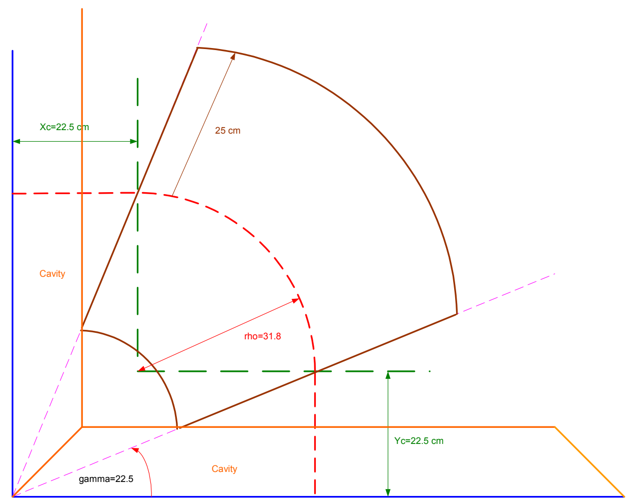

Figure 1: Sketch showing a single cell of the cooling ring.

Table 1: Parameters that describe the muon cooling ring

Parameter	Value
Dipole Field	1.8 T
Number of Cells	4
Reference Momentum	172.12 MeV/c
Ring Circumference	3.81 m
X Aperture	±20 cm
Y Aperture	±10 cm
P_z Acceptance	±10 MeV/c
Minimum β_X	38 cm
Maximum β_X	92 cm
Minimum β_Y	54 cm
Maximum β_Y	66 cm
Hydrogen Gas Pressure	40 Atm @ 300° K
RF Frequency	201.25 MHz
RF Gradient	10 MV/m
Total RF Length	0.8 m
Total Turns	100

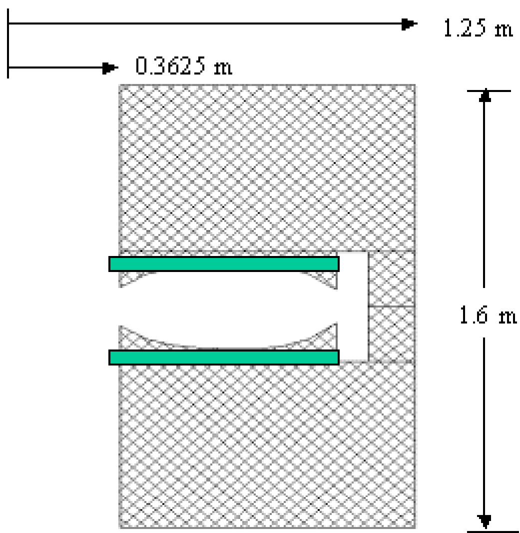

Figure 2: Vertical cross section of the magnet along the symmetry plane.

The magnet poles are shaped to provide a more uniform field with some additional quadrupole added. The additional quadrupole is needed to supplement the focusing provided by the magnet edges. Fig 3 shows the effect of shaping the poles to provide an extended and more uniform field as a function of the radial distance. This will improve the horizontal dynamic aperture. The field quality needs to be reasonably good over a region ±20 cm about the reference orbit.

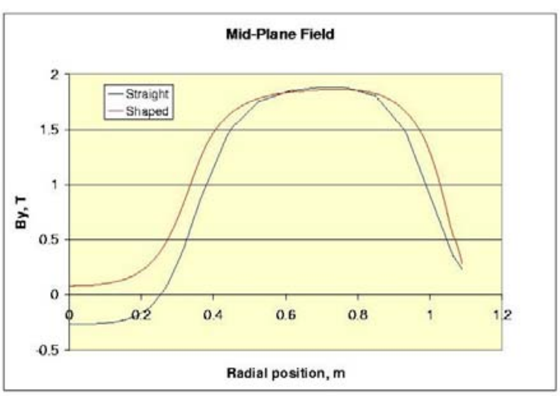

Figure 3: B_y in the mid-plane of the vertical symmetry plane. The red curve shows the field with the shape pole. The blue curve shows the field for flat poles.

FIELD COMPUTATION

The field from this magnet is computed using the finite-element program TOSCA [5]. A single magnet is modeled and the presence of the other magnets is taken into account with the boundary conditions. TOSCA provides the ability to track particles within the program. By launching muons at various start positions on a symmetry plane, a closed orbit can be found. Using a mid-plane field map from TOSCA, the field and its harmonics can be calculated along the closed orbit path. Fig 4 shows the field harmonics along the closed orbit path for two cells. The harmonics shown are determined from the variation of the mid-plane field in the plane transverse to the closed orbit path at each point along the path.

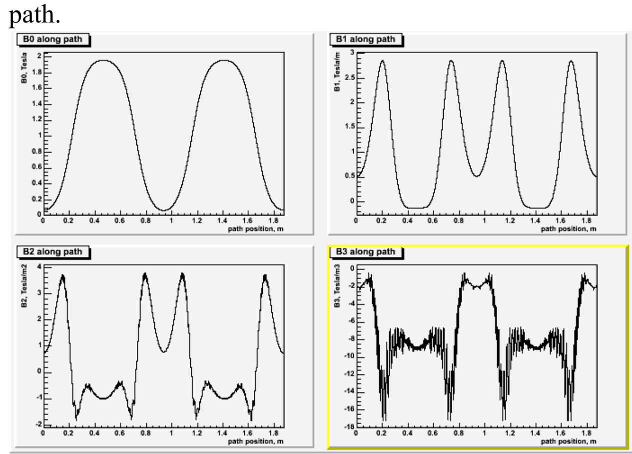

Figure 4: Field harmonics along the closed orbit path.

The ICOOL simulation program can accept the field as a Fourier decomposition of each of these harmonics along the closed orbit path. The field can be reconstructed from an expansion in variables of the local coordinate system defined by the closed orbit path [6]. The errors in the field calculated in this manner are expected to grow with distance from the closed orbit. TOSCA also provides a field map where the field errors are more uniformly distributed in space. This field map is used in a parallel study using the GEANT program [7] for tracking.

STORAGE RING SIMULATION

The muon beam will occupy a large fraction of the available aperture. Tracking muons through the ring in a storage ring mode without rf and hydrogen gas can be used to establish the dynamic aperture of the ring. Muons are launched at different start distances from the closed orbit on a symmetry plane between two cells. These muons are tracked through the storage ring using the ICOOL simulation program. The particle positions and angles are sampled as each particle passes a symmetry plane between two cells. Fig. 5 shows x vs. P_x (y vs. P_y) for muons launched at 1 cm intervals along the x-axis (y-axis). The horizontal dynamic aperture of this magnet is essentially as large as if a hardedge uniform field were used. The shaping of the transverse profile of the pole is responsible for the large horizontal aperture. The figure shows that the vertical aperture is limited in size. The vertical focusing of the magnet is dependent on the field falloff in the longitudinal direction. The field falloff is determined by both the vertical aperture, which must be large in order to contain the beam, and the distance to the next magnet, which is dictated by the size of the rf cavity. Varying the pole shape in the longitudinal direction to improve the vertical focusing is difficult because it would

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cut severely into the vertical aperture. Another approach to improve the vertical aperture would be to vary the field along the longitudinal direction with smaller dipole coils. This has not yet been investigated.

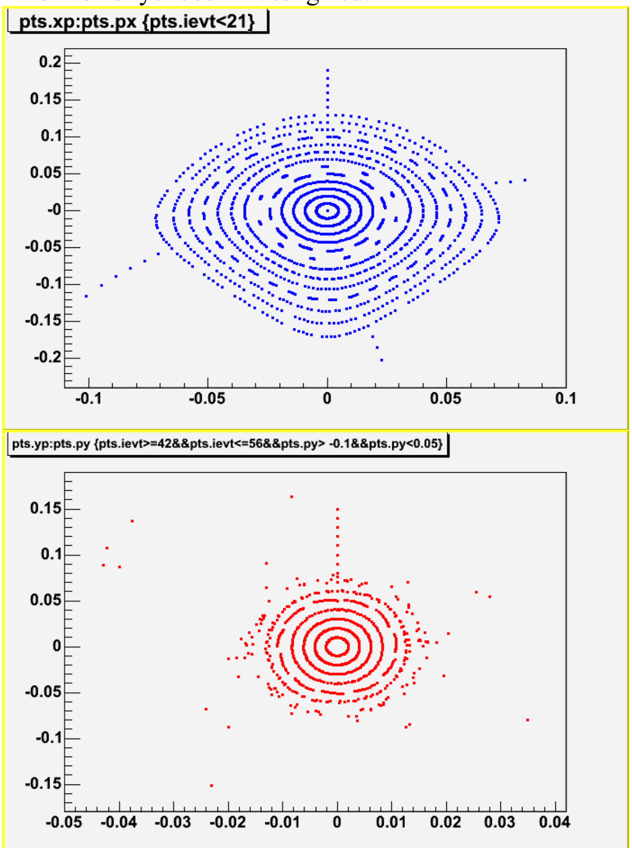

Figure 5: x vs. P_x and y vs. P_y phase space plots illustrating how the dynamic aperture is calculated. The spacing between initial muon trajectories is 1 cm.

Table 2 summarizes the size of the horizontal and vertical apertures by counting the ellipses of trajectories that survive 32 turns in this cooling ring. Table 2 also shows for comparison the dynamic aperture achieved with a hardedge field with constant vertical field. The actual acceptance of the ring should vary as the square of the dynamic aperture size.

Phase Space	All Harmonics	No High Order Harmonics	Hardedge
x P_x	11	13	12
y P_y	7	7	14

Table 2: Estimate of the size of the dynamic aperture.

The cooling ring was designed using SNYCH lattice design program. This program treats the dipole magnets with a hardedge constant field with local edge focusing provided at the entrance and exit to the magnet. Using realistic fields calculated with TOSCA and reconstructing the field from harmonics may affect the lattice parameters. The lattice variables are sensitive to the amount of quadrupole present. Although the magnet edge angles determined by SYNCH should have provided sufficient focusing, the lattice parameters determined from the ring using this magnet have moved from the SYNCH design. Since there is no room in the lattice to increase the edge angles without interfering with the rf cavity, it was necessary to add extra integrated quadrupole to the dipole magnet by tilting the pole faces. Minimizing the deviation from the SYNCH values and maximizing the useful aperture determined the amount of quadrupole that was added. Table 3 shows lattice parameters from the SYNCH model and for the nominal magnet with the edge angles determined from SYNCH and the magnet with the optimized tilted pole faces to add additional quadrupole.

Table 3: Comparison of lattice parameters for the original SYNCH ring model with those determined from the designed storage ring.

	Synch	Nominal	Added Quad
μ_x	99.88º	117.58º	101.68º
β_x	37.85 cm	27.01 cm	32.91 cm
α_x	0	-0.00315	-0.0039
μ_y	92.63º	68.11º	94.95º
β_y	56.89 cm	81.96 cm	62.70º
α_y	0	0.00092	0.00180

ACKNOWLEDGEMENTS

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