TRANSVERSE IMPEDANCE OF TWO-LAYER TUBE

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Abstract

The exact analytical expressions for the multipole longitudinal and transverse impedances of two-layer tube with finite wall thickness are obtained. The numerical examples for the impedances of the vacuum chamber with laminated walls are given.

INTRODUCTION

The knowledge of the vacuum chamber impedance in accelerators is an important issue to provide the stable operation of the facility from the machine performance and beam physics point of view [1,2]. To adjust the technical (high vacuum performance, reduction of static charge etc) and beam physics (resistive instability) issues, the laminated walls of vacuum chamber parts are often used in accelerators.

The two-layer circular tube is a good model for the small-gap undulator vacuum chamber with thin covered walls. The exact solution for the monopole longitudinal impedance of two-layer tube has been derived in [3]. In this paper explicit analytical solution for longitudinal and transverse multipoles has been obtained. The numerical example for copper-NEG (non-evaporated getter) two-layer tube impedance is given.

MULTIPOLE EXPANSION

Consider the ultrarelativistic point-charge moving parallel to the axis of the uniform circular-cylindrical structure. The transverse position of the charge is given by the offset r_s and the polar angle $\varphi_s = 0$. The m-th multipole term of longitudinal impedance is given by [2]

$$\bar{Z}_{\parallel,\, m}(\omega, r, r_s) = \left(r r_s / b^2\right)^m Z_{\parallel}^{(m)}(\omega) \cos m\varphi \qquad (1)$$

where b is the tube radius, r, φ are the radial and polar coordinates of the observation point and $Z_{\parallel}^{(m)}(\omega)$ is the frequency-dependent term of multipole, sometimes also identified as an impedance. The transverse mode is given by Panofsky-Wenzel theorem [4].

THE PROBLEM

Let us consider the relativistic $(v < c)$ plain disk (disk radius $a_1 = r_s$) moving with velocity v along the uniform, circular-cylindrical two-layer tube of inner radius a_2 (Fig.1). The disk centre coincides with the tube axis and the disk radius corresponds to the charge offset.

The charge density in frequency ω domain is then given by $(\delta_0 = 1,\ \delta_{m>0} = 2)$:

$$\rho^{(m)} = \frac{q\delta_m}{\pi r_s^2 c} e^{-j\omega z/v} (m+1)\left(r/r_s\right)^m \cos m\varphi \qquad (2)$$

The boundary between two layers is located at $r = a_3$ and the outer radius of the tube is a_4. Outside of the tube is vacuum. In analogous to [1] the frequency-depend part of impedance is independent from the disk radius and hence valid for the point-like charge.

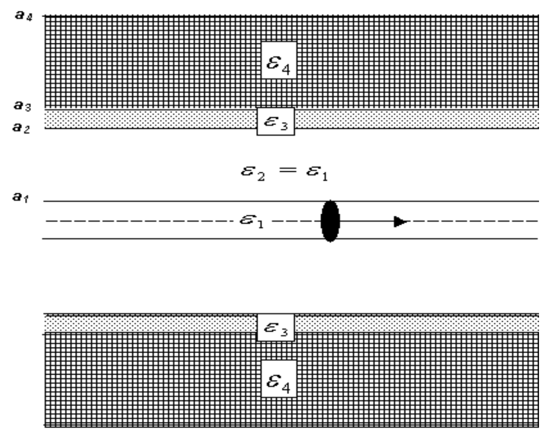

Figure 1: Geometry of the problem.

SOLUTION

The cross section of the tube is divided into the five concentric regions: 1) $0 \le r \le a_1 = r_s$ (vacuum), 2) $a_1 \le r \le a_2 = b$ (vacuum), 3) $a_2 \le r \le a_3$ (first layer), 4) $a_3 \le r \le a_4$ (second layer) and 5) $r \ge a_4$ (vacuum). Due to current axial asymmetry the fields radiated in the tube have all six components $E_z, H_z, E_\varphi, H_\varphi$, and E_r, H_r. The frequency domain wave equation for longitudinal electrical and magnetic components E_z, H_z in each region can be written as:

$$\begin{aligned} \Delta_\perp E_z^{(i)} - \chi_i^2 E_z^{(i)} &= j\,\rho_i \chi_i^2 / k\varepsilon_i \\ \Delta_\perp H_z^{(i)} - \chi_i^2 H_z^{(i)} &= 0 \end{aligned} \qquad (3)$$

where $\rho_1 = \rho^{(m)}$ is a charge density, $\rho_{i>1} = 0$, ε_i are the dielectric permeability, χ_i are the radial propagation constants, $k = \omega / v$. In vacuum regions ($i = 1,2,5$) $\varepsilon_i = \varepsilon_0$ and $\chi_i = k/\gamma = \lambda$ with ε_0 - the vacuum

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dielectric constant and γ - the Lorenz factor. Transverse components are expressed by the longitudinal ones:

$$\begin{aligned}\vec{E}_\perp &= jk\chi_i^{-2}\left\{\vec{\nabla}_\perp E_z - jv\mu_0\vec{e}_z\times\vec{\nabla}_\perp H_z\right\}\\ \vec{H}_\perp &= jk\chi_i^{-2}\left\{\vec{\nabla}_\perp H_z + jv\varepsilon_i\vec{e}_z\times\vec{\nabla}_\perp E_z\right\}\end{aligned} \quad (4)$$

where $\vec{\nabla}_\perp$ is a transverse part of the gradient operator, and μ_0 is a magnetic permeability of vacuum. The rhs of the equation (3) for the electrical component vanishes everywhere except the beam region with non-zero charge density $\rho^{(m)}$. Therefore the non-zero partial solution exists only in the beam region. For the charge density given by (2), it is simply $E_z = G_1 = j\rho^{(m)}/k\varepsilon_0$. The solution of the homogeneous wave equation in the beam region, which includes the axis $r = 0$, is only modified Bessel functions of first kind since those of the second kind diverge for argument zero. The longitudinal electric and magnetic fields in the beam region are then

$$\begin{aligned}E_z^{(1)}(r) &= F_1 I_m(\lambda r)\cos m\varphi + G_1\\ H_z^{(1)}(r) &= P_1 I_m(\lambda r)\sin m\varphi\end{aligned} \quad (5)$$

In the subsequent regions 2,3,4, the longitudinal field components are given by superposition of modified Bessel functions of both kinds:

$$\begin{aligned}E_z^{(i)} &= F_i R_i(r) + G_i S_i(r)\\ H_z^{(i)} &= P_i R_i(r) + Q_i S_i(r)\end{aligned}, \quad i = 2,3,4 \quad (6)$$

with F_i, G_i, P_i, Q_i unknown coefficients and the functions $R_i(r), S_i(r)$ combined as

$$\begin{aligned}R_i(r) &= K_m(\chi_i a_i)I_m(\chi_i r) - I_m(\chi_i a_i)K_m(\chi_i r)\\ S_i(r) &= K'_m(\chi_i a_i)I_m(\chi_i r) - I'_m(\chi_i a_i)K_m(\chi_i r)\end{aligned} \quad (7)$$

Transverse components (4) expressed with the help of functions

$$\begin{aligned}R'_i(r) &= K_m(\chi_i a_i)I'_m(\chi_i r) - I_m(\chi_i a_i)K'_m(\chi_i r)\\ S'_i(r) &= K'_m(\chi_i a_i)I'_m(\chi_i r) - I'_m(\chi_i a_i)K'_m(\chi_i r)\end{aligned} \quad (8)$$

In the outer region ($i = 5$) that extends to infinity, only modified Bessel functions of the second kind are admissible. The longitudinal fields in outer region is then:

$$E_z = F_5 K_m(\lambda r),\ H_z = P_5 K_m(\lambda r) \quad (9)$$

The unknown coefficients F_i, P_i ($i = 1,\ldots 5$), and Q_i, G_i ($i = 2,3,4$) are defined by the matching conditions. From five field components $E_z, H_z, E_\varphi, H_\varphi$ and H_r which should be matched at transition boundaries, the four ones have been chosen for basic equations system composition, providing the matching at $r = a_i$ ($i = 1,2,3,4$):

$$\begin{cases}E_z^{(i)} = E_z^{(i+1)},\ H_z^{(i)} = H_z^{(i+1)}\\ H_\varphi^{(i)} = H_\varphi^{(i+1)},\ H_r^{(i)} = H_r^{(i+1)}\end{cases} \quad (10)$$

The fifth E_φ component matching is follows automatically.

LONGITUDINAL MULTIPOLES

The system (10) contains 16 linear equations with 16 unknown parameters. The common solution of this system has a complex form and doesn't present here. Nevertheless, after putting $m = 0$ and proceeding the ultra-relativistic limit it transfers to the already obtained results for monopole longitudinal mode [3]. Here we are presenting the ultra-relativistic form of coefficient F_1, valid for any $m > 0$:

$$F_1 = -j\frac{2^m m!}{\pi\varepsilon_0 k\chi_1^m}\left(\frac{2}{a_1^{m+2}} + \frac{a_1^m}{a_2^{2m}}U^{-1}\right) \quad (11)$$

with

$$U = \frac{a_2^2}{m+1} + \frac{m}{\chi_3^2}\frac{\varepsilon_{03}}{\varepsilon_0} - \frac{a_2}{\varepsilon_0\chi_3^2}\left(\chi_3\varepsilon_{03}\frac{R'_3}{R_3} + V\right) \quad (12)$$

where $\varepsilon_{03} = \varepsilon_0 + \varepsilon_3$ and the function V is the combination of the Bessel functions of the first and second kinds that can be analytically evaluated. The analytical presentation of function V is omitted in this paper due to space limit.

The longitudinal component of electric field E_z is obtained by substituting F_1 (11) into (9) and taking the ultrarelativistic limit of modified Bessel function:

$$E_z = -jqr^m a_1^m a_2^{-2m}(\pi\varepsilon_0 kU)^{-1} \quad (13)$$

The mth multipole mode of the longitudinal impedance is then given by:

$$Z_\parallel^{(m)}(k) = -jZ_0(\pi kU)^{-1} \quad (14)$$

with $Z_0 = 377\ \Omega$. For the single-layer tube with infinity wall thickness ($a_4 \to \infty$, $\varepsilon_3 = \varepsilon_4$, $\chi_3 = \chi_4$) the impedance is modified to

$$U = \frac{a_2^2}{m+1} + \frac{\varepsilon_0 + \varepsilon_4}{\varepsilon_0 \chi_4^2}\left(m - \frac{a_2\chi_4 K_m'(a_2\chi_4)}{K_m(a_2\chi_4)}\right) \quad (15)$$

which for $|a_2\chi_4| >> m$ turns to the well-known point-charge truncated longitudinal multipole mode [1]:

$$U = a_2^2(m+1)^{-1} + a_2(\varepsilon_0 + \varepsilon_4)(\chi_4\varepsilon_0)^{-1} \quad (16)$$

On Fig.2 the distributions of the longitudinal modes for $m = 1,2,3$ for stainless-steel (SS) tube with thin inner copper cover $(\Delta = 100nm)$ are presented. The geometry of the tube is: $a_2 = 2mm$, $a_3 = a_2 + \Delta$ and $a_4 = 4mm$.

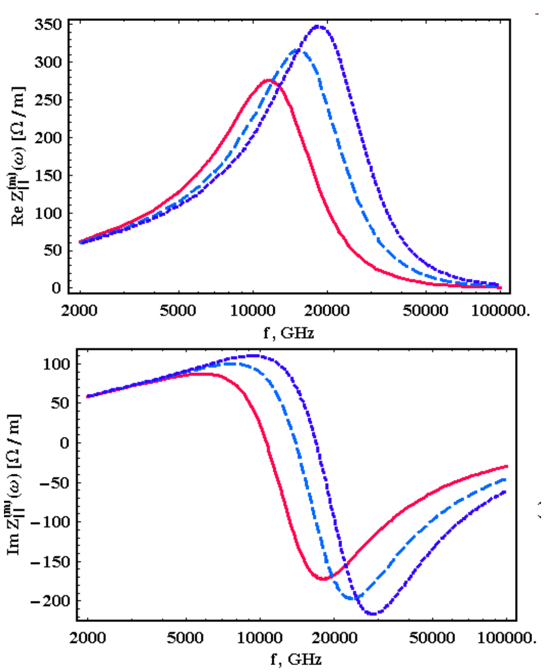

Figure 2: Distributions of real (top) and imaginary (bottom) parts of the longitudinal modes for $m = 1$ (solid), 2 (dashed) and 3 (dotted) for SS-copper tube.

TRANSVERSE MULTIPOLES

Transverse impedance is determined using Panofsky-Wenzel theorem [4]:

$$\vec{Z}_{\perp,m} = k^{-1} Z_{\parallel}^{(m)}(\omega)\vec{\nabla}_{\perp}\left\{\frac{r^m r_s^m}{b^{2m}}\cos m\varphi\right\} \quad (17)$$

The transverse multipole mode frequency dependence is described by the function $Z_{\parallel}^{(m)} k^{-1}\,[\Omega]$. In the ultra-relativistic limit transverse mode is expressed via the coefficient F_1 (11). The distribution of the frequency dependent part of the dipole $(m = 1)$ transverse impedance is given in Fig.3. For comparison shown the transverse impedances for copper and stainless-still tubes.

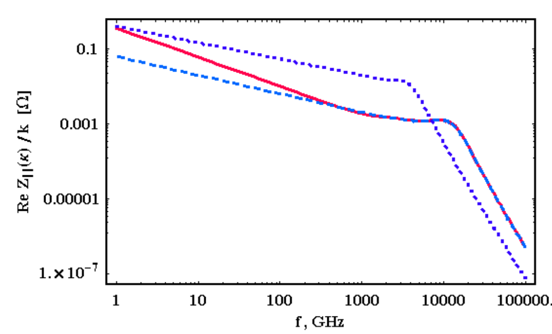

Figure 3: Distribution of the real part of transverse dipole mode (solid). Also shown: copper (dashed) and stainless-steel (dotted) tubes truncated impedances.

As it follows from the Figure 3, the behaviour of the transverse dipole mode impedance is the same as ones for the longitudinal monopole mode [3] and conditioned by the skin depth of inner layer. For the low frequencies the impedance tends to the stainless-steel tube impedance and in the opposite case closes to the copper tube impedance.

CONCLUSION

An exact solution for the longitudinal and transverse impedance multipoles of the point-like charge in two-layer circular tube with finite wall thickness is obtained. The limiting cases for the single layer tube and tube with infinity wall thickness are discussed as well. The solution is valid for both thin and thick layers with arbitrary materials and wall thickness. These results can be used for the small-gap undulator laminated vacuum chamber calculation.

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