IMPEDANCE CALCULATION FOR FERRITE INSERTS*

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Abstract

Passive ferrite inserts were used to compensate the space charge impedance in high intensity space charge dominated accelerators. We study the narrowband longitudinal impedance of these ferrite inserts. We find that the shunt impedance and the quality factor for ferrite inserts are inversely proportional to the imaginary part of the permeability of ferrite materials. We also provide a recipe for attaining a truly passive space charge impedance compensation and avoiding narrowband microwave instabilities.

INTRODUCTION

High intensity high power hadron accelerators serve important functions for neutron sources and muon and neutrino factories. They also have industrial applications in power amplification and atomic transmutation. Particle beams in high intensity accelerators encounter large longitudinal and transverse space charge forces. The transverse space charge force can be alleviated by careful design of the accelerator lattice and a proper choice of betatron tune [1]. On the other hand, the longitudinal space charge force of the beam tends to spread out the bunch distribution to minimize the peak current. If longitudinal space charge is not properly controlled, the beam bunch can fill the beam gap, and this in turn leads to electron cloud instability for high intensity beams.

At the PSR, experiments with such inductive inserts have been performed [2]. With the inserts installed, the resulting longitudinal impedance becomes

$$\frac{Z_\parallel}{n} = -j\left[\frac{Z_0 g_0}{2\beta\gamma^2} - \omega_0 L\right], \quad L = \frac{\mu\ell}{2\pi}\ln\frac{b}{a}, \quad (1)$$

where $Z_0 = 377\Omega$ is the impedance of free space, g_0 is the geometry factor of the space charge impedance, L is the effective inductance of the inductive inserts, a and b are the inner and outer radii of the inductive inserts, μ is the permeability of the ferrite material at low frequency, and ℓ is the total length of inductive inserts.

In 1999 three modules were installed in the PSR, and the longitudinal microwave instability was observed to peak at 72.67 MHz or the $n = 26$ harmonic. Later it was found that heating the ferrite to a temperature of $125°C$ effectively alleviates this instability [3]. Two heated ferrite modules are routinely used in high intensity operation. Although heating can change the properties of ferrite and provide a solution to mitigate the narrowband impedance, it is not the most desirable solution. It would be preferable to provide a truly passive device to counteract the space charge impedance. For this purpose, an analytic understanding of the narrowband impedance of these ferrite cavities would be helpful. This paper will carry out analytic calculation of the TM_{010} mode impedance for these ferrite cavities.

FERRITE INSERTS

For this analytical model and its resulting calculations, we follow the design of the ferrite modules installed in the PSR. A module consists of 30 ferrite rings with inner diameter 12.7 cm, outer diameter 20.3 cm, and thickness 2.54 cm. The ferrite cores line up end to end, so that one module looks like a hollow cylinder of ferrite without end faces. These modules can be treated as pillbox cavities.

The ferrite ring is cylindrically symmetric, we thus use the cylindrical coordinate system. Since only the longitudinal particle motion concerns us, we consider only the fundamental TM_{010} mode, where the electric field is independent of the longitudinal coordinate s. In a uniform isotropic medium, the electromagnetic wave with $e^{j\omega t}$ obeys Maxwell's equation:

$$\frac{\partial^2 E_s}{\partial r^2} + \frac{1}{r}\frac{\partial E_s}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_s}{\partial \phi^2} + \frac{\partial^2 E_s}{\partial s^2} = -\omega^2\mu\epsilon E_s$$

in both free space and the ferrite region. Here, μ and ϵ are permeability and permittivity of the uniform and isotropic medium. The ferrite cavity can be divided into two regions transversally: (1) free space in $r \in (0, a)$, ferrite region in $r \in (a, b)$. A cylindrical conducting beam pipe at $r = b$ encases the ferrite rings and is assumed to be a perfect conductor.

In vacuum, the wavenumber is $k = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c}$, and the electromagnetic fields are

$$E_s = E_0 J_0(kr), \quad H_\phi = j\frac{E_0}{\mu_0 c}J_1(kr).$$

The longitudinal electric and azimuthal magnetic fields depend only on radial distance for the 010 mode.

In the ferrite region, the wavenumber k becomes

$$k_c = \omega\sqrt{\mu\epsilon} = k\sqrt{\epsilon_r(\mu' - j\mu'')}, \quad (2)$$

where ϵ_r is the relative permittivity and μ' and μ'' are the real and imaginary parts of the complex relative permeability, respectively. The intrinsic characteristic impedance of the ferrite medium is $Z_c = \sqrt{\frac{\mu}{\epsilon}} = Z_0\sqrt{\frac{\mu' - j\mu''}{\epsilon_r}}$. The fields in the ferrite are

$$E_s = AH_0^{(1)}(k_c r) + BH_0^{(2)}(k_c r),$$

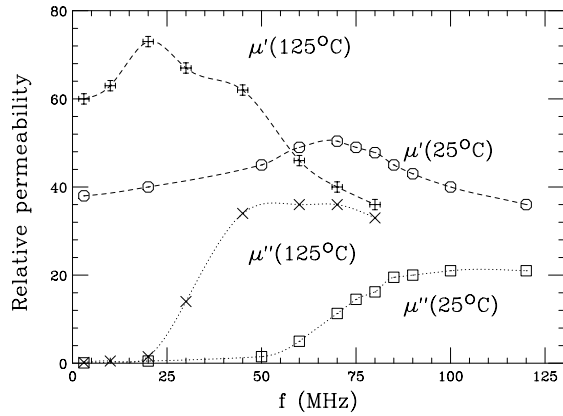$$H_\phi = \frac{j}{Z_c}[AH_1^{(1)}(k_c r) + BH_1^{(2)}(k_c r)],$$

Figure 1: Real and imaginary parts of the complex permeability at different frequencies (From Chris Beltran [3].)

where $H_n^{(1)}$ and $H_n^{(2)}$ are Hankel functions with asymptotic waves $e^{jk_c r}$ and $e^{-jk_c r}$ respectively. The parameters A, B and k_c are determined by the boundary conditions:

$$AH_0^{(1)}(k_c b) + BH_0^{(2)}(k_c b) = 0$$
$$E_s(a^-) = E_s(a^+), \quad H_\phi(a^-) = H_\phi(a^+).$$

To calculate the longitudinal impedance, the general formula is $\Delta V = -IZ_\parallel = -E_s \ell$, where E_s is the longitudinal electric field and ℓ is the total length. Using Ampere's law, $I = \oint H dl = 2\pi a H_\phi$, we obtain

$$\frac{Z_\parallel}{\ell} = -\frac{E_s}{2\pi a H_\phi} = -\frac{\mu_0 c J_0(ka)}{2\pi a j J_1(ka)} \qquad (3)$$

Here the relation between $J_0(ka)$ and $J_1(ka)$ is obtained by the continuity conditions. This formula gives the impedance per unit length on the axis of the cavity as a function of frequency, which is embedded in the k_c.

In general, the permeability of all ferrite materials is a complicated function of frequency. Figure 1 shows the "derived" relative permeability as a function of frequency for Ni-Zn ferrite cores M4C21A [3]. C. Beltran has obtained these values of the complex permeability by fitting the measured S_{11} parameter of the two-port network driven by source frequencies spanning 0 to 120 MHz [3]. Since the complex permeability is measured and analyzed at a discrete number of frequencies, we approximate the intermediate frequencies by linear interpolation as shown in Fig. 1.

Figure 2 compares the impedance per unit length and per unit harmonic of our model and those obtained by the MAFIA code [3]. The real and imaginary curves from the model and from C. Beltran agree quite well in shape and peak location but differ slightly in magnitude. Overall there is good agreement.
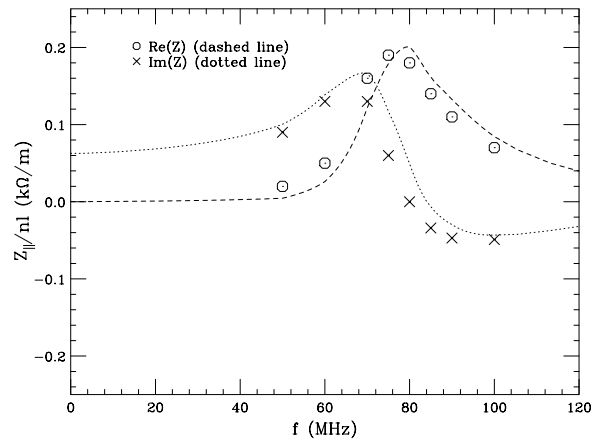

Figure 2: Comparison of real and imaginary impedances from the model and Chris Beltran's MAFIA calculations shown as circles and X's. In this calculation, the ferrite ring parameters are $a = 6.35$ cm, $b = 10.15$ cm, at room temperature of 25°C.

PROPERTIES OF THE IMPEDANCE FOR THE FERRITE INSERT

Since our model calculation agrees well with the numerical calculation of MAFIA, we can study general properties of the ferrite inserts. With the definition of the Hankel function, the impedance can be expressed as

$$\frac{Z_\parallel}{\ell} = j\frac{Z_c}{2\pi a}\frac{J_0(k_c b)Y_0(k_c a) - J_0(k_c a)Y_0(k_c b)}{J_0(k_c b)Y_1(k_c a) - J_1(k_c a)Y_0(k_c b)}, \qquad (4)$$

where $J_n(z)$ and $Y_n(z)$ are Bessel and Neumann functions and $k_c = (\omega/c)\sqrt{\epsilon_r(\mu' - j\mu'')}$. The relative permittivity of the ferrite is fixed at $\epsilon_r = 15$, and the permeability is shown in Fig. 1. The impedance in Eq. (4) has a maximum at a resonance condition given by the zeros of the denominator. Figure 3 shows the impedances for the outer radii $b = 10$ cm, 11 cm, 12 cm, and 12.5 cm respectively. We note that as the outer radius increases, the resonance frequency is shifted lower, and the peak of impedance becomes much larger.

Near the resonance frequency, the magnitude of the numerator is nearly constant, while the real part of the denominator vanishes at the resonance. The resulting impedance can be fitted by an RLC-circuit model:

$$\frac{Z}{n\ell} = \frac{\omega_0}{\omega}\frac{R_{\rm sh}}{1 + jQ(\omega/\omega_r - \omega_r/\omega)}. \qquad (5)$$

Both the shunt impedance $R_{\rm sh}$ and the quality factor Q are inversely proportional to the imaginary part of the permeability.

APPLICATIONS

The passive inductive insert concept is a very useful method to combat the large space charge impedance for
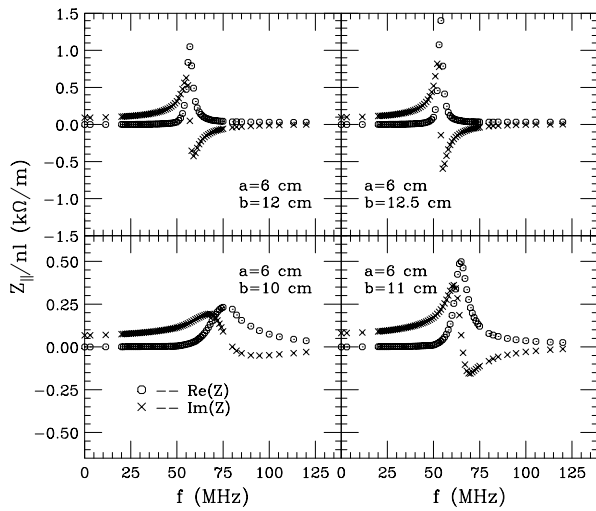
Figure 3: Impedance per unit length per harmonic is calculated for ferrite cores with inner radius 6 cm and outer radii 10, 11, 12, and 12.5 cm. Note that the peak frequency is lowered, and the magnitude of the impedance becomes larger as the outer radius of the ferrite core increases.

Figure 4: Stability comparison of different core thickness for a Gaussian beam. The U' and V' parameters for the 3 room-temperature ferrite inserts (2.286 m) are plotted as circle symbols. The U' and V' of the 2 heated ferrite-inserts (1.524 m) are plotted as cross symbols. The dotted line corresponds to a 2.0 m ferrite insert of $b = 9.0$ cm and 10 cm in 5:1 ratio. The blue solid line corresponds to a de-Qed 3:2:1 combination of 9.0 cm, 9.5 cm, and 10 cm core inserts of a total length of 2.0 m. The red dashed line corresponds to an instability growth rate of 0.4 ms. The red line indicates the stability region.

high intensity low energy accelerators. Since the actual experiment indicates that the narrowband impedance can induce microwave instability and cause beam loss, it becomes important to find a possible solution to de-Q these inductive inserts.

We note that the ferrite cavities installed in the PSR have an impedance of about $R_{sh} \approx 6.5$ kΩ and $Q \approx 3.5$. To de-Q these cavities, we can install cavities with different geometry. We note that the cavities with outer radii b larger than 11 cm have Q values much too large for compensation. Furthermore, the resonance frequency for b larger than 11 cm does not vary as much and thus does not de-Q as easily. The most beneficial solution is to install cavities with 10 cm, 9.5 cm and 9.0 cm outer radii.

To avoid microwave instability induced by a narrowband resonance, the stability threshold of the UV diagram is a useful indicator, where one defines

$$U' + jV' = \frac{eI_0(Z_{||}/n)}{\beta^2 E \delta_{\text{FWHM}}^2 |\eta|} = \Gamma \frac{Z_{||}}{n}. \qquad (6)$$

Using the parameters of PSR: beam energy $E = 1.736$ GeV ($\beta = 0.84$), the peak current $I_0 = 74$ A for 9.0 μC circulating charge with 290 ns bunch length, $V_{\text{rf}} = 14$ kV, the phase slip factor of $|\eta| = 0.185$, the FWHM beam momentum spread for a Gaussian beam of $\delta_{\text{FWHM}} = \sqrt{8 \ln 2} \sigma_\delta$ with $\sigma_\delta = 2.66 \times 10^{-3}$, we obtain $\Gamma = 8.35 \times 10^{-3}$. Assuming the geometry factor $g_0 = 3$, the space charge impedance is $Z_{\text{sc}} = -j196$ Ω. Figure 4 plots V' vs. U' parameters for various geometric combination of ferrite inserts. Points that fall inside the stability curve will be stable, while points that fall outside the curve will be unstable.
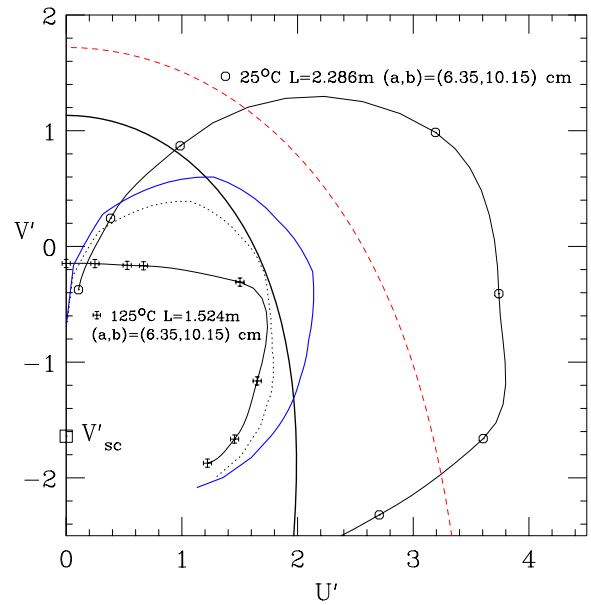
CONCLUSION AND DISCUSSIONS

With this analytic model, we have conducted a systematic study of the properties of ferrite inserts with different geometries. We find that the shunt impedance and the quality factor of the TM$_{010}$ mode are inversely proportional to the imaginary part of the permeability. We also find that the resonance condition is approximately given by $|k_c|(b-a) \approx 2$, where $k_c = (\omega/c)\sqrt{\epsilon_r(\mu' - j\mu'')}$. In fact, these properties can be used to determine the permeability of ferrite materials by carrying out impedance measurements of ferrite cores. Careful calculation before implementing the space charge compensation would be important in minimizing agonizing side effects of passive compensation encountered in the PSR. It is indeed possible to produce fixed ferrite inserts, and install them in a high intensity low energy ring for passive space charge compensation without heating the ferrite.

REFERENCES

[1] See e.g. S. Cousineau, Ph.D. Thesis, Indiana University (2002); and references therein.

[2] M.A. Plum et al., PRST-AB, **2**, 064201 (1999).

[3] C. Beltran, Ph.D. thesis, Indiana University (2003).