THE CODE MBIM2 FOR THE CALCULATION OF THE ARBITRARY MULTIBUNCH BEAMS LONGITUDINAL COHERENT OSCILLATIONS STABILITY (IN THE CASE OF LONG BUNCHES)

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Abstract

The presented code is an advanced version of the code MBIM1 also presented at this conference and dealing with short bunches. The code MBIM2 analyses the stability of longitudinal coherent motion for arbitrary multibunch beams in storage rings without limitations on the bunch length or RF cavities wavelength, which is especially important for higher types of multipole synchrotron oscillations. The code implies also the possibility to consider coupling between different types of multipole synchrotron oscillations and Landau damping. In considered approach, the problem reduces to the eigenvalue problem for the linear algebraic equation system. The order of this system is equal to the number of bunches times numbes of multipole types times approximation order wich appears to be small (a few units) in most cases.

INTRODUCTION

The code MBIM1 (Multibunch Beam Instability, Multipole oscillations, version 1 for short bunches) for calculation of growth rates of coherent instabilities presented in [1] has two important restrictions. The first one is a short bunch approach, which allows to consider only surrounding structure impedances with wavelengthes much greater than the bunch length. The second one is the approach of small coherent frequency shifts in comparison with unperturbed frequency of coherent oscillations, which allows to consider different multipole types of synchrotron oscillations independently from each other and therefore, not considering bunch lengthening.

In order to overcome these two restrictions the method used in [1] was developed for long bunches and realized in the code MBIM2 (version 2 - for long bunches) for the stability analysis of arbitrary multibunch beams multipole synchrotron oscillations (including the case of counterrotating beams).

All details of derivation are given in [2].

INITIAL POINTS

As in [1], we follow here a method developed in [3] for symmetric beams, using a continuum model with the same restrictions as in [3], omitting the approach of short bunches: the sinusoidal oscillations in the absence of excitation are small; the perturbations of the distribution functions due to the interaction are small (as compared with the undisturbed distribution); the amplitude dependence of the synchrotron frequency is taken into account in the first approach of small amplitudes; the undisturbed distribution functions of all bunches are identical - gaussian, with the same length for all bunches, but their currents can be different; smooth focusing with the same betatron tune.

FINAL LINEAR SYSTEM OF EQUATION

We start here from the system of integral equations for all harmonics of the synchrotron frequency for perturbations of distribution functions of all bunches as in [1] (eq.(1)), but keeping coupling between different multipole modes, which was neglected in [1]:

$$(s+in\Omega(J))F_n^l(J,s)-$$
$$-\sum_{j=1}^{N_b}\sum_q\int K_{nq}^{lj}(J,J',s)F_q^j(J',s)dJ'=0, \qquad (1)$$
$$l=1,...,N_b,$$

where $F_n^l(J,s)$ is the n-th multipole harmohic of synchrotron oscillation for the perturbation distribution of the l-th bunch.

The kernel of this system is

$$K_{nq}^{lj}(J,J',s)=e\frac{\partial f_0^l}{\partial J}I_j\sum_m\left\{\frac{n}{m}Z(s-im\omega_0)\times\right.$$
$$\left.\times e^{im(\theta_l-\theta_j)}J_n\left(m\phi_0\sqrt{J/J_0}\right)J_q\left(m\phi_0\sqrt{J'/J_0}\right)\right\} \qquad (2)$$

where $f_0^l(J)$ is the undisturbed distribution function of the l-th bunch, independent from time and phase, which is supposed to be the same (gaussian) for all bunches: $f_0^l(J)=(1/2\pi J_0)\exp(-J/J_0)$; $\phi_0=\sigma_b\sqrt{2}/R$, R is the radius of the storage ring, σ_b is the r.m.s. bunch length (the same for all bunches); $Z(s-im\omega_0)$ is the total impedance of the cavity reduced to the narrow gap; ω_0 is the revolution frequency; I_j is the average current of the j-th bunch; $\Omega(J)$ in the frequency of synchrotron oscillations (at zero current).

But now, unlike [1], the factor $(s+in\Omega(J))$ is not moved into the denominator of the kernel (2), but will remain the factor at $F_n^l(J,s)$ in (1), which simplifies further derivations.

The system (1) will be transformed to a linear algebraic system, using the expansion of $F_n^l(J,s)$ via Laguerre polinomials $L_k^{(|n|)}(x)$ [6], like in [5], for which the weight function is proportional to the gaussian unperturbed distribution:

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$$F_n^l(J) = e^{-x}\sum_{k\geq 0} a_k^{nl} f_k^{|n|}(x), \;\; x = J/J_0,$$

$$f_k^{|n|}(x) = A_k x^{|n|/2} L_k^{(|n|)}(x),$$

$$\int_0^\infty e^{-x} f_k^{|n|}(x) f_m^{|n|}(x) dx = \delta_{km}.$$

Using orthogonality of expansion terms and the Bessel function expansion via Laguerre polinomials [6] (eq.(22.9.16)), taking into account the amplitude dependence of synchrotron frequency $\Omega(J) = \Omega_0(1-\xi J/J_0)$, one can get the following system of linear algebraic equations for the coefficients of distribution function perturbation expansion $D_k^{nl} = (a_k^{nl} - a_k^{-nl})$:

$$\left\{s^2 - (in\Omega_0)^2(\hat{E} - \xi\hat{M}^n)^2\right\}\vec{D}^{nl} -$$

$$-2An\sum_{j=1}^{n_0} I_j \sum_m Z_m(s - im\omega_0) e^{im(\theta_l - \theta_j) - \frac{m^2\varphi_0^2}{2}} \times$$

$$\times \sum_{q>0} \hat{B}^{|n||q|}(-iq\Omega_0)(\hat{E} - \xi\hat{M}^q)\vec{D}^{qj} = 0, \qquad (3)$$

$$B_{k'k}^{|n||q|}(m) = \frac{m(m\phi_0/2)^{2(k+k')+|q|+|n|-2}}{\sqrt{(|n|+k')!k'!(|q|+k)!k!}}, \qquad (4)$$

$$M_{km}^n = \int x W(x) f_k^{|n|}(x) f_m^{|n|}(x) dx =$$

$$= (|n|+2k+1)\delta_{km} - \sqrt{k(|n|+k)}\delta_{k-1,m} -$$

$$-\sqrt{m(|n|+m)}\delta_{k,m-1},$$

$$A = \frac{\Omega}{2q_{rf}V_{rf}\sin\phi_{s0}},$$

where q_{rf}, V_{rf} and ϕ_{s0} are the RF harmonic number, the RF voltage amplitude and the synchronous phase, θ_j are the angular positions of bunches (for the case of counterrotating bunches θ_j are defined as in [1]).

NECESSARY NUMBER OF TERMS

Considering the coefficients $B_{k'k}^{|n||q|}(m)$ (4), one can see that due to factorials in the denominator, the matrix elements decrease rather quickly with increasing k', k and for any upper boundary $\omega_{max} = m_{max}\omega_0$ of the considered RF spectrum the necessary order of approximation k_{max} can be estimated as

$$k_{max} \approx (m_{max}\phi_0/2)^2. \qquad (5)$$

For short bunches, at $m_{max}\phi_0 << 1$, the zero order of approximation $k_{max} = 0$ gives the accurate result - the same as in [1] (without Landau damping).

Note that the elements of the matrix $\hat{M}^n$ describing the effect of amplitude dependence of the synchrotron frequency, do not decrease as well with increasing k. One can show (see [2]) that neglecting interaction with RF elements and considering the solutions of (3) taking into account only $\xi\hat{M}^n$, we can get with increase k_{max} the more wide and the more dense spectrum of solutions, which evidently describes incoherent motion with continuous spectrum. But in the case of coherent oscillation, the term $\xi\hat{M}^n$ only gives an correction to the main solution determined by interaction with RF system, and the necessary value of k_{max} can be defined by (5) or should be increased if coherent frequency shifts are of the order or less than $\xi\Omega_0$.

THE CODE MBIM2

The method given above is realized in the computer code MBIM2 (MultiBeam Instability, Multipole oscillations, version 2 for long bunches). The code solves the problem of longitudinal coherent oscillations for the case of long bunches, taking into account coupling of neighbour multipole types of synchrotron oscillations and Landau damping.

For resonant eigen modes of RF cavities spectrum the method of analytical summation of the series over azimuthal harmonics given in [7] is applied, in order to improve the accuracy of summation, especially for 1)low quality factors; 2)different charges of bunches; 3)at considering internal bunch motion.

The order of the problem (that is the order of the considered equation system) is equal to the number of bunches with a nonzero charge times approximation order times number of different multipole types considered simultaneously.

AN EXAMPLE

Consider a simplest case of interaction of the symmetric multibunch beam with only one RF cavity mode, in dependence on its quality factor.

For a symmetric beam, the whole matrix of the problem splits for N_b independent matrixes for each normal symmetric mode. If the width of the considered impedance $\Delta\omega_r$ is much more than $N_b\omega_0$, then the solutions for all symmetric modes coincide and the multibunch beam has the same growth rates and the same threshold current of one bunch $I_1 = I_N / N_b$ as that of the singlebunch beam I_0. If $\Delta\omega_r < N_b\omega_0$, then the solutions for symmetric modes are splitted, the threshold currents for some modes decrease and for others - increase in comparison with the single-bunch beam. Finally, for the narrow resonance $\Delta\omega_r << N_b\omega_0$, exciting only one of symmetric modes, the threshold current of the whole multibunch beam I_N coincides with the threshold current of the single-bunch beam I_0. This dependence is shown on the Fig.1 (for $\omega_r/\omega_0 = 1000$, $N_b = 5$).

COMPARISON OF THE SOLUTION WITH LANDAU DAMPING WITH THE SHORT BUNCH APPROACH

When $\sigma_b << \lambda_{min}$ and the calculated complex frequency shifts are much less than the synchrotron frequency, both codes (MBIM1 [1] and MBIM2) could be used. In this case, the dependence on the impedance remains only in the matrix elements of the zero order of approximation, with $(k,k') = (0,0)$. But for accurate description of Landau damping, we should keep higher order of the matrix M.

Fig. 2 shows the lines of equal maximal growth rates (for a simplest case of one bunch and dipole synchrotron oscillations) on the complex plane of the variable $\lambda = A\sum_m mZ_m(-i\Omega - im\omega_0)$, in units of the synchrotron frequency spread $\xi\Omega$. This picture is shown for the large approximation order $k_{max} = 40$ and for the model of short bunches [1].

The higher the order of approximation, the more similar is the picture to the case of short bunches, at small growth rates. Such high approximation order is necessary only for $|\lambda| \sim \xi\Omega$ and the greater $|\lambda|$ the lesser approximation order is sufficient for good approximation. At large growth rates the same result can be obtained with much smaller order of approximation.

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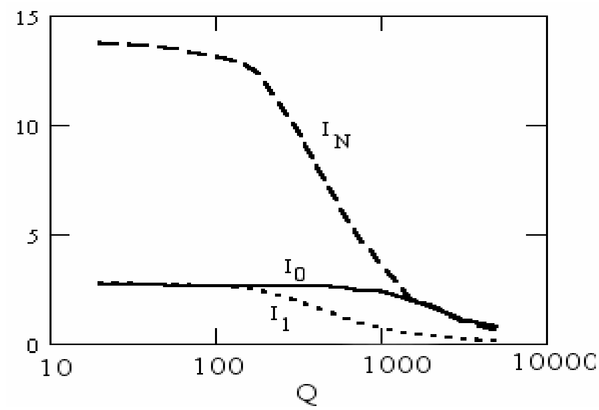

Figure 1: The dependence of the threshold current of the whole multibunch beam I_N, of its one bunch I_1 and of the singlebunch beam (I_0) on the quality factor Q.

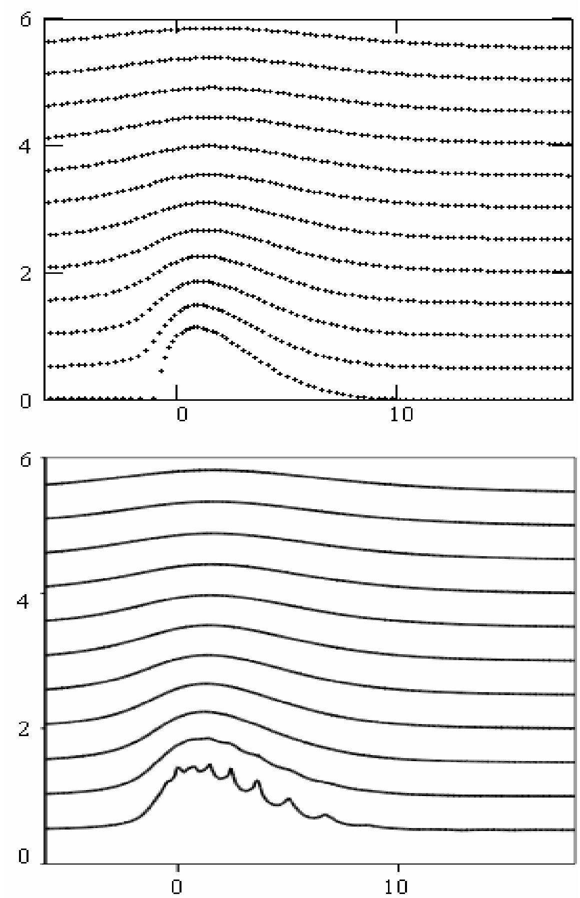

Figure 2: The lines of equal maximal growth rates on the complex plane of the variable $\lambda = A\sum_m mZ_m(-i\Omega - im\omega_0)$, obtained with the approach of short bunches (above) and with the method of present paper, for approximation orders $k_{max} = 40$ (below).