SIMULATION OF THE EFFECT OF AN IN-VACUUM UNDULATOR ON THE BEAM DYNAMICS OF ALS*

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Abstract

The femtoslicing project at the Advanced Light Source (ALS) requires that a short period (3 cm) and narrow gap (5 mm) in vacuum undulator to be installed. The combination of the short period and the narrow gap raised concern of the impact on the beam dynamics. A 3D field model was established based on numerical data using 8 longitudinal and 4 transverse harmonics. At first fourth-order symplectic integrator was used. It was to our surprise that the dynamic aperture decreased by 30%. To understand the cause of the drastic change in the dynamic aperture, the field model was implemented in a differential algebraic code and the Taylor map of the undulator was obtained. Tracking result using the Taylor map showed little change in the dynamic aperture, which was latter corroborated using the symplectic integrator with 150 slices per period (as opposed to 10 before). Yet it is simply too time consuming to use the symplectic integrator with such thin slices. For this case, Taylor proves to be a much faster alternative.

INTRODUCTION

With the advancement of ultrafast science, new ways of reaching shorter and shorter time scale are being developed, one of which is the femtoslicing project under commissioning at the Advanced Light Source (ALS). The basic idea is that a thin slice ($\sim$50 fs) of the electron is energy modulated in a wiggler through interaction with a femtosecond laser pulse and generates an ultra-short X-ray pulse in a down stream bending magnet or undulator where dispersion is present [1]. The feasibility of this concept has been experimentally demonstrated at the ALS using a bending magnet to produce the X-ray pulse [2]. The femtoslicing project utilizes an in-vacuum undulator as the radiator and poses many challenges to the operation of the ALS [3], one of which is the effect of the new insertion devices on the beam dynamics of the storage ring. Based on the experience at SSRL [4], great attention has been paid to minimizing transverse field roll off during the design stage [7]. Simulation study of the 11 cm period wiggler showed that the precaution taken at the design stage ensured that it has little effect on the beam dynamics (for result using a crude field model see ref. [5]; subsequent unpublished study using the same model described in this work confirmed the earlier conclusion.).

In the present paper, we will report the simulation results of the impact of the 3 cm period undulator on the beam dynamics, along with technical improvements that convinced us the validity of those results. In particular, we will discuss the analytical field model that fits the numerical data well in a large volume and the Taylor map method that, compared to the more traditional symplectic integration method, greatly speeds up the tracking simulation and allows integration through the undulator with much smaller step size.

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FIELD MODEL

Since the in-vacuum undulator has a narrow pole width, a narrow gap, a relatively high field (close to 1.5 T) and a short period, both the transverse and the longitudinal roll off are rather steep, which means strong high order harmonics in both the transverse and the longitudinal dependence of the field. As a result, we decided to adopt an approach to find an analytical model of the numerical field data similar to that developed by Sagan *et. al.* [6]. Due to the fact the undulator contains 50 periods, the effect of the ends should be relatively weak compared to that of the main body. Hence the end is modeled using a half period sinusoidal undulator without transverse roll off and with the period half of the main body, which is 1.5 cm. The result is an analytical model with fewer harmonics. Specifically, the model can be expressed as

$$\frac{B_y}{B_0} = -\sum_{m,n} h_{xm} h_{zn} \cos(k_{xm}x)\cosh(k_y y)\cos(k_{zn}z)$$

$$\frac{B_x}{B_0} = \sum_{m,n} \frac{h_{xm} h_{zn} k_{xm}}{k_y} \sin(k_{xm}x)\sinh(k_y y)\cos(k_{zn}z)$$

$$\frac{B_z}{B_0} = \sum_{m,n} \frac{h_{xm} h_{zn} k_{zn}}{k_y} \cos(k_{xm}x)\sinh(k_y y)\sin(k_{zn}z)$$

where $k_y = \sqrt{k_{xm}^2 + k_{zn}^2}$, $k_{xm} = mk_{x0}$ and $k_{zn} = (2n-1)k_{z0}$. Note that this model assumes the ideal geometry of the magnet blocks and hence the phase of each harmonic is zero and only odd harmonics appear in the z dependence. It should also be noted that the function of the field on the transverse coordinates is not periodic. Yet we can still obtain a reasonable fit within the domain of interest where the field only change a few percent. Furthermore, the degree of freedom of the amplitude of the harmonics is restricted to the tensor product of two vectors, which implies that the horizontal and longitudinal dependence are more or less separable.

In the case of the in-vacuum undulator, we found that a model with 4 horizontal and 8 longitudinal harmonics is sufficient. Together with k_{x0}, there are 13 free parameters. Due to the symmetry, we only have to fit our model to the numerical data within the volume of one quadrant in the transverse plane times one quarter of the period. The size of

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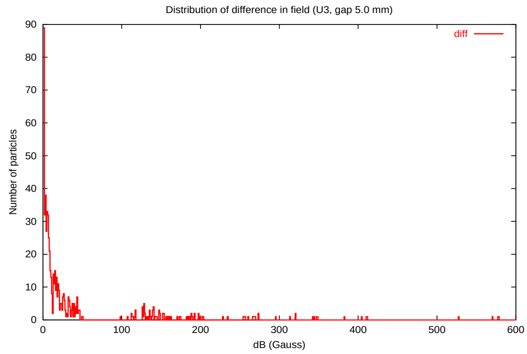

Figure 1: Histogram showing the difference between the numerical and fitted analytical field data.

the volume is 1.5 cm horizontally, 2 mm vertically and 0.75 cm longitudinally, containing 620 data points. The result is shown in Fig. 1 and the rms of the difference between the analytical model and the numerical data is 20 G.

SIMULATION TOOLS

The main simulation tool used to study single particle dynamics of the ALS is frequency map analysis, which has been validated experimentally [9, 10]. Specifically, we take advantage of the MATLAB environment of the symplectic tracking code Accelerator Toolbox [11] and attached the frequency map analysis code as a post processor [10]. For the in-vacuum undulator, we initially adopted the fourth-order symplectic integrator developed by Wu *et. al.* [8] and subsequently developed the code that computes the Taylor map of the undulator and tracks electrons through it, for reasons that will be discussed below. The code that computes the Taylor map was written as an addition to the code COSY INFINITY [12], which contains a DA package that computes the Taylor map and a seventh order Runge-Kutta integrator with automatic step size control that ensures adequate precision [13]. After the field model described above and the equations of motion in the Cartesian coordinates were implemented, we found that, for similar number of steps, the Taylor map computed in the Cartesian coordinates is slightly more symplectic than that obtained in the curvilinear coordinates. Yet the main reason for using the Cartesian coordinates is that it does not assume midplane symmetry, which restricts the orbit to a plane. The code that tracks electrons through a Taylor map was developed for the Accelerator Toolbox so that the infrastructure can be reused, bringing together the strengths of both packages.

RESULTS AND DISCUSSION

During the simulation study, the ALS lattice is set up in the way that closely reflects the reality of the routine operation. The quadrupole errors due to the setting errors of the power supplies and orbit were obtained through response matrix measurement; skew quadrupoles are set to the values used in user operation to blow up vertical beam size through vertical dispersion; two pairs of quadrupoles next to the undulator are adjusted to minimize beta-beating; and the tunes and chromaticities are brought back to the nominal values of (14.25, 8.20) and (0.4, 1.4). The only thing that is not included is the large vertical dispersion bump across the undulator which is required by the femtoslicing technique [3] so that the effect of the undulator itself can be studied first.

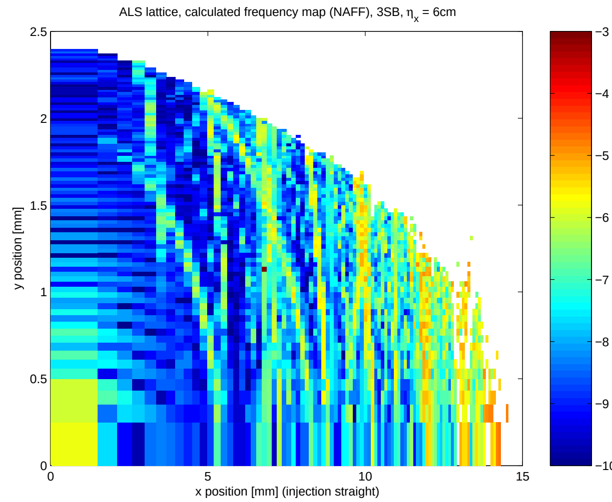

Figure 2: On-momentum frequency map analysis with the undulator off.

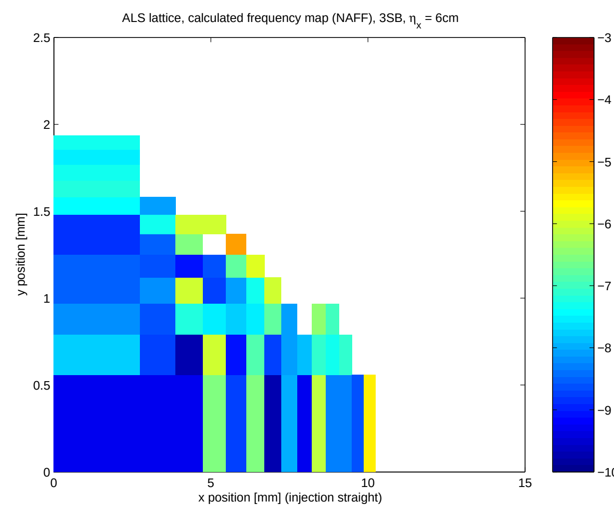

Figure 3: On-momentum frequency map analysis with the undulator on, using the 3D fitted analytical model.

The results are presented in Fig. 2-6, showing the frequency maps of the electrons of the design energy (1.9 GeV). Fig. 2 shows the case when the undulator is off, where the dynamic aperture is about 15 mm. Since the injection septum is about 9 mm of the center of the design orbit, the dynamic aperture is large enough for efficient injection from the booster ring. When the undulator is turned on, the dynamic aperture is reduced to around 10 mm, as shown in Fig. 3. In order to determine the source of the reduction of the dynamic aperture, transverse roll off of the field was turned off and the dynamic aperture remain essentially unchanged (see Fig. 4), indicating that the longitudinal roll off is the source of reduction. Yet the question remains whether the reduction is real or not. Since the highest harmonic is the 15th and the step size is one tenth of a period, we felt that the same study should be done with smaller step size. The symplectic integration method, on the other hand, was too slow to get an answer in a few days, since it took more than 20 hours to track 600 thousand particle turns with 10 steps per period (20 minutes without the undulator). Hence the Taylor map method was implemented and it takes less than 30 minutes to compute the 7th order map with more than 5000 steps per period and 10 hours to track 10 million particle turns. The result, as shown in Fig. 5, clearly demonstrated that the reduction of the dynamic aperture is an artifact of the numerical simulation. This conclusion was confirmed by the simulation using the symplectic integration method with 150 steps per period (Fig. 6). From hindsight, we believe that cause of the artificial reduction is due to the aliasing problem, where the 9th and 11th harmonic are close to be in phase with the integration steps, although more detailed studies is needed to understand it clearly.

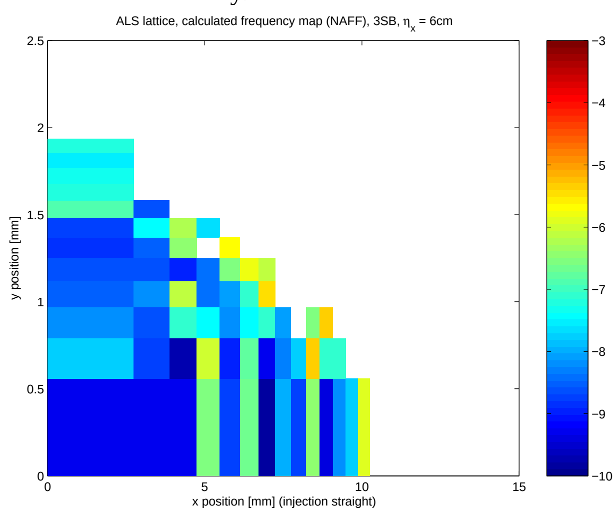

Figure 4: On-momentum frequency map analysis with the undulator on but no transverse field roll off.

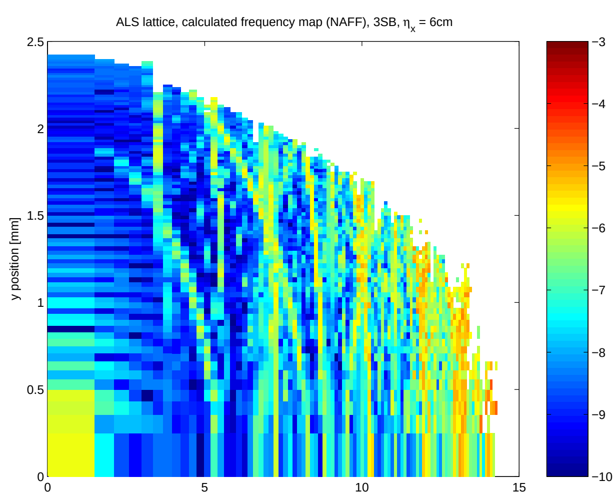

Figure 5: On-momentum frequency map analysis with the undulator on, using the 3D fitted analytical model and the 7th order Taylor map.

In conclusion, a 3D analytical model of the magnetic field was developed for the in-vacuum undulator installed recently at the ALS. Using the Taylor map of the undulator as an alternative to the traditional symplectic integrator, we found that the simulation was sped up by 2 orders of magnitude. Finally, the adoption of the Cartesian coordinates removes the constraint on the orbit and opens up future possibility of computing the maps of more complicates devices such as the elliptically polarized undulators (EPU).

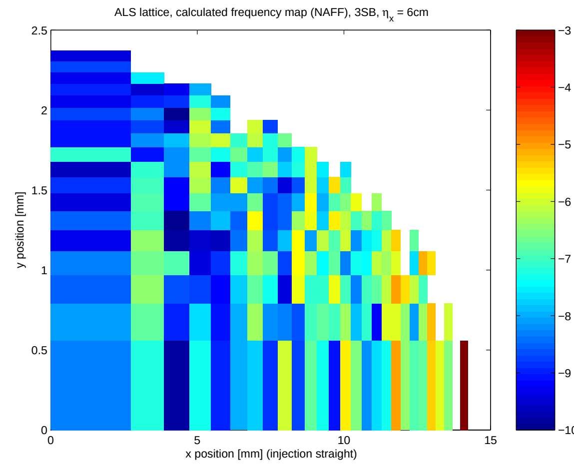

Figure 6: On-momentum frequency map analysis with the undulator on, no transverse field roll off and 150 slices per period.

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