EFFECTS OF THE PASSIVE HARMONIC CAVITY ON THE BEAM BUNCH

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Abstract

In this paper, we present a computer tracking code, which can investigate the bunch length, energy spread and the threshold current of Robinson instability under the influence of the passive harmonic cavity. The effects of the radiation damping, quantum excitation and the beam loading of the harmonic cavity are included in the computation. The calculated result shows that the beam has a constant energy spread and blows up as the beam current increases from below to over the threshold current of the Robinson instability. It also indicates that the shunt impedance of the harmonic cavity is critical for whether the harmonic cavity can reach the designed goal, a stable and lengthening beam at the design beam current.

INTRODUCTION

The harmonic cavity has been known as a approach to lengthen the bunch and increase the beam Tousheck life time since it was proposed two decades ago. And its positive results have also been demonstrated in several light source facilities [1].

The analytic model [2] can describe the mechanism that makes the beam lengthening and keeping constant energy spread in the main and harmonic cavities. In order to explain the side effect that was observed in the harmonic rf system, the computer simulation [3] was introduced to study the Robinson instability that is caused by the harmonic cavity.

In the previous study, each bunch is modelled as a macro-particle. In this paper, we present a particle tracking code that describes each bunch as made-up by a number of micro-particles. The influences of the radiation damping, quantum excitation and the beam loading of the harmonic cavity are included in the calculation. By tracing the time and energy deviations of the micro-particles in the synchrotron motion, we can calculate the energy spread and the bunch length of different beam currents and then determine the threshold current of the Robinson instability at a tuning angle setting for the passive harmonic cavity. The equations used in the calculation are represented in this paper. And a example, calculated with the equations, are demonstrated here to explain the effects of the harmonic cavity on the beam bunch.

FORMALISM FOR CALCULATION

The mainly influence of the harmonic cavity is on the longitudinal motion of the beam bunch, not transverse motion. We can obtain the effects of the harmonic cavity by adding the harmonic voltage term to the equations of the longitudinal motion of the beam bunch.

RF Voltage Induced by Beam Bunches in a Cavity

As a charge particle moves through a cavity, the charge particle will loss its energy and excite the rf field of the fundamental and the high order modes. If we consider the fundamental mode only, the energy loss of the charge particle, passed a cavity, is

$$\Delta E = -\frac{R_s \omega_c}{2Q_0} q^2 = V \bullet q \tag{1}$$

where Q_0 is the unloaded quality factor, R_s is the shunt impedance, ω_c is the cavity resonant frequency, V is the voltage acting on the charge. it is important to note that according the fundamental theorem of beam loading [4], only half of the induced voltage acts back on the charge itself, and that the induced voltage will decay exponentially with a time constant,

$$T_d = 2\frac{Q_L}{\omega_c} \tag{2}$$

where Q_L is the loaded quality factor of the cavity. After a charge passed through the cavity, the rf voltage can be expressed as

$$\overrightarrow{V_q}(t) = -\frac{R_s \omega_c}{Q_0} \bullet q \bullet \exp(\imath \omega_c t - \frac{t}{T_d}) \tag{3}$$

From (3), We can calculate the rf voltage, which is induced in a cavity by the passed charge and is seen by the passing charge, by summing the excited rf voltage from infinite time ago [5],

$$\overrightarrow{V}(t_m) = -\frac{\omega_c R_s}{Q_0}\{\sum_{n=-\infty}^{n<m} q_n \exp[\imath\, \omega_c (t_m - t_n)] \cdot \exp(-\frac{t_m - t_n}{T_d})\} - \frac{\omega_c R_s}{2Q_0} q_m \tag{4}$$

Difference Equations for the Synchrotron Motion

The longitudinal motion of a beam bunch in a storage ring can be described by the parameters, the energy and time deviations. The equations of the longitudinal motion are expressed below:

$$\frac{d\varepsilon}{dt} = \frac{q}{T_0}[V_{acc}(t_s + \tau) - U_{rad}(\varepsilon)] \tag{5}$$

$$\frac{d\tau}{dt} = \alpha \frac{\varepsilon}{E_s} \quad (6)$$

where T_0 is the revolution period of the synchronous charge, V_{acc} is the total accelerating voltage of the main and the passive harmonic cavity, α is the momentum compaction factor, E is the electron energy, the subscript s represents the quantity for the synchronous charge. U_{rad} is the radiation loss. The radiation loss in a single turn is given by

$$U_{rad}\,[KeV] = 88.46 \frac{E\,[GeV]^4}{\rho}\,[m] \quad (7)$$

$$= 26.520308 E\,[GeV]^3\,B\,[T] \quad (8)$$

where ρ is the radius of curvature of the orbit in the bending magnet and B is the bending magnetic field. (5) and (6) indicate that a beam bunch with a more positive energy deviation will radiates more energy and take more time in one revolution. it is that makes the bunch having damping effect in the longitudinal motion. The radiation loss expressed as (7)is based on the classic view, which supposes the synchrotron radiation is emitted with a continuous spectrum of frequency. If we consider that the electromagnetic radiation occurs in the quanta of discrete energy, then we need to add a additional term to (5) for the quantum excitation effect [6]. By adding the quantum excitation effect and rewriting (5) and (6) for one turn period, we can obtained the equations for describing the longitudinal motion of the charge particles turn-by-turn:

$$\varepsilon_{j+1} = \varepsilon_j + eV_{acc}(t_s + \tau_j) - eU_{rad}(\varepsilon_j) + 2\sigma_e \left(\frac{3U_s}{2E_s}\right)^{\frac{1}{2}} R_j \quad (9)$$

$$\tau_{j+1} = \tau_j + \alpha T_0 \frac{\varepsilon_{j+1}}{E_s} \quad (10)$$

where σ_e is the nature electron energy spread and R_j is the random number, whose average is zero, root mean square is equal to 1, the subscript j represents the revolution number. The accelerate voltage $V_{acc}(t_s + \tau_j)$ here is composed by the voltages of the main and the harmonic cavity,

$$V_{acc}(t_s + \tau_j) = V_M(t_s + \tau_j) + V_H(t_s + \tau_j) \quad (11)$$

Here we use the subscript M and H to represent the parameters that are related to the main cavity and the harmonic cavity respectively.

If we do not consider the beam loading and the noise sidebands of the main cavity, then we can express the voltage of the main cavity as a regular sinusoidal signal,

$$\overrightarrow{V_M}(t_m) = |\overrightarrow{V}_M| \exp\{\imath[(\omega_{rf} t_m) + \phi_s]\} \quad (12)$$

The initial harmonic voltage can be set as that induced by the synchronous charge, which can be calculated from (4) or be derived from the equivalent circuit of a cavity [7]:

$$\overrightarrow{V}_H(\tau = 0) = -\frac{2 I_b R_s}{1+\beta} \cos(\psi) \exp(\imath\psi) \quad (13)$$

Where β is the coupling factor of the harmonic cavity and ψ is the tuning angle of the cavity. which is a measure for the phase difference of the excited rf field and the rf generator current. It can be written as

$$\tan(\psi) = -2Q_L \frac{\omega_{rf} - \omega_c}{\omega_c} \quad (14)$$

where ω_{rf} is the rf harmonic frequency.

The synchrotron phase ϕ_s in (12) is set by the condition: the radiation loss of a synchronous particle is exactly compensated by the accelerating voltage,

$$V_{acc}(t_s) = U_{rad}(\varepsilon = 0) \quad (15)$$

EFFECTS OF THE HARMONIC VOLTAGE

The harmonic cavity displays its effect by causing the change in the energy and phase of the charge particles of the bunches. As we calculate the deviations of the energy and time with (9) and (10), we model each beam bunch as the composition of a number of micro-particles. But for simplicity, we treat each beam bunch as a macro-particle as we calculate the harmonic voltage with (4).

In the following sections, we present a example to demonstrate the calculation for the effects of the harmonic cavity. The parameters for the example are listed below:

Table 1: Parameters of the example

Beam energy (GeV)	1.5
Main rf frequency (MHz)	500
Main rf voltage (kV)	800
Radiation loss / turn (keV)	168
Natural energy spread ratio, σ_e/E_0	6.6×10^{-4}
Momentum compaction factor	6.768×10^{-3}
Revolution frequency (MHz)	2.5
Harmonic cavity frequency (MHz)	1500
Harmonic cavity impedance ($M\Omega$)	$3/\sqrt{3}$
Harmonic cavity quality factor	$36000/\sqrt{3}$
β of the harmonic cavity	0

Energy Spread and Robinson Instability

Figure 1 presents that the threshold of the Robinson instability increases with the tuning angle of the harmonic cavity as the resonant frequency of the harmonic cavity is tuned to above the rf harmonic frequency. And the threshold value becomes to be higher as the harmonic cavity is tuned to other side, negative tuning angle. That is consistent with the suggestion from the study about the Robinson instability: the beam will be stable if the cavity is tuned to below the beam harmonic frequency.

The analytic model predicts that the harmonic cavity can lengthen the bunch without increasing the energy spread. That is shown in figure 1 for the beam current that is below the threshold of the Robinson instability.

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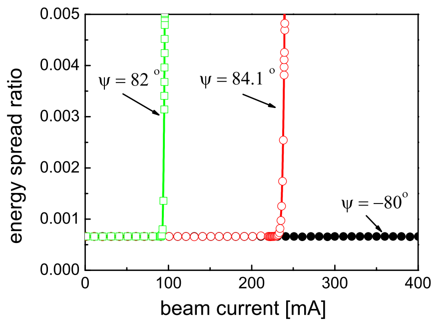

Figure 1: The energy spread ratio at different tuning angle for the currents, which are below and over the threshold of the Robinson instability. The number of the harmonic cavity is one. The bunch number is one.

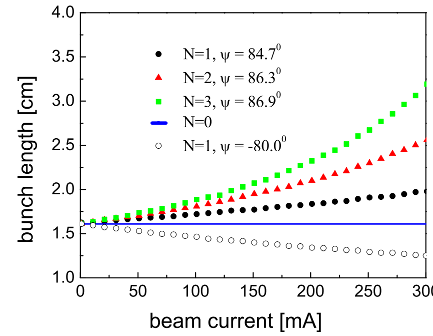

Figure 2: Bunch length versus the beam current for different N, the number of the harmonic cavities. They are calculated by (16) and are compared with the solid line that is calculated by (17).

Bunch Length

From the time deviation of the micro charge particles of the bunch in the synchrotron motion, we calculate the bunch length with the following equation,

$$\sigma_z = 2.0[\frac{1}{M}\sum_{i=1}^{i\leq M}(\tau(i)-\tau_0)^2]^{0.5} \bullet c \qquad (16)$$

where c is light velocity, τ_0 is the time displacement of the bunch center. The strength of the effect of the harmonic cavity is current dependant. The influence of the harmonic cavity is strengthened with the beam current. As the beam current is close to zero, the bunch length with the harmonic cavity should be equal to that calculated by (17), which do not include the harmonic voltage[8],

$$\sigma_z = \frac{2.0\alpha}{\Omega E_0}\sigma_\varepsilon \bullet c \qquad (17)$$

where Ω is the synchrotron oscillation frequency in radius.

From the analytic model, we know that the bunch will be lengthen if the tuning angle is set to positive value and will be shorten if set to other side. That can be seen in figure 2.

The bunch can be maximally lengthened and the bunch charge density can be minimized as the voltage of the main and harmonic cavities has a widest flattened region around the synchronous phase. That condition need the harmonic cavity or cavities that can induce the voltage, which is high enough to cancel the slop of the main rf voltage at the operation current, and keep the beam in stable. More harmonic cavities has higher shunt impedance to excite the required voltage at a larger tuning angle. It makes more number of the harmonic cavities, as shown in figure 2, owing the advantage to obtain a lengthened and stable beam.

CONCLUSIONS

This paper has presented a method to calculate the bunch length and energy spread under the influence of the passive harmonic cavity. It provides a way to evaluate whether the total shunt impedance of the harmonic cavities is high enough to achieve its design goal: a stable beam with the required bunch lengthening.

From the calculation, we can obtain the detail charge distribution of the bunch on the longitudinal dimension, which may be helpful for improving the accuracy in the calculation of the Tousheck life time. Furthermore if we add a phase or amplitude modulation to the voltage of the main cavity, then we can use this tracking code to examine the influence of the noise modulation on the effects of the harmonic cavity.

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