MEASUREMENT AND CORRECTION OF NONLINEAR CHROMATICITY IN RHIC*

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Abstract

To improve luminosity in RHIC by using smaller β^*, higher order chromatic effects may need to be corrected [1]. Measuring of higher order chromaticities is discussed and compared to a model of RHIC, showing agreement. Assuming round beams, four families of octupoles are used to correct the second order chromaticities while keeping under control the amplitude dependent betatron tune spread in the beams. We show that the octupoles can reduce the second order chromaticity in RHIC, but they have insufficient strength for complete correction.

THEORY

Chromaticity is due to the tunes in an accelerator changing with the beams momentum [2]. This relationship can be expressed as:

$$\nu = \nu_0 + \xi_1\delta + \xi_2\delta^2 + \cdots$$

for either plane where $\delta = \Delta p/p$, p is the longitudinal momentum of the beam, ξ_1 is the linear chromaticity, ξ_2 is the second order chromaticity, plus higher order terms. Here, one assumes that the beams motion is uncoupled in both planes.

Second order chromaticity can be changed with either sextupoles or octupoles. The next section describes the using four families of octupoles.

Octupole Correction

The contribution to the Hamiltonian due to the octupoles is:

$$\Delta H = [\frac{1}{4}x^4 - \frac{3}{2}x^2y^2 + \frac{1}{4}y^4]b_3$$

where b_3 is the strength of the octupole.

Applying the following change of variables to the Hamiltonian:

$$x = \sqrt{2J_x\beta_x}\cos(\phi_x) + \delta\eta_x$$

and

$$y = \sqrt{2J_y\beta_y}\cos(\phi_y) + \delta\eta_y$$

where $\eta_{(x,y)}$ are the dispersion functions, $\beta_{(x,y)}$ are the beta functions, $J_{(x,y)}$ are the actions and $\phi_{(x,y)}$ are the phases. The tune changes – to linear order in the octupole strength – are as follows:

$$\Delta\nu_{(x,y)} = \frac{1}{2\pi}\oint\frac{\partial\Delta H}{\partial J_{(x,y)}}ds$$

* Work performed under Contract Number DE-AC02-98CH10886 with the auspices of the US Department of Energy.

This leads to (with $J = J_x = J_y$ for round beams):

$$\Delta\nu_x = \frac{1}{2\pi}[\frac{3}{4}J\langle(\beta_x^2 - 2\beta_x\beta_y)b_3\rangle + \frac{3}{2}\langle\beta_x(\eta_x^2 - \eta_y^2)b_3\rangle\delta^2]$$

$$\Delta\nu_y = \frac{1}{2\pi}[\frac{3}{4}J\langle(\beta_y^2 - 2\beta_x\beta_y)b_3\rangle + \frac{3}{2}\langle\beta_y(\eta_y^2 - \eta_x^2)b_3\rangle\delta^2]$$

where $\langle\cdots\rangle$ denotes integration for one revolution and terms that have odd powers of the phase terms were assumed to cancel to zero.

Thus, the four families of octupoles satisfy the following four equations to introduce a change in the second order chromaticity:

$$\Delta\xi_{x_2} = \frac{3}{4\pi}\langle\beta_x(\eta_x^2 - \eta_y^2)b_3\rangle$$

$$\Delta\xi_{y_2} = \frac{3}{4\pi}\langle\beta_y(\eta_y^2 - \eta_x^2)b_3\rangle$$

$$0 = \langle(\beta_x^2 - 2\beta_x\beta_y)b_3\rangle$$

$$0 = \langle(\beta_y^2 - 2\beta_x\beta_y)b_3\rangle$$

MEASUREMENT

RHIC consists of two rings named Blue and Yellow. In the Blue ring, the beam goes clockwise and for the Yellow ring, the beam is counter-clockwise. The chromaticites were measured in both rings. To measure the chromaticity,

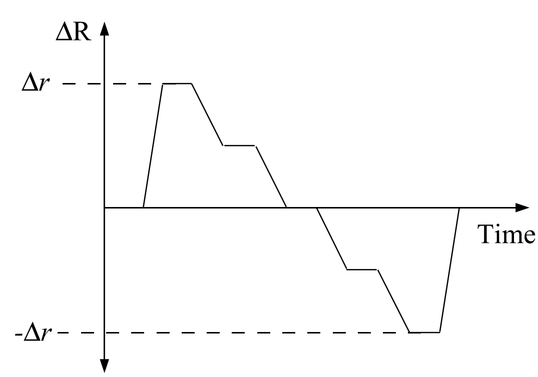

Figure 1: A schematic for the changes to the radial steering during a measurement. A tune measurement is made at each plateau. The number of steps is adjustable. Δr is the maximum size of the radial shift.

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Table 1: Second order chromaticity comparison

Run#	Ring	Δr [mm]	X [meas]	X [model]	Y [meas]	Y [model]
4207	Yellow	2.0	12 ± 26	137	1332 ± 28	1141
			-13 ± 44		1661 ± 119	
			94 ± 27		1563 ± 32	
			448 ± 33		1452 ± 25	
4454	Blue	2.0	410 ± 19	844	855 ± 24	940
			346 ± 27		838 ± 62	
			365 ± 17		924 ± 48	
			292 ± 31		871 ± 21	
Averaging measured values for a comparison						
4207	Yellow		134 ± 179	137	1460 ± 105	1141
4454	Blue		361 ± 40	844	870 ± 26	940

the tunes are measured at different momenta. These tunes are then fit to a polynomial as a function of the momenta using linear regression. A statistical error is also computed. The momentum of the beam is changed by changing the frequency of the RF cavities. In RHIC, one can request a particular radial steering, the RF cavity frequencies are then changed to produce this radial shift. A schematic for radial steering is shown Fig. 1. For aperture reasons we have not used more than 2*mm*.

In a constant magnetic field, the change in the momenta can be found from the radial shift using [2]:

$$\frac{\Delta R}{R} = \alpha \frac{\Delta p}{p}$$

where $\alpha = 1/\gamma_T^2$ is the momentum compaction factor and γ_T is beam γ at transition. For RHIC at store: $\gamma_T = 23.32$.

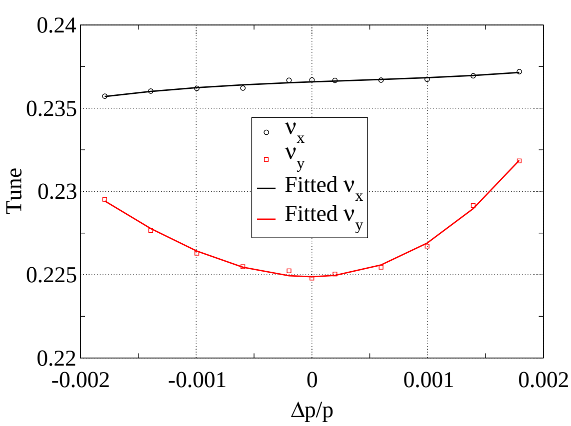

Figure 2: Tunes vs momentum with 11 data points in the Yellow ring. Fitting the data to a third order polynomial gives: $\xi_x = 0.25$, $\xi_{x_2} = -49$ and $\xi_y = 0.04$, $\xi_{y_2} = 1800$. There is significant vertical 2^{nd} order chromaticity, but the horizontal is nearly linear.

The next section describes the tune-meter system.

The PLL Tune System [3]

The tune-meter must have enough precision to be able to distinguish tune changes caused by the higher order chromaticities. For this reason we chose the PLL tune-meter system which promises tune resolutions of $\delta\nu = 0.0001$. Using a radial steering shift of $2mm$, the 2^{nd} order chromaticity will shift the tune by almost 0.002 units as seen in Fig. 3.

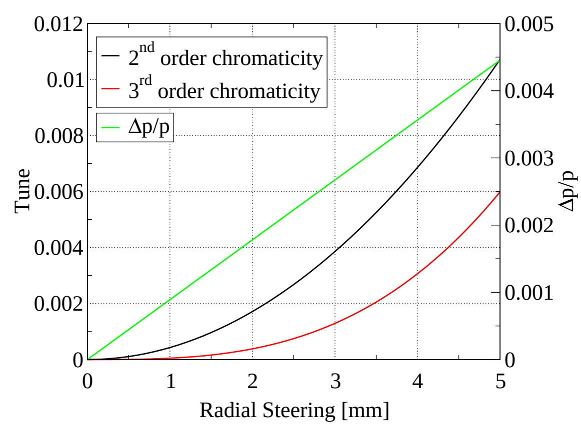

Figure 3: The sensitivity of the tunes to changes in the radial steering. Note, for the third order chromaticity, the tune shift is too small to be measured for a normal RHIC store. We assumed $\xi_2 = 540$ and $\xi_3 = 67700$ for determining tune-meter requirements.

In the next section, we compare the measurements with the model.

Comparison with Model

To compare the measurements to the model, we use the design optics and adjust the tunes and linear chromaticities close to their measured values. The second order chromaticities are the calculated using MAD [4]. Here, we assume the major source of the 2^{nd} order chromaticity are the main quadrupoles and sextupoles. Some of the results are shown in Table 1. The largest difference is in the *Blue*

Table 2: Second order chromaticities with octupoles

Case	octf (1)	octd (1)	octf (2)	octd (2)	ξ_{2_x}	ξ_{2_y}
1	0	0	0	0	-53	1953
2	2	0	0	0	-3	1739
3	2	-1	0	0	-144	1670
4	4	-1	0	0	13	1445
5	4	-1	0	2	-15	1802
6	4	0	-1	-4	102	1696
7	4	2	-1	-4	486	1575
8	4	4	-1	-4	1029	1481

ring's horizontal measurement. This difference could be due to the real optics being different from the design optics. Additionally, we have not taken into account other sources of multipoles, orbit errors, coupling, etc.

CORRECTION

To correct the 2^{nd} order chromaticities, we use the four available families of octupoles in RHIC. Two of the families are in the arcs, while the other two are in low dispersion regions of the insertions. Table 3 shows the linear contribution of each octupole family to the 2^{nd} order chromaticities and tunes.

Table 3: Effect of the RHIC octupoles on 2^{nd} order chromaticity. There are four families of octupoles: (1) are in the arcs and (2) are in the insertions.

	octf (1)	octd (1)	octf (2)	octd (2)
$\Delta\xi_{x_2}$	165.9	11.7	2.4	0.3
$\Delta\xi_{y_2}$	-37.0	-47.4	-0.6	-0.5
$d\Delta\nu_x/dJ$	388.5	-288.5	135.8	-105.9
$d\Delta\nu_y/dJ$	-278.4	339.2	-111.4	127.6

There is a maximum strength that the octupoles can be set (4 units). This limits the magnitude of the 2^{nd} order chromaticity that can be corrected. Table 2 shows the measured results when the octupoles are excited. From Table 3, we can see that there is not enough strength in the octupoles to correct the vertical 2^{nd} order chromaticity and keep the tune spread minimized. However, we can reduce eliminate the horizontal and reduce the vertical 2^{nd} order chromaticity.

SUMMARY

The 2^{nd} order chromaticity was measured in both RHIC rings. Since the tune change is small, we needed a tune-meter with sufficient resolution. We chose to use the PLL tune-meter. In comparing the measured results with a RHIC model – except for one case – the results are near the model's, although, not within the error bars. Furthermore, the model used was the design model which may not represent the real machine.

The next step was to correct the 2^{nd} order chromaticity using four families of octupoles. Two of the families are used for the correction and the other two are used to reduce the amplitude dependent tune spreads (at least to linear order in the octupole strength). As shown, the octupoles can reduce the 2^{nd} order chromaticity, but there is insufficient strength for complete correction.

REFERENCES

[1] S. Tepikian, H. Huang, W. MacKay, F. Pilat, V. Ptitsyn, T. Satogata, D. Trbojevic, J. van Zeijts, "High Luminosity β^*=$0.5m$ RHIC Insertions", Particle Accelerator Conference (2003).

[2] M. Conte, W. W. MacKay, "An Introduction to the Physics of Particle Accelerators", World Scientific Publishing Company (1991) and S. Y. Lee, "Accelerator Physics", World Scientific Publishing Company (1999).

[3] P. Cameron, J. Cupolo, W. Dawson, C. Degen, A. DellaPenna, M. Kesselman, A. Marusic, J. Mead, C. Schultheiss, R. Sikora, K. Vetter, "RHIC Third Generation PLL Tune System", Particle Accelerator Conference (2003).

[4] F. Schmidt, "MAD-X Home Page", http://mad.home.cern.ch/mad .

 0-7803-8859-3/05/$20.00 ©2005 IEEE