SYNCHROTRON SIDEBANDS OF A LINEAR DIFFERENTIAL COUPLING RESONANCE

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Abstract

Sidebands of a linear differential coupling resonance are observed in the tune survey of the SPring-8 storage ring. The vertical beam size and the Touschek beam lifetime blow up at a distance by synchrotron tune from the linear differential resonance. The synchrotron sidebands of a linear betatron coupling resonance are excited by the vertical dispersion at sextupole magnets. Although the vertical dispersion of the SPring-8 storage ring is well reduced to a small value, it still generates the sidebands of the coupling resonance in virtue of the strong sextupole magnets. By means of the tracking simulation based on the ring model obtained by the response matrix measurement, we confirm the existence of the synchrotron sidebands of a linear differential coupling resonance. In order to incorporate synchrotron motion, the full six-dimensional code is essential in the simulation.

INTRODUCTION

The SPring-8 storage ring is a high brilliance light source facility for hard x-ray experiments. In order to provide highly brilliant x-ray beams to users, various efforts have been made since the construction phase. The focusing magnets were aligned with extremely high precision by means of the two-stage alignment method [1], which significantly reduces error fields generated by the magnet misalignment. In addition to the precise alignment, the proper closed orbit correction [2] achieved a pretty small betatron coupling ratio even without skew quadrupole correction [3, 4, 5].

The major beam parameters of the SPring-8 storage ring are listed in Table 1. The original optics of the storage ring is double bend achromat with strong focusing magnets. As the optional operation mode the low emittance optics, which has nonzero values of dispersion function at straight sections to lower the natural emittance, is available. The small coupling ratio of the storage ring contributes to achieve the high brilliance of the synchrotron radiation.

For the purpose of monitoring the status of the dynamical stability or the resonance excitation of the storage ring, we routinely pursue the tune survey of the Spring-8 storage ring. In the tune survey we scan the horizontal tune with fixed vertical one to cross the nearest linear differential resonance. At that time we observed the unexpected resonances at the distances by synchrotron tune from the linear differential coupling resonace, which correspond to the synchrotron sidebands. In this paper we report the driving mechanism of the sidebands and the simulation results. In order to deal with the synchrotron motion as well as linear coupling, in the simulation of the synchrotron sidebands we used the six-dimensional multi-particle tracking code developed in house [6].

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Table 1: Parameters of the SPring-8 storage ring.

Energy [GeV]		8
Revolution Frequency [kHz]		208.78
Horizontal Tune		40.15
Vertical Tune		18.35
Emittance [nmrad]	(Achromat)	6.6
	(Low-emit.)	3.4
Coupling Ratio		0.002
Synchrotron Frequency [kHz]		2.033

EXPERIMENTS

Since the vertical beam size is primarily determined by the vertical dispersion and the betatron coupling, it is a proper indicator to describe the status of the ring dynamics. In a single resonance approximation it is written as [7]

$$\epsilon_y = \frac{\kappa}{1+\kappa}\epsilon_{xr} + \frac{1}{1+\kappa}\epsilon_{yr}, \tag{1}$$

where $\epsilon_{x(y)r}$ is the horizontal (vertical) emittance excited by the radiation in the non-zero dispersion region, and κ is the coupling ratio of the betatron oscillations. Here the coupling ratio is described as

$$\kappa = \frac{|C|^2}{|C|^2 + 2\Delta^2}, \tag{2}$$

where C is the coupling driving term and Δ the distance from the resonance. The driving term of the difference resonance is given by

$$C = \frac{1}{2\pi}\oint dsK(s)\sqrt{\beta_x(s)\beta_y(s)} \times e^{i[\phi_x(s)-\phi_y(s)-(2\pi s/L)(\nu_x-\nu_y-m)]}, \tag{3}$$

where $K(s)$ is the strength of the skew quadrupole field at location s, $\beta_{x,y}(s)$ the horizontal and vertical betatron functions, $\phi_{x,y}(s)$ the betatron phases, $\nu_{x,y}$ the betatron tunes, and L the circumference of the ring. The resonance condition is

$$\nu_x - \nu_y = m, \tag{4}$$

where m is an integer. Assuming longitudinal and transverse motions are independent of each other, the vertical beam size is given by

$$\sigma_y(s) = \sqrt{\beta_y(s)\,\epsilon_y + \eta_y^2(s)\,\sigma_\delta^2}, \quad (5)$$

where $\eta_y(s)$ is the vertical dispersion function, and σ_δ the r.m.s. momentum spread. At the SPring-8 storage ring the vertical dispersion function is well reduced to a small value, around 1 mm in r.m.s., by the dispersion correction by using the skew quadrupole magnets [4]. In Fig. 1 the blue and red circles denote the measured vertical dispersion function at beam position monitors before and after the correction, respectively. The vertical beam size is then dominated by

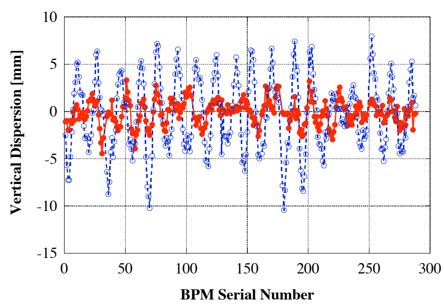

Figure 1: The vertical dispersion function at the SPring-8 storage ring.

the betatron coupling (2) and possesses the prominent peak on the resonance like Lorentz distribution.

Because the coupling of horizontal and vertical oscillations generates new eigen modes of oscillation, the measured betatron tunes are described by using the unperturbed ones $\nu_{x,y}$ and the coupling driving term C as follows

$$\nu_{1,2} = \frac{1}{2}\left[\nu_x + \nu_y - m \mp \sqrt{(\nu_x - \nu_y - m)^2 + |C|^2}\right]. \quad (6)$$

The measured fractional tunes are shown in Fig. 2, where the circles denote the measured betatron tunes and the solid lines are the fitting results with Eq. (6). At the SPring-8 storage ring the precise alignment of the magnets [1] and the proper correction of the COD [2] make the skew quadrupole error or the coupling driving term extremely small. In Fig. 2 one can find that the separation of the measured tunes, *i.e.* the coupling driving term is about 0.002.

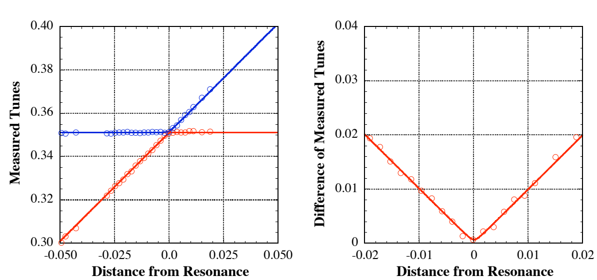

Figure 2: The measured betatron tunes (left) and the difference (right) in the tune survey.

At the SPring-8 storage ring the beam size is measured by the visible light interferometer [8] and the x-ray beam profile monitor [9] by using synchrotron radiation. In Fig. 3 the measurements of the vertical beam size in the tune survey are shown. It is difficult for the interferometer to measure the beam size on resonance since it is optimized for the measurement of the very small size at the regular working point. It should be emphasized that there obviously exist the sidebands on both sides of the differential resonance. The distances of the sidebands from the resonance are about 0.01, which just corresponds to the synchrotron tune of the SPring-8 storage ring. Note that there is no higher order resonance corresponding to the sidebands up to the fifth order.

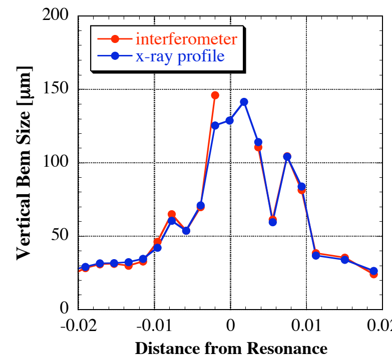

Figure 3: The vertical beam size in the tune survey.

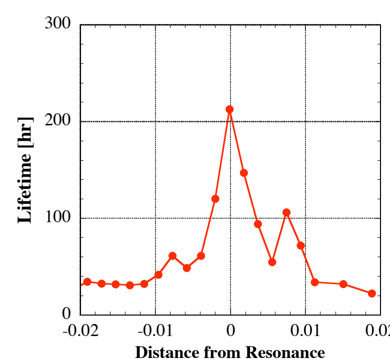

Figure 4: The Touschek dominant beam lifetime in the tune survey.

Although the electron energy of the SPring-8 storage ring is relatively high, the emittance and the coupling ratio are so small that the Touschek effect significantly contributes to the beam lifetime, especially in the operation with high current par bunch. The Touschek lifetime is proportional to the bunch volume or to the vertical beam size so that we also measure it in the tune survey. Then the tune survey is performed in the Touschek effect dominant condition of 1 mA per bunch. In Fig. 4 the measurements of the beam lifetime in the tune survey are shown, which also possesse the sidebands of the differential resonance as

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seen in the vertical beam size. Thus we have confirmed the appearance of the sidebands of the coupling resonance.

THEORY AND SIMULATIONS

Piwinski pointed out [10] that there exist the synchrotron sidebands of the betatron coupling resonances excited by the vertical dispersion at sextupole magnets. The Hamiltonian H including sextupoles can be written as

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + G_x x^2 + G_y y^2\right) + \frac{1}{3!} S\left(x^3 - 3xy^2\right), \tag{7}$$

where $G_{x,y}$ are the force exerted on the particles by the magnetic gradients, and S the strength of a sextupole magnet. Provided δ is the relative momentum deviation $\Delta p/p$, the lowest perturbing term at sextupole is given by

$$H_1(x, p_x, y, p_y) = S\left[\frac{1}{2}\eta_x\left(x^2 - y^2\right) - \eta_y xy\right]\delta, \tag{8}$$

where $\eta_{x,y}$ are the horizontal and vertical dispersion functions, respectively. The first term in Eq. (8) is used to compensate the chromaticity of the betatron motion. If there is a vertical dispersion in sextupole magnets, the synchrotron oscillation excites the sideband of the betatron coupling resonance through the second term of Eq. (8). Specifically speaking, the perturbing hamiltonian

$$U(x, p_x, y, p_y) = -S\eta_y \delta xy, \tag{9}$$

is the skew quadrupole field with varying strength as the synchrotron motion, which can generate the synchrotron sidebands of the equilibrium emittance with the betatron coupling and radiation. Writing a synchrotron oscillation as

$$\delta = De^{2i\pi\nu_s s/L} + \bar{D}e^{-2i\pi\nu_s s/L} \tag{10}$$

with ν_s the syncrotron tune, and D the amplitude, we find that the following satellite resonances of the betatron couplings

$$\nu_x - \nu_y = m \pm \nu_s \tag{11}$$

are excited.

In order to confirm the existence of the synchrotron sidebands, the simulations were done for the ring model of the SPring-8 storage ring. In the simulation we use the full six-dimensional multi-particle tracking code developed in house [6] to incorporate the synchrotron motion with the betatron coupling ones. We have analyzed the error distribution along the ring by using model calibration method [11, 12]. The model of the SPring-8 storage ring is thus reconstructed from the error distribution obtained by the beam response method.

In the simulation the beam emittance is calculated from an equilibrium distribution of 500 particles after 4000 revolutions. Figure 5 shows the simulation results of the vertical beam size at the source point of the beam size monitors. In Fig. 5 the six calculation results with different vertical betatron tunes are shown. Provided the sidebands are generated by the synchrotron motion, they should appear at the same distance by the synchrotron tune from the betatron coupling resonance as shown in Fig. 5. The simulation results thus suggest the existence of the synchrotron sidebands of the differential coupling resonance.

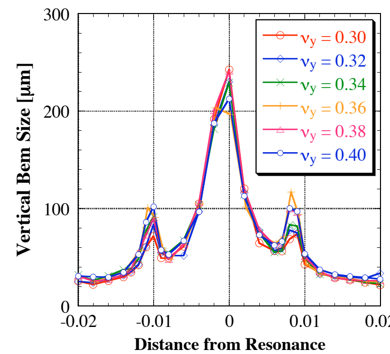

Figure 5: The simulation of the vertical beam size in the tune survey.

It is astonishing that while the vertical dispersion function of the SPring-8 storage ring is considerably small as mentioned above it still contributes to the sidebands of the coupling resonance. As the small skew error field of the Spring-8 storage ring can excite the significant coupling resonance, the equivalently small vertical dispersion to the skew field may give rise to the satellite resonances. It is known that the linear coupling resonance causes the higher order resonance and then significantly affects the dynamics of a particle motion in a wide range. Since there might be a similar influence on the beam dynamics caused by the synchrotron sidebands, the full six-dimensional code is indispensable to accurately investigate a particle motion at an actual ring.

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