NON LINEAR ERROR ANALYSIS FROM ORBIT MEASUREMENTS IN SPS AND RHIC*

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Abstract

Recently, an "action and phase" analysis of SPS orbits measurements proved to be sensitive to sextupole components intentionally activated at specific locations in the ring. In this paper we attempt to determine the strength of such sextupoles from the measured orbits and compare them with the set values. Action and phase analysis of orbit trajectories generated by RHIC models with non linearities will also be presented and compare with RHIC experiments.

INTRODUCTION

Most of the techniques to study non linear errors in accelerators yields global estimations of such errors rather than local. Even though this is enough for most of the cases, the ever increasing need for luminosity and hence smaller beams sizes has led to design interaction regions where the smallest non linearity can have very harmful effects on the beam. It is then important to be able to measure non linear errors at those particular regions of an accelerator.

Studies at the SPS [1] has shown that local measurements of sextupoles errors, intentionally placed in the accelerator, are possible using the Resonance Driving Terms Method. In this paper we show a different technique that is also based on orbit measurements and differs from the conventional one in that the former uses the design lattice functions and assume that all the magnetic errors show up in the action J and the phase φ of the particle trajectory rather than in the beta functions. Such analysis can be done taking pairs of adjacent measurements, x_i and x_{i+1}, of the particle trajectory and applying [2],

$$J = \frac{\xi_i + \xi_{i+1} - 2\xi_i\xi_{i+1}\cos(\phi_{i+1}-\phi_i)}{2\sin^2(\phi_{i+1}-\phi_i)} \tag{1}$$

$$\tan\varphi = \frac{\xi_i \sin\phi_{i+1} - \xi_{i+1}\sin\phi_i}{\xi_i\cos\phi_{i+1} - \xi_{i+1}\cos\phi_i}$$

to all pairs of such measurements in the accelerator.Here, i runs from the orbit measurement done at the beginning of the ring to the measurement done at the end of the ring, ϕ_i and ϕ_{i+1} are the corresponding phase advances, and ξ_i is defined as the relation between the horizontal position x_i and the beta function at the place where the measurement is done. As result, plots of action and phase can be obtained and jumps in such plots indicate the places where magnetic errors are present.

* Work supported by DOE and DINAIN

This technique has been successfully used to estimate skew and gradient errors at RHIC [5] [4]. The technique has also shown to be sensitive to non linear errors [6]. Numerical estimation of such errors will be given in this paper.

SIMULATION OF SEXTUPOLE ERRORS IN RHIC ORBITS

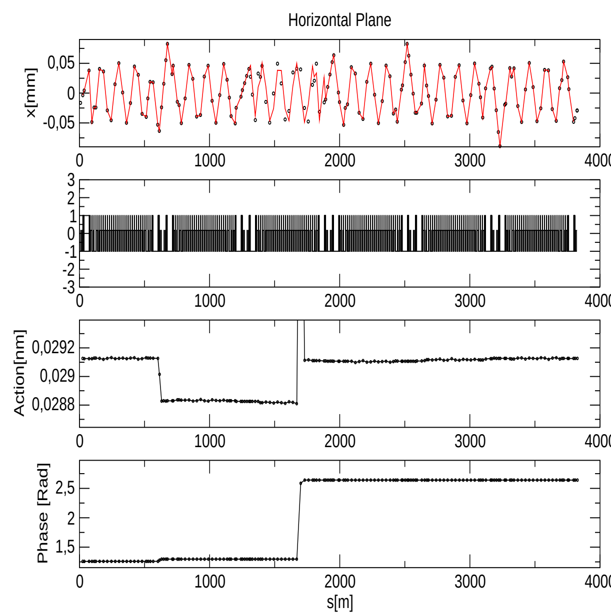

Figure 1: Action and phase analysis on a RHIC simulated orbit. One sextupole error at $s = 600m$ has been intentionally introduced in the simulation.

Simulated orbits of RHIC can be generated using the Methodic Accelerator Design program (MAD). A sextupole error was introduced in such simulations around $s = 600m$. Plots of action and phase (Fig 1) show a clear jump at this place. The other jump in the plots is due to the dipole corrector that must be turn on in order to produce a large amplitude oscillation. Action and phase to each side of the sextupole and Eq. 2 allow to estimate the magnetic kick $\Delta x'$ that the particle experience at s_0, the position of the sextupole [6]:

$$\Delta x'(s_0) = \sqrt{\frac{\left(J_x^L + J_x^R - 2\sqrt{J_x^L J_x^R}\cos(\psi_x^L - \psi_x^R)\right)}{\beta_x(s_0)}} \tag{2}$$

where J_x^L, J_x^R, ψ_x^L and ψ_x^R correspond to the action and phases for $s < s_0$ (superindice L) and $s > s_0$ (superindice R) respectively.

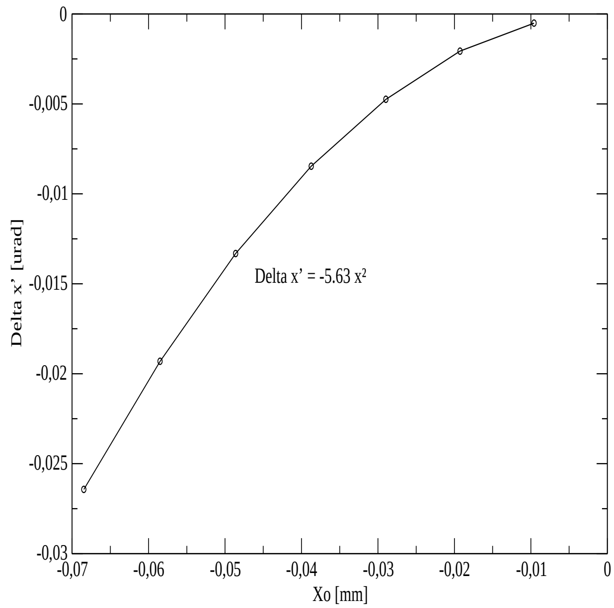

Figure 2: Strength of the magnetic kick experienced by the particle as it transverses the sextupole. The fit of the curve gives the integrated sextupole strength.

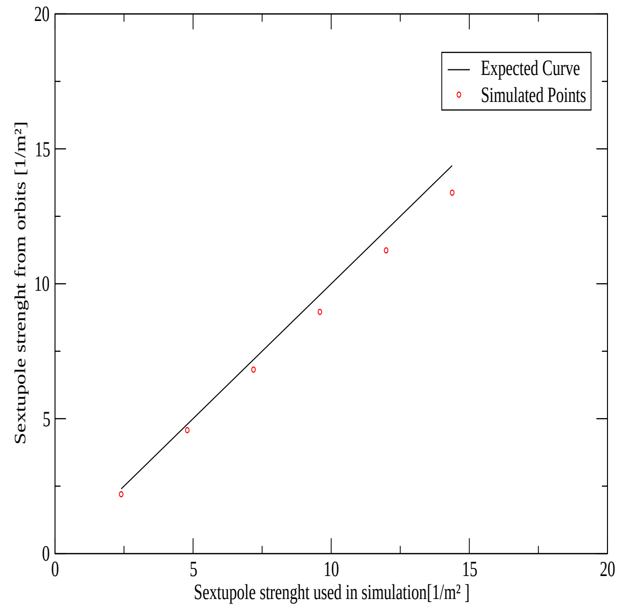

Figure 3: Strength of the magnetic kick experienced by the particle as it transverses the sextupole. The fit of the curve gives the integrated sextupole strength.

On the other hand, $\Delta x'(s_0)$ can also be expressed as function of B_1 , the gradient errors present at s_0, and non linear components like B_2, the normal sextupole errors. Such expression is given by (assuming no vertical orbit):

$$\Delta x' = -B_1 x_0 - B_2 x_0^2 \tag{3}$$

where x_0 is the horizontal position of the beam at s_0.

It is then possible to evaluate the coefficients of Eq. 3 if a set of points $\Delta x'$ vs x_0 are available. Fig 2 shows a plot of a set of such points obtained from Eq. 2 using simulated RHIC orbits kicked at different horizontal amplitudes. An integrated sextupole strength, $\frac{B''l}{B\rho}$, equal to $11.24\ \frac{1}{m^2}$ was added to the simulation which lead to $B_2 = 5.62\ \frac{1}{m^2}$ since [3]:

$$B_2 = \frac{B''l}{2B\rho} \tag{4}$$

Set of similar points were generated for different sextupole strengths and after doing the fitting of the resultant $\Delta x'$ vs x_0 curves, a relation between the sextupole strengths used in the MAD simulations and the corresponding strengths estimated using the action and phase analysis is found (see Fig. 3). The discrepancy between the values used in the RHIC lattice (MAD simulations) and the ones obtained by applying the action and phase analysis on the simulated orbits were about 8%.

SEXTUPOLE ERRORS FROM REAL RHIC ORBITS

During the RHIC 2003 dAu run, non linear experiments were conducted by turning on a sextupole corrector at the 8'clock Interaction Region.

For this experiment, a set of orbits with different amplitudes were taken while the sextupole strength was kept constant in order to obtain plots like Fig. 2. The non linear components were found by non linear fitting of the plots. This experiment was repeated 4 times, each time with a different sextupole strength. The relation between the sextupole strengths used for each series and the one found by the method were shown in a previous paper [6].

It was concluded that even though the scattering of the points were significant, the average behavior was as expected. It is still necessary to do additional experiments at RHIC to see if the scattering of the points reach the simulated limit of about 8%.

SEXTUPOLE ERRORS FROM REAL SPS ORBITS

The action and phase method can take advantage of any particle trajectory produced by a strong magnetic dipole kick as long as a baseline orbit (orbit with no kick) is available. The baseline is necessary to produce the so called difference orbit.

Since experiments conducted at SPS for resonance driving terms studies [1] use large orbits and baselines are available in the first turns of the orbits, action and phase analysis can also be used in these orbits. However, while action and phase analysis in RHIC experiments were

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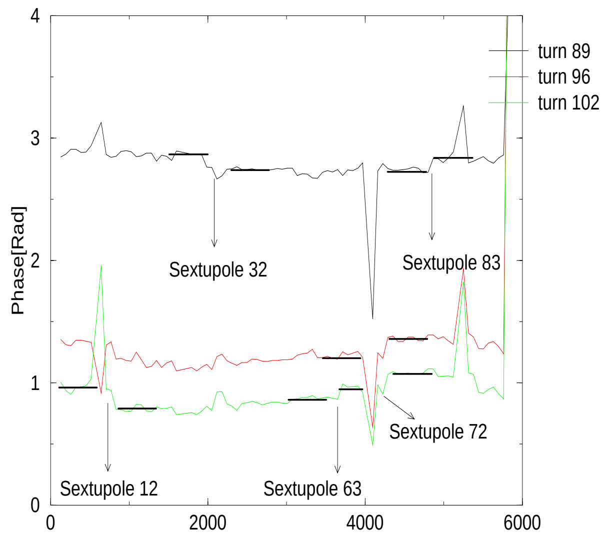

Figure 4: Phase analysis of SPS orbits. The sextupoles that were introduced intentionally in the accelerator can be clearly identified by the jumps in phase.

mainly done on closed orbits in SPS the analysis will be done on the available multi turn trajectories.

The SPS orbits were taken with 8 strong sextupoles activated along the ring. Action and phase analysis of those orbits reveals jumps at the locations were those sextupoles are. As can be seen in Fig. 4 the jumps appears in different turns of the particle orbit. This is due to the fact that the orbit is not always a maximum at the place where a specific sextupole is. Since the tunes are no integer values there will be a turn for which the orbit will be a maximum at the place of that specific specific sextupole. Hence, this particular turn will be optimum to estimate the sextupole strength.

In this paper we evaluate the strength of the sextupole located at s = 3646 m for which turn 97 seems to be the optimum trajectory. The procedure to obtain the sextupole strength from the action and phase plots is the same as before: Obtain plots of $\Delta x'$ vs horizontal position (Fig 5) and the no linear fit will give the value of the $B2$ coefficient from which the strength can be extracted. From the plot $B2 = 0.219 \pm 0.016 \frac{1}{m^2}$ is obtained which according to Eq. 4 lead to an integrated sextupole strength of $0.438 \pm 0.032 \frac{1}{m^2}$. The error bars in the plot are estimated with the different orbits (usually 3) that were taken with the same dipole kick strength. The set sextupoles integrated strengths during the SPS experiment were $2000 * 2.23145 * 10^{-4} = 0.446 \frac{1}{m^2}$ in good agreement with the sextupole strengths obtained from this analysis

CONCLUSIONS

Action and phase analysis for non linear errors has been validated with MAD simulations. Such simulations indicate that magnetic sextupole components can be determined within 8% of uncertainty.

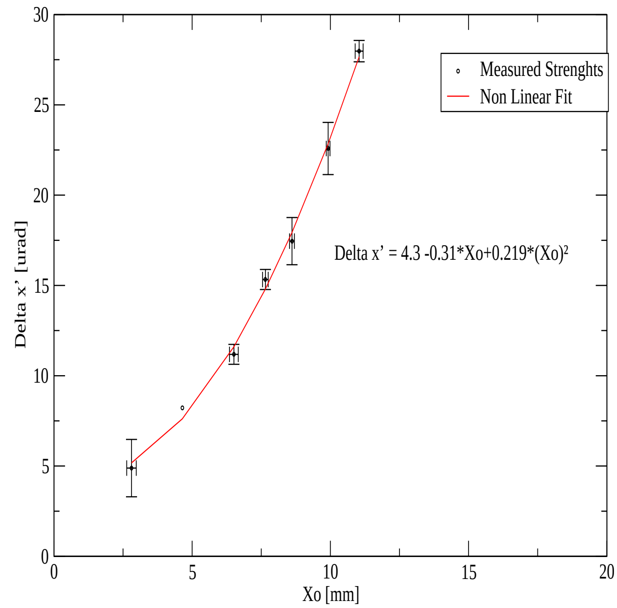

Figure 5: The non linear fit of the curve give an integrated sextupole strength in agreement with the strength used for the experiments

Experiments in RHIC to determine magnetic sextupole components exhibit the right trend but they are not conclusive due to the significant scattering of the corresponding measurements.

The application of the method to the SPS orbits have been successful in determining a normal sextupole component at specific place on the ring within an acceptable uncertainty.

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