CONTROL OF DYNAMIC APERTURE FOR SYNCHROTRON LIGHT SOURCES*

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Abstract

A summary of how modern analytical- and numerical techniques enable one to construct a realistic model of state-of-the-art synchrotron light sources is provided. The effects of engineering tolerances and radiation are included in a self-consistent manner. An approach for utilizing these tools to develop an effective strategy for the design- and control of the dynamic aperture for such dynamical systems is also outlined.

INTRODUCTION

The dynamic aperture (DA) problem dates back to the design of the first strongly focusing synchrotrons. The classical approach is based on [1, 2]:

- harmonic analysis of the periodic Hamiltonian,
- identification of the slowly varying terms, i.e. structural resonances, and
- first- and second order perturbative solutions of the equations of motion.

The strategy has been to reduce the:

- driving terms for the structural resonances,
- momentum dependence of the optics functions,
- tune shift with amplitude,
- and nonlinear chromaticity

by minimizing some heuristic merit function. However, due to the complexity of a realistic model, analytical studies are in general limited to a simplified model that:

- ignores the effects of engineering tolerances and radiation.
- is qualitatively different from the model used for numerical simulations.

A more comprehensive treatment requires a change of mindset, from Hamilton's equations to the Poincarè map, i.e. represented by a Lie series with the generator a power series in the multipole strength [3]. In particular, early work at the SSC used the rms variation of the linear invariant- and tune shift with amplitude to guide the design. This was initially accomplished by numerical simulations [4], but eventually advanced to a corresponding analytical model; including the effects of engineering tolerances and orbit [5].

CHROMATIC CORRECTION

Traditional design guidelines for (linear) chromatic correction are:

I. Avoidance (weakly focusing rings with high periodicity): with two chromatic families and a choice of working point away from structural resonances.
II. Anti-symmetry (strongly focusing large rings) e.g. [6]: introduce pairs of sextupole separated by modulo-π in horizontal- and vertical phase advance.
III. Second order[1] achromat (strongly focusing periodic rings) e.g. [7]: introduce a unit cell with phase advance such that the second order geometric aberrations are cancelled over N cells.

A robust DA has been achieved for the SLS which has:

- a low emittance lattice,
- with strongly focusing optics,
- but only 3-fold periodicity (due to the short-, medium-, and long straights),

through the generalization of the achromat guideline (III) above by careful modeling- and control of the [8-14]:

- first- and second order sextupolar modes,
- tune shift with amplitude,
- non-linear chromaticity,
- orbit in the sextupoles,
- and engineering tolerances.

The dynamical model is based on the Hamiltonian [10]

$$H(\vec{x};s) = -\left(1+h_{ref}(s)x\right)\left(1+\delta-\frac{p_x^2+p_y^2}{2(1+\delta)}+\frac{q}{p_0}A_s(s)\right) + O\left(p_x^4, p_y^4\right)$$

with the phase-space coordinates

$$\vec{x} \equiv \left[x, p_x, y, p_y, \delta, c_0\Delta t\right],$$

and multipole expansion of the magnetic fields

$$\frac{q}{p_0}A_s(s) = -\mathrm{Re}\left\{\sum_{n=1}^{\infty}\frac{1}{n}\left(ia_n(s)+b_n(s)\right)(x+iy)^n\right\}$$

obtained by the simplifications[2]:

- the ultra-relativistic limit ($c_0 \to \infty$): $\delta = E$,
- piece-wise constant fields, i.e. no fringe-fields[3],
- δ can be treated as a slowly varying parameter[4].

Numerical evaluations (TRACY-2) are implemented by [15-18]:

- a 4th order symplectic integrator,
- with classical radiation modeled by modifying the kick[5],
- a Galilean transformation before- and after each element to introduce misalignments,
- and first order[6] Truncated Power Series Algebra to obtain the corresponding (linear) maps

to calculate lattice functions, radiation damping,

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[1] In the phase-space coordinates.
[2] For medium size rings (~10^2 m) at ~3 GeV, not expanded in δ.
[3] This leads to a small error in the (linear) chromaticity.
[4] The adiabatic approximation.
[5] By generalizing from a Hamiltonian- to a vector flow.
[6] For numerical efficiency.

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equilibrium emittance, and tracking; with a self-consistent treatment of engineering tolerances and radiation.

In the pursuit of lower emittance, the NSLS-II requirements are pushing the envelope to a level where considering effects up to 2nd order is no longer sufficient. In particular, tune shift with amplitude up to J^3 (to 6th order in the sextupole strength) needs to be considered. Accordingly, the analytical framework has been extended.

"CONTROL THEORY" FOR THE DYNAMIC APERTURE PROBLEM

An intuitive[7] approach to control the DA based on the (formal) Lie series representation of the Poincarè map has been pursued [10]

$$\mathcal{M}^k = \left(e^{:h:}\mathcal{M}_{\text{linear}}\right)^k = \mathcal{A}^{-1}\left(e^{:h_3+h_4+\ldots h_8+\ldots:}\mathcal{R}\circ\Delta\mathcal{R}\right)^k\mathcal{A}$$

where e.g.

$$h_3 \sim \sum_{i=1}^{N}\left(b_3 L\right)_i \beta_{xi}^{m_x/2}\beta_{yi}^{m_y/2} e^{i\left(n_x\mu_{xi}+n_y\mu_{yi}\right)}$$

In other words, the one-turn map is factored into:

1. A (linear) transformation to normalized phase-space,
2. a single, non-linear kick (by parallel-transport of all the thin kicks to the start of the lattice),
3. and a major- and minor phase-space rotation.

It is clear that the general stability problem depends strongly on: h, $\mathcal{R}\circ\Delta\mathcal{R}$, and the initial conditions. Intuitively, one may argue that the DA can be improved by bringing the map closer to the linear approximation (for which stability has been established as part of the optics design) by:

- reducing the magnitude of the power series coefficients for the Lie generator h,
- and re-optimizing the working point $\mathcal{R}\circ\Delta\mathcal{R}$.

The non-linear effects generated by the sextupoles introduced for (linear) chromaticity correction are:

- 27 geometric modes to 3rd order see Table 1.
- 12 tune shift with amplitude terms to 6th order,
- and 13 chromatic terms to 6th order

for a total of 52 terms.

Table 1: Sextupolar Geometric Modes to Third Order.

h	$n_{x,y}$								
h_3	1,0	3,0	1,-2	1,2					
h_4	2, 0	0,2	4,0	0,4	2,-2	2,2			
h_5	1,0	3,0	1,2	1,2	5,0	1,4	1,4	3,-2	3,2

To optimize the DA for NSLS-II the following approach is applied:

1. Start from the 2 chromatic families.
2. Cancel the first order generators by symmetry, according to guideline (III) above, see Table 2. It follows that a robust design requires a cell with flexible optics.
3. Extend to 9 families so that the working point can be adjusted without exciting the first order generators. Then include the higher order terms, i.e. one obtains a 52×9 non-linear (6th order) system for the appropriate sextupole strengths. Solve the system, in a least-square sense, by introducing a suitable set of weights; obtained, heuristically, from tracking.
4. Optimize the tune by selecting a grid of working points, and for each point:
   a) adjust the cell phase advance with the quadrupoles in the straights, i.e. ignoring the perturbations on the beta functions,
   b) determine the sextupole strengths by minimizing the 52×9 system,
   c) and evaluate the DA in normalized phase space by tracking.
5. Move to the desired working point by re-optimizing the cell optics.
6. Iterate.

Table 2: Canceling of the First Order Sextupolar Modes.

Cell	ν_x,ν_y	$2\nu_x$	$2\nu_y$	ν_x	$3\nu_x$	ν_x-$2\nu_y$	ν_x+$2\nu_y$
1	8/5,3/5	3.2	1.2	1.6	4.8	0.4	2.8
2		6.4	2.4	3.2	9.6	0.8	5.6
…							
5	8,3	16	6	8	24	2	14

Remarks:

1. It is feasible to minimize the highly over constrained non-linear system of equations only because:
   a) the higher order terms are generated by cross terms (commutators) of the lower,
   b) and the initial system had all the first order terms cancelled by symmetry.
2. The second order chromaticity [10]

$$\frac{\xi_{x,y}}{\partial\delta} = -\frac{1}{2}\xi_{x,y} \pm \frac{1}{8\pi}\sum_{i=1}^{N}\left[2\left(b_3 L\right)_i\frac{\partial\eta_{xi}}{\partial\delta}\beta_{x,yi} \mp \left(\left(b_2 L\right)_i - 2\left(b_3 L\right)_i\eta_{xi}\right)\frac{\partial\beta_{x,yi}}{\partial\delta}\right]$$

   depends strongly on the sextupole location as shown in Figure 1. Similarly, $\beta_{x,y}\partial\eta_x/\partial\delta$ needs to be considered as well.
3. Because low emittance lattices tend to have a small (linear) momentum compaction α_1, they have the potential to generate "alpha-buckets" [19, 20], i.e. α_2 needs to be controlled in order to maintain the RF bucket. Figure 2 and 3 shows a validation of our numerical model against the analytical. Moreover, due the non-linear dynamics, it is important to track for at least one synchrotron oscillation, with radiation, to determine the actual momentum aperture.

[7] Since the KAM theorem only applies for miniscule perturbations, i.e. is of limited use for the design of a strongly non-linear system.

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4. Frequency Map Analysis is a valuable technique for analyzing the phase-space structure [21], which may be applied to gain better insight into how to adjust the weights in the merit function.

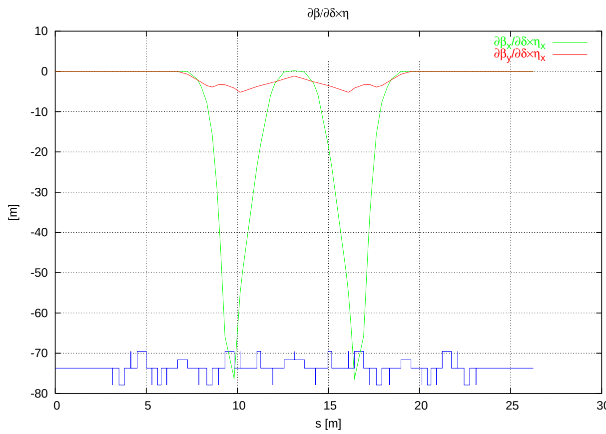

Figure 1: Second Order Chromaticity: $\eta_x \, \partial\beta_{x,y} / \partial\delta$.

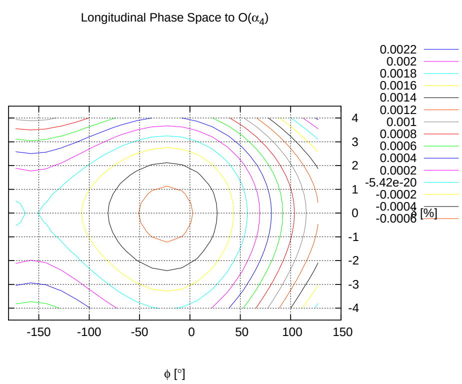

Figure 2: Longitudinal Phase Space for NSLS-II (analytical model: $\alpha_1 = 1.7 \times 10^{-4}$, $\alpha_2 = 1.4 \times 10^{-3}$).

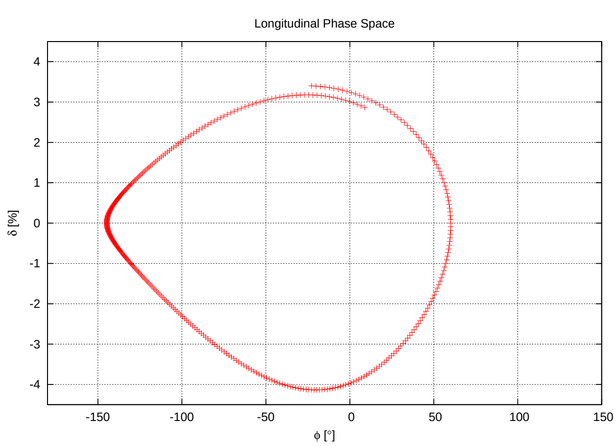

Figure 3: Longitudinal Phase-Space for NSLS-II (TRACY-2 with radiation).

LOOKING FORWARD: MODULAR OPTICS

For the design of a high performance lattice, the different requirements for:

- low emittance => large natural chromaticity => strong sextupoles,
- high brightness insertion devices => low beta,
- robust injection => high beta,
- robust DA => carefully selected cell phase advance and working point,
- low emittance => low α_1 => control of α_2 ,
- …

are likely to produce a set of conflicting constraints on the optics. An approach to decouple these would be to implement a modular lattice design. In particular [22]:

- a straight section tailored for injection,
- and a phase trombone for adjusting the working point [23]

will be introduced to simplify the control of the optics. This approach may enable the optimization of a robust high performance lattice and facilitate the commissioning of the real accelerator.

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