PARTICLE DISTRIBUTION FUNCTION FORMING IN NONLINEAR SYSTEMS

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Abstract

Modern ion-optical systems are used in different fields of beam physics both independent facilities as consisting of large machines. One of these destination is to create beams with a desired distribution of beams particles. Often there is a need to ensure a homogeneous distribution for a terminal beam phase portrait in a transverse configuration space. This is one of problems of nonlinear aberrations management. It is known that nonlinearity properties inhere to any beam lines. Such these nonlinearities have unremovable character, and their influence can be remove using only special nonlinear lattice elements, which are introduced artificially into the beam line. In this paper we suggest a procedure to find necessary nonlinear correcting control elements for purposive forming of beam particle distribution functions.

INTRODUCTION

In this report we suggest an approach which allows to simulate nonlinear effects and calculate parameters of elements, which correct the resulting distribution function on demand. For the distribution function described the beam particle at some initial moment one can uses two types of presentations: the first of them is based on a symbolic form for distribution function, generated form experimental data or some another information. For example, in the case of a symmetric character of particle distribution we can use the following pseudo-normal distribution

$$f_0(\mathbf{X}) = Q_{2m}(\mathbf{X})e^{-P_{2n}(\mathbf{X})}, \qquad (1)$$

where $Q_2m(\mathbf{X})$, $P_2n(\mathbf{X})$ are polynomials of the $2m$-th and $2n$-th order correspondingly. Coefficients of these polynomials are determined from experimental data fully or partly. In the last case the remainder coefficients can be determined from other information or used as control parameters.

The second type uses two types of following Taylor series expansions. the first form is used in general case

$$f(\mathbf{X}) = f^0 + \sum_{k=1}^{\infty} \mathbf{F}_k^* \mathbf{X}^{[k]}, \qquad (2)$$

where $\mathbf{F}_k^*$ are vectors of expansion coefficients.

The second (in the case of elliptical symmetry of the beam phase portrait) has the form

$$f(\mathbf{X}) = f^0 + \sum_{k=1}^{\infty} a_k \kappa_{2k}^*(\mathbf{X}). \qquad (3)$$

Here $\kappa^2(\mathbf{X})$ is a quadratic form, described the phase ellipsoid.

Let

$$\frac{d\mathbf{X}}{ds} = \mathbf{F}(\mathbf{X}, \mathbf{U}, \mathbf{B}, s) \qquad (4)$$

be a particle motion equation here we should note that s is a length measured along the optical axis od a beam line. Here $\mathbf{U}$ and $\mathbf{B}$ are a control function vector and a control parameters vector correspondingly. These vectors describe the beam line external field and installation parameters (elements lengths, apertures and so on).

It is known that Eq. (4) generates a beam line propagator $\mathcal{M}(\mathbf{F}; s|s_0)$, associated with the function $\mathbf{F}$, between two moments: s_0 and a current s:

$$\mathbf{X}(s) = \mathcal{M}(\mathbf{F}; s|s_0) \circ \mathbf{X}_0. \qquad (5)$$

According to the algebraic Lie methods (see, for example, [1]) one can write for the current distribution function as a result of Eq. (5)

$$f(\mathbf{X}, t) = f_0\left(\mathcal{M}^{-1}(\mathbf{F}; s|s_0) \circ \mathbf{X}\right). \qquad (6)$$

Using the matrix formalism [2] we write

$$\mathbf{X} = \sum_{l=1}^{k} \mathbb{M}^{1k}(\mathbf{F}; s|s_0)\mathbf{X}_0^{[k]}, \qquad (7)$$

where $\mathbf{X}_0$ is a initial phase vector, $\mathbb{M}^{1k} = \mathbb{M}^{1k}(\mathbf{F}, s|s_0)$ are two dimensional matrices describing aberrations k-th order, generated by beam line under study known, and $\mathbf{X}^{[k]}$ are vectors of k-th order phase moments, which consist in all monomials of k-th order). These vectors form so called Poincare–Witt basis).

The corresponding matrices for the inverse propagator $\mathcal{T} = \mathcal{M}^{-1}$ can be evaluated the generalized Gauss algorithm (we have to note that in this case only $\mathbb{M}^{11}$ should be inverted. The rest block matrices are evaluated using usual matrix operations: multiplication and summation. So one can write the following matrix presentation for eq. (6):

$$f(\mathbf{X}, t) = f_0\left(\sum_{k=0}^{\infty} \mathbb{T}^{1k}(\mathbf{F}; s|s_0)\mathbf{X}^{[k]}\right). \qquad (8)$$

TARGET SETTING

As it is mentioned above there are two types of distribution functions: using some formula presentation (see, for example, (1)) or a Taylor expansion (see (2), (3)). According to our goal we should introduce an information on a

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terminal distribution function in the form similar Eq. 1) either similar Eqs. (2), (3). Let denote the desired terminal distribution function as $f^{\mathrm{des}}(\mathbf{X}, s_{\mathrm{term}})$, s_{term} — a terminal value of the independent variable s.

The main our aim is to ensure the following equality

$$f(\mathbf{X}, s_{term}) = f_0\left(\mathcal{M}^{-1}\left(\mathbf{F}; s_{\mathrm{term}}|s_0\right) \circ \mathbf{X}\right) \tag{9}$$

for a given initial distribution function f_0 and some control vectors $\mathbf{U}$ and $\mathbf{B}$. These vectors we note as $\mathbf{U}^{\mathrm{opt}}$ and $\mathbf{B}^{\mathrm{opt}}$.

The Choice of a Control Vector

The guiding external electromagnetic field in beam lines is distributed along the optical axis of the system and in most cases admits the following expansion (for example, in the case of magnetostatics)

$$\mathbf{B}(x, y, s) = \sum_{k,j=0}^{\infty} \mathbf{B}_{kj}(s) \frac{x^k}{k!}\frac{x^j}{j!}, \tag{10}$$

where x, y are transverse coordinates in some coordinate chart, associated with the beam line optical axis. After substituting Eq. (10) into the Maxwell equations only some independent coefficients are remained in the in the expansion (10). For example, for a focusing system, consisting in quadrupole and octupole lenses we can obtain

$$\begin{aligned} B_x = \; & y\left(k_x - \frac{k''_x}{12}(3x^2 + y^2)\right) + \\ & + \eta y(x^2 - 3y^2)/3 + \mathcal{O}(5), \end{aligned}$$

$$\begin{aligned} B_y = & -x\left(k_x - \frac{k''_x}{12}(x^2 + 3y^2)\right) + \\ & + \eta x(y^2 - 3x^2)/3 + \mathcal{O}(5), \end{aligned}$$

$$B_s = \; k'_x xy + \mathcal{O}(5).$$

Here $k_x = qG/m_0 c\beta\gamma$ for a magnetic quadrupole lens, $G = \partial B_x/\partial y|_{x=y=0} = \partial B_y/\partial x|_{x=y=0}$, and as a octupole lens force $\eta = (q/m_0 c\beta\gamma)\, \partial^3 B_y/\partial x^3\big|_{x=y=0}$.

In this case we have two control functions $u_1(s) = k_x(s)$ and $u_2(s) = \eta(s)$, which form the control vector $\mathbf{U}(u_1, u_2)^*$. The s dependence of u_i are determined by the character of fringe fields for control elements (in our example, quadrupole and octupole lenses). We can approximate the fringe fields using some model functions. Here there are several possibilities:

- the rectangular approximation (or piece-wise approximation)

$$u(s) = \Big\{ u_i,\; s_i \le s < s_{i+1},\; i = \overline{1,n}$$

- a special model approximation functions $f(\mathbf{A}, s)$, where $\mathbf{A}$ is a parameter vector, determining the corresponding function.

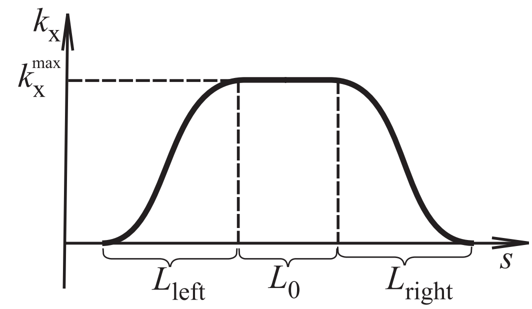

Figure 1: An example of lens force distribution.

In the first case instead of a single control function $u(s)$ one obtains a vector of parameters $\mathbf{U} = \{u_i\}_{i=\overline{1,n}}$. In the second variant we use the vector $\mathbf{A}$ as a control parameter vector $\mathbf{U} = \mathbf{A}$.

In the both cases the number of control parameters is determined from an approximation condition. For example, using the model function of the sinus-like form we have only four control parameters

$$u_{\mathrm{fringe}}(s) = a * sin(\alpha * s + \beta) + b,$$

which can be expressed using the maximal value of the lens force $u_{\max}$, the length of the fringe field sectors (L_{left} and L_{right} and the location of this control element. We should note that in this case we present $u(s) = k_{\mathrm{x}}(s)$ in the form (see fig. 1).

$$u(s) = \begin{cases} u_{\mathrm{left}}, & s \in \text{[domain of the left fringe field]} \\ u_{\max}, & s \in \text{[domain of the central part field]} \\ u_{\mathrm{right}}, & s \in \text{[domain of the right fringe field].} \end{cases}$$

So in any case one reduce the search problem of control function to search problem of control parameters, and our problem is reduced to nonlinear programming one.

The Solution Algorithms

So we have some collection of control parameters, which provide a beam evolution. These parameters should be found from the condition (9) using some nonlinear programming methods. In this paper we use (including some additional equality and inequality constraints) so called flexible tolerance method (see, for example, [3]).

Let us consider (as a simple example) the problem of distribution function uniformity (in the transverse configuration space $\{x, y\}$). In this case one should find the correcting octupole lenses, which guarantee the uniform distribution function for a nuclear microprobe [4].

As an initial distribution function we choose a normal function

$$f_0(\mathbf{X}) = f_0 e^{(-\kappa^2/2\sigma)},\;\; \kappa^2 = \mathbf{X}^*\mathbb{S}\mathbf{X}, \tag{11}$$

where $\mathbb{S}$ is the correlation matrix (so called σ-matrix). As a starting focusing channel for the nuclear microprobe we consider a system of four quadrupole lenses, but for all that we take into account nonlinear aberrations up to third order. In this case the matrix formalism gives for the phase vector $\mathbf{X}$.

$$\mathbf{X}(s_{\mathrm{term}}) = \mathbb{M}^{11}(s_{\mathrm{term}}|s_0)\mathbf{X}_0 + \mathbb{M}^{13}(s_{\mathrm{term}}|s_0)\mathbf{X}_0^{[3]}.$$

This transformation leads to some distortions of the initial (normal) distribution. As a desired terminal distribution we choose almost uniform distribution. For the solution the formulated problem it is more convenient to use the Taylor expansions of two described types (see eq. (2), (3)). In the case of simultaneous influence of the quadrupole and octupole lenses the last equation can be rewritten in the following form

$$\mathbf{X}(s_{\mathrm{term}}) = \mathbb{M}^{11}(s_{\mathrm{term}}|s_0)\mathbf{X}_0 + \\ + \left(\mathbb{M}^{13}_{\mathrm{quad}}(s_{\mathrm{term}}|s_0) + \mathbb{M}^{13}_{\mathrm{oct}}(s_{\mathrm{term}}|s_0)\right)\mathbf{X}_0^{[3]}. \quad (12)$$

This presentation allows us to apply different approaches. Here we consider two different methods. the first of them is based on enough traditional methods of nonlinear programming (see, for example, [3]). The second — on the correction procedure [5], which uses all advantages of the matrix formalism.

Let us give some primary features of our approach. On the first step one should to single out some supreme distortions, leading to nonhomogeneity of the resulting distribution function (at least in the frame of some approximation). On this step an investigator must study what kind of elements m^{13}_{ik} of the aberration matrix $\mathbb{M}^{13}_{\mathrm{oct}}$ play a vital part. Let be the number of these equal to M. In this case (if this number is not too large) we have to introduce M correcting elements (in our case — octupole lenses).

On the next step, according to the correction procedure [5], we compute some auxiliary matrices $\mathbb{M}^{13}_{\mathrm{oct}\,j}$, $j = \overline{1, \mathrm{M}}$ and $\mathbb{M}^{13}_{\mathrm{quad}}$, which computed in the case of *durante absentia* of octupole correcting lenses. The matrix $\mathbb{M}^{13}_{j}$, $j = \overline{1, \mathrm{M}}$ is computed on conditions that: all octupole lens forces are equal zero with the exception of octupole lens force of the j-th lens, for it we suppose $\eta_j = 1$. After this procedure we build a new a vector and matrix:

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{\mathrm{M}} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1\mathrm{M}} \\ a_{21} & a_{22} & \dots & a_{2\mathrm{M}} \\ . & . & \dots & . \\ a_{\mathrm{M}1} & a_{\mathrm{M}2} & \dots & a_{\mathrm{MM}} \end{pmatrix}.$$

Here $a_{i,k}$, $i, k = \overline{1, \mathrm{M}}$ are equal to the selected matrix elements of the M^{13} according to next rule: the number of matrix line i is corresponds to number of octupole lens with nonzero force $\eta_i = 1$. Another words the i-th line consists on selected matrix elements (arranged according to usual lexicographical order) of the matrix $\mathbb{M}^{13}_{\mathrm{oct}}$ in the case of $\eta_k = 0$, $\forall k \neq j$, $\eta_j = 1$. For searching of the desired correcting octupole forces η_i, $i = \overline{1, \mathrm{M}}$ we should solve the next linear algebraic equation

$$\mathbb{A}\boldsymbol{\eta} = \mathbf{B}, \quad \boldsymbol{\eta} = (\eta_1, \dots, \eta_{\mathrm{M}})^* .$$

COMPUTATIONAL EXPERIMENTS

The corresponding computational experiments were carried out both for the first approach based on the nonlinear programming methods and on the above described correcting procedure. The first approach does not need previous investigation, what elements of the aberration matrix $\mathbb{M}^{13}$ exert influence on a character of the distribution function at the moment s_{term}. But at the same time in this case computational burden are more sufficient.

The second approach is more convenient from computational point of view but it demands some previous computational experiments. Here we should note, that as computational experience becomes available the preliminary costs become progressively smaller. On the fig. 4 one can see the result of correction procedure for some model of a nuclear microprobe using four correcting octupole lenses. Here we did not reach good quality of the focusing properties as in [5]. As one can see from comparison of the last figures (in this case demagnification coefficient is equal to 6. If this demagnification is insufficient, then we have to introduce into the correcting system some additional octupoles.

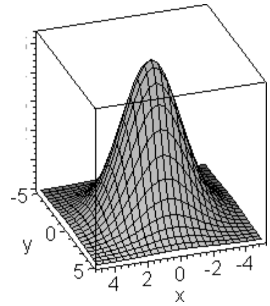

Figure 2: The initial normal distribution function in the transverse configuration space $\{x, y\}$.

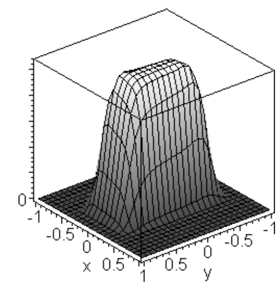

Figure 3: Resulting almost uniform distribution function in the transverse configuration space$\{x,y\}$.

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