DATA BASE EXTENSION FOR THE ENSEMBLE MODEL USING A FLEXIBLE IMPLEMENTATION*

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Abstract

To guarantee an adequate design and a proper functionality of various machine components it is of primary importance to perform detailed studies of the charged particle transport. However, it is often not necessary to initiate individual kinetic simulations based on the discrete particle movements.

When the time evolution of such integral quantities like average or rms dimensions, total energy or projected emittances is of the research interest, it is worth treating an investigated particle ensemble as a whole and applying a macroscopic formulation.

Based on the moment method a fast C++ code capable to handle various beam line elements has been implemented. The present paper treats the implementation issues of the code and discusses the simulation results for such axial magnetic multipoles like quadrupoles and sextupoles under the influence of fringe field effects.

INTRODUCTION

A collision-less kinetic approach allows a straightforward derivation of the moment method model from the well-known VLASOV equation. Instead of solving directly this equation to obtain the phase space density of the real particle distribution, one can use moments of the density function obtained by means of an averaging process over the entire space of coordinates and momenta [1].

This formalism had been implemented into the beam dynamics simulation program *V-Code* and a fundamental database of various beam line elements like cavities, drift spaces, solenoids, quadrupoles, steerers and bending magnets was set up [2].

The particular aim of extending the data base to higher order axial magnetic multipoles and the obvious requirement to handle all the involved quantities effectively necessitate the development of flexible realization for the automatical determination of all the desired moment equations up to an arbitrary order.

For this purpose a symbolic algebra program is used to clearly represent all the derived update equations whereas the C++ code is needed to control the time critical affairs like the management of the beam line elements, the time integration process and the graphical output of beam parameters to the user interface.

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ENSEMBLE MODEL

The ensemble model is based on the moment method and uses a discrete set of moments of the particle distribution function instead of the particle distribution itself.

In statistical calculations different types of moments are usually used as the degrees of freedom. Apart from the first order raw moments, which describe the centre of mass/charge with averaged position and momentum there are also centralized moments of higher order which describe the shape of the distribution. The more moments are involved, the more precisely the beam can be modeled and the more accurately the simulations are carried out. However, the analytical and numerical efforts to be invested increase significantly.

The ensemble model is derived from the VLASOV equation and leads after rearrangement of formulas to a governing set of ordinary differential equations which can be given in the compact form

$$\begin{aligned}\frac{\partial <\mu>}{c\,\partial t} &= <\mathrm{grad}_{<\vec{r}>}(\mu)> \cdot <\frac{\vec{p}}{\gamma}> \\ &+ <\mathrm{grad}_{<\vec{p}>}(\mu)> \cdot <\frac{\vec{F}}{m_0c^2}> \\ &+ <\mathrm{grad}_{\vec{r}}(\mu) \cdot \frac{\vec{p}}{\gamma}> + <\mathrm{grad}_{\vec{p}}(\mu) \cdot \frac{\vec{F}}{m_0c^2}> .\end{aligned}$$

The averaging process is performed in the 6D space Ω spanned by the 3D position and the 3D momentum subspaces and is described by

$$<\mu> = \int_\Omega \mu\ f(\vec{r},\vec{p})\ \mathrm{d}r^3\mathrm{d}p^3$$

where f denotes the 6D particle distribution function and it is usually abbreviated using the angle brackets [3].

The time dependent parameters μ used in the ensemble model are chosen in Cartesian coordinates such that

$$\begin{aligned}&\mu \in \{x, y, z, p_x, p_y, p_z\} \qquad \text{for the first and} \\ &\mu \in \{(x-<x>)^{l_x} \cdot (y-<y>)^{l_y} \cdot (z-<z>)^{l_z}, \ldots\}\end{aligned}$$

for the higher order moments.

All parts on the right hand side of the fundamental equation including the momentum expression $\vec{p}/\gamma$ have to be applied permanently whereas those parts including $\vec{F}/m_0c^2$ are changed depending on whether internal space charge forces are considered or not and which kind of external forces are currently taking effect on the model. In what follows the influence of only axial magnetic multipoles are considered and electrical and space charge forces are omitted.

AXIAL MAGNETIC MULTIPOLES

Various kinds of magnetic fields are used for guiding charged particle beams. Their principal behaviour can be characterized by means of a multipole expansion either of the field itself or of a corresponding scalar or vector potential [4].

Fundamental Properties

On account of the MAXWELL equations

$$\text{curl}\,\vec{H} = 0, \quad \text{div}\,\vec{B} = 0 \quad \text{and} \quad \vec{B} = \mu_0 \vec{H}$$

which are valid for the considered sourcefree regions inside the vacuum tube one can introduce a magnetic scalar potential V_m and/or a magnetic vector potential $\vec{A}$ such that

$$\vec{B} = -\text{grad}\,V_m \quad \text{and/or} \quad \vec{B} = -\text{curl}\,\vec{A}$$

are fulfilled. Furthermore it is possible to combine both differential relations and deduce a complex function with the longitudinal component of the vector potential as the real part and the scalar potential as the imaginary part.

Series Expansion

The potential formulations then constitute the CAUCHY-RIEMANN conditions of an analytic function which allows a power series expansion in the transversal plane. The circular region of convergence can be chosen such that the beam tube is completely covered and all source terms are located anywhere outside the region of interest.

It is sufficient to concentrate on the magnetic scalar potential

$$V_m(r,\phi) = \sum_{n=0}^{\infty} \Big(a_n \cos(n\phi) + b_n \sin(n\phi) \Big) \cdot r^n$$

which describes the magnetic field in the transverse plane. Alternatively, the LAPLACE equation $\Delta V_m = 0$ can be solved applying the BERNOULLI separation technique in the form $V_m = \sum R(r) \cdot \Phi(\phi)$.

Extensions which allow to include the dependency in longitudinal direction lead with $V_m = \sum R(r) \cdot \Phi(\phi) \cdot Z(z)$ to the expression

$$\begin{aligned} V_m(r,\phi,z) &= \sum_{n=0}^{\infty} \Big(a_n(z) \cos(n\phi) + b_n(z) \sin(n\phi) \Big) \cdot r^n \\ &= \sum_{n=0}^{\infty} A_n(z) \cdot \cos\big(n\phi + \phi_n(z) \big) \cdot r^n \end{aligned}$$

which can be further simplified to

$$V_m(r,\phi,z) = \sum_{n=0}^{\infty} A_n(z) \cdot \cos\big(n(\phi - \phi_0) \big) \cdot r^n$$

if the field is not twisted along the longitudinal axis and the coordinate system is aligned in a way that only pure cos-parts remain. Unfortunately, the divergence field property as mentioned above can only be maintained using higher order series expansion.

Axial Multipoles

For ideal $2n$-pole magnets the expressions for the magnetic scalar potential and all field components can be given as

$$V_m = \frac{1}{n}\, r^n\, A(z)\, \cos\big(n(\phi - \phi_0) \big)$$

and

$$\begin{aligned} B_r &= -r^{n-1}\, A(z)\, \cos\big(n(\phi - \phi_0) \big) \\ B_\phi &= r^{n-1}\, A(z)\, \sin\big(n(\phi - \phi_0) \big) \\ B_z &= -\tfrac{1}{n} r^n\, A'(z) \cos\big(n(\phi - \phi_0) \big), \end{aligned}$$

consequently.

The evaluation of the field information is crucial for particle tracking whereas the knowledge of the scalar potential behaviour is mainly used to get an impression of the pole shoes shape during the magnet design phase.

V-CODE IMPLEMENTATION

In order to transfer a real magnetic multipole field including the edge effects to *V-Code* one needs to have the variation of the gradient along the longitudinal axis. This can be achieved either via measurements using a real magnet or via magnet simulations. From this the magnitude function $A(z)$ can be derived as a function of the longitudinal coordinate.

With all the available field components in the entire calculational domain, the time evolution of each ensemble parameter can be deduced from the solution of the fundamental set of differential equations. The essential initial conditions arise from such bunch properties like energy and shape specified at that time instance when the ensemble enters the corresponding beam line element.

Since the implementation of the governing set of differential equations prove to be extremely subtle due to the vast amount of analytical transformations which have to be carried out, a symbolic algebra program was used. This program is organized in a way that any polynomial functional dependencies of the applied forces can be handeled. With a proper series expansion of the inverse relativistic factor $1/\gamma$ any polynomial dependency of the electric field strength $\vec{E}$ and magnetic flux density $\vec{B}$ can be processed so that the moment equations up to an arbitrary order are generated and converted into a proper C++ notation.

The time integration process is performed in C++ by means of RUNGE-KUTTA methods. Additionally, a linear implicit ROSENBROCK method is available where the required JACOBI matrix can also be extracted from the symbolic algebra program. A high processing speed can be achieved using an adaptive time stepping procedure where the optimal time steps are deduced from the difference between a higher and a lower order solution in conjunction with a user specified precision tolerance.

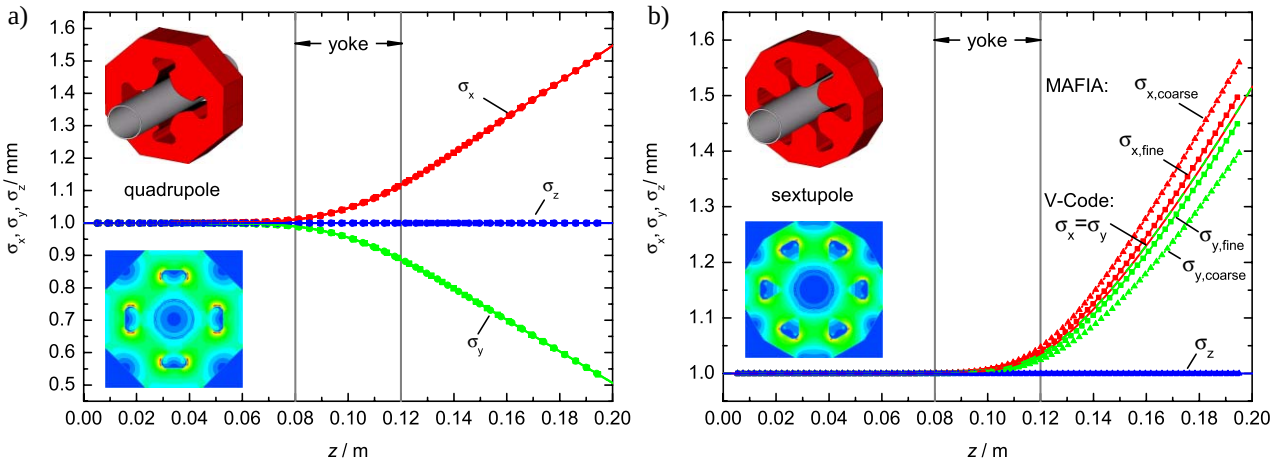

Figure 1: Geometry information and simulation results for a) magnetic quadrupole and b) magnetic sextupole.

SIMULATION

To check whether the implementation was carried out correctly two classic examples consisting of a single quadrupole and a single sextupole were chosen. The calculation results obtained by *V-Code* are compared with the well established electromagnetic field simulation program *MAFIA* [5] which is also capable of handling charged particle beams. Since *V-Code* is not able to simulate static magnetic fields, the gradient data along the longitudinal coordinate are transferred from a preceding *MAFIA* run during a postprocessing operation.

All tests are performed for a low charged bunch so that the space charge routines were switched off both in *MAFIA* and in *V-Code* in order to check exclusively the magnetic multipole routines. The simulation domain is stretched over a distance of $l = 200$ mm and the magnets are placed exactly in the center. The bore diameter as well as the thickness of the yoke are set to $d = 40$ mm for the test models.

In both programs the intrinsic simulation parameters like the grid size in *MAFIA* and the order of moments in *V-Code* were changed in order to get an impression of their influence to the calculation results. The transversal grid sizes were slightly varied in two steps from 0.375 mm for the coarse one to 0.25 mm for the finer one while keeping the longitudinal grid constant to 1.0 mm. Further refinement results in a tremendous amount of memory consumption.

Quadrupole

In Fig. 1a) the simulation results for a single magnetic quadrupole applying *MAFIA* and *V-Code* are shown. The quadrupole is operated in a way that it acts defocusing in x and focusing in y direction. The simulated rms sizes of the particle distribution vary in the micrometer range although different grid sizes or different orders of moments have been used. Within the chosen plot scale no difference between the various schemes can be recognized.

Sextupole

In contrast to the perfect agreement of the simulation results in case of a quadrupole geometry, the adequate comparison is not possible for a sextupole magnet. This is due to the fact that the sextupole geometry cannot be modeled exactly in the *MAFIA* Cartesian grid. Different material approximations of the yoke in the vicinity of the current excitation lead to a slightly unsymmetric magnetic flux density distribution inside the beam tube thus causing asymmetry even in the final bunch dimensions. With a higher grid resolution these effects vanish. Taking this into account one can conclude from Fig. 1b) that the results obtained by *V-Code* are perfectly enclosed in the *MAFIA* solutions.

CONCLUSION

The proposed flexible implementation of the ensemble model allows to deduce proper differential equations which can be used by a subsequent time integration procedure in *V-Code* without further treatment. It has been demonstrated that this technique works well for such magnets as quadrupoles and sextupoles. Additionally, the symbolic algebra program is able to handle all types of magnetic multipoles, so extensions to even higher order fields are possible.

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