FROZEN BEAMS

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Abstract

In general, the temperature of a charged particle beam travelling in an accelerator is very high. Seen from the rest frame of the beam, individual particles randomly oscillate about the reference orbit at high speed. This internal kinetic energy can, however, be removed by introducing dissipative interactions into the system. As a dissipative process advances, the beam becomes denser in phase space or, equivalently, the emittance is more diminished. Ideally, it is possible to reach a "zero-emittance" state where the beam is Coulomb crystallized. The space-charge repulsion of a crystalline beam just balances the external restoring force provided by artificial electromagnetic elements. This paper briefly reviews the dynamics of coasting and bunched crystalline beams circulating in a storage ring. Results of molecular dynamics simulations are presented to demonstrate the nature of various crystalline states. A practical method to approach such an ultimate state of matter is also discussed.

INTRODUCTION

Consider a dynamical system consisting of many identical particles of mass *m*. The average kinetic energy is given by $k_B T = (\langle p_x^2 \rangle + \langle p_y^2 \rangle + \langle p_z^2 \rangle)/2m$ where k_B is the Boltzmann constant, $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum of a particle, and $\langle X \rangle$ stands for taking the average of the quantity *X* over the whole phase space. *T* is commonly referred to as the *temperature* of the system. When the center of mass is moving at a certain speed, we define the temperature in the rest frame, subtracting the contribution from the centroid motion. Even after such subtraction, we find that the temperature of a charged-particle beam propagating in a typical accelerator is still quite high due to the betatron and/or synchrotron oscillations driven by external electromagnetic forces. This means that most beams naturally have rather large energy spread in all three degrees of freedom. A question is how much this thermal motion can be suppressed in principle.

In order to reduce the beam temperature or, in other words, to improve the beam quality, we have to introduce some dissipation into the system. Suppose that a sufficiently strong dissipative force is available. Then, is it possible to realize a completely "frozen" state in which $T = 0$? If the beam focusing force is uniform along the reference orbit, the answer is yes; a frozen beam can be generated, at least, in theory. At $T \approx 0$, all particles are fixed at specific coordinates and never move in the rest frame. Schiffer and co-workers numerically studied such an ideal system, employing the molecular dynamics (MD) approach [1]. Particles forming a frozen beam are spatially ordered, so that the internal Coulomb repulsion exactly balances the external artificial force. It is, however, not obvious whether an analogous crystallized state can be established in an actual accelerator where the beam is exposed to nonuniform focusing forces. This important subject was examined by Wei, Li and Sessler who incorporated complex lattice structures into MD simulations [2]. They showed that it is still possible to form various ordered configuration, although *T* is now non-zero unlike uniform focusing situations. As we can easily understand, any beam must execute a breathing motion owing to the discrete nature of a modern strong focusing lattice [3]. It is thus only approximately feasible to produce a frozen beam in reality.

CRYSTALLINE BEAMS

Hasse and Schiffer have theoretically investigated the structural transition of infinitely long Coulomb crystals, assuming the time-independent linear potential for particle confinement [4]. According to their analysis, the spatial configuration of a crystal can be determined by the dimensionless parameter

$$\lambda = N a_{WS} \tag{1}$$

where *N* is the number of particles per unit length, and a_{WS} is the Wigner-Seitz radius. At very low line density, a *string* crystalline structure is formed; namely, all particles are exactly on axis at the same intervals (see the upper panels in Fig. 1). By increasing the line density, we can transform a string into a two-dimensional (2D) zigzag crystal as depicted in the middle panels in Fig. 1. The density threshold of the string-to-zigzag transition is given by $\lambda = 0.709$. At higher density, a three-dimensional (3D) *shell* crystal can be attained (the lower panels in Fig. 1). Table 1 lists the transition density predicted in Ref. [4]. Although this theoretical prediction is based on the harmonic potential model, it approximately explains the actual transition of crystalline structures exposed to a strong focusing force.

Table 1: Structures of Coulomb crystals [4]

Density	Crystal Structure
$0 < \lambda < 0.709$	String (1D)
$0.709 < \lambda < 0.964$	Zigzag (2D)
$0.964 < \lambda < 3.10$	Single shell (3D)
$3.10 < \lambda < 5.7$	Single shell + String (3D)
$5.7 < \lambda < 9.5$	Double shells (3D)

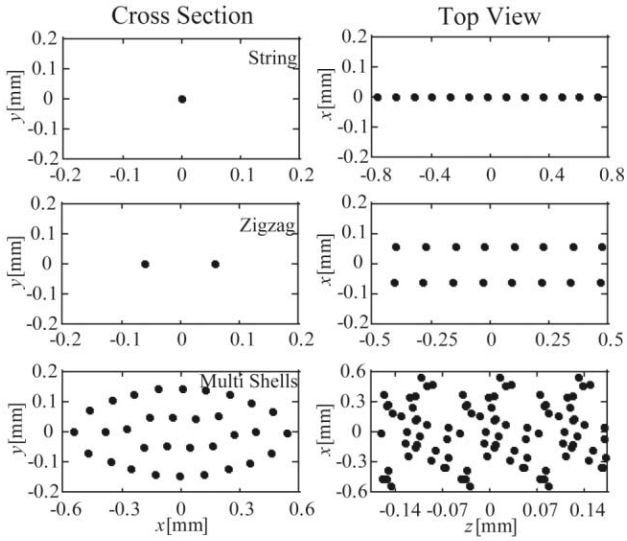

Figure 1: Real-space configurations of typical coasting crystalline beams. These results are obtained from MD simulations in which the lattice of the cooler storage ring S-LSR has been assumed [5]. The horizontal and vertical *bare* betatron phase advances per lattice period have been both set at 86.4 degrees in all three examples. Each dot (•) in the pictures corresponds to a single $^{24}Mg^{+}$ ion circulating in S-LSR at the kinetic energy of 35 keV. The symbols (*x*, *y*, *z*) represent, respectively, the horizontal, vertical and longitudinal spatial coordinates in the beam rest frame. The ordered structures are stable without the cooling force.

Coulomb crystalline states similar to the numerical examples in Fig. 1 have already been realized in many ion traps around the world by using the laser cooling technique [6-8]. This advanced technique is currently the only means for us to make Coulomb crystals because of its high damping rate and very low limiting temperature. Considering a physical analogy between a Paul trap and a linear beam transport channel [9], it should theoretically be possible to crystallize even a fast ion beam if its orbit is linear. Laser cooling is, however, applied only to a circulating beam in a storage ring, such that the ions interact with laser photons in one or more straight sections every turn [10,11]. This technical requirement for extending the effective cooling region has been known to cause a serious trouble. As discussed later, bending magnets peculiar to a circular machine plays a crucial role in crystalline states.

The lattice of a storage ring aiming at beam crystallization must satisfy a couple of conditions [2,12]. First of all, in order to form an ordered structure, the ring has to operate below the transition energy γ_T:

$$\gamma < \gamma_T, \tag{2}$$

where γ is the energy of the reference particle. Secondly, the following condition is required to maintain a crystalline beam:

$$\max(\nu_x, \nu_y) < \frac{N_{sp}}{2\sqrt{2}}, \tag{3}$$

where ν_x and ν_y are horizontal and vertical betatron tunes, and N_{sp} is the lattice superperiodicity of the ring. According to a recent understanding, this second condition is necessary but not sufficient from a practical point of view [13].

SINGLE-PARTCLE ORBIT

In a coasting crystalline beam as displayed in Fig. 1, the trajectories of all particles are proportional to each other [14]. The transverse motion of any single particle in the beam can be expressed, with universal orbit functions D_x and D_y, as

$$x = C_x D_x, \quad y = C_y D_y, \tag{4}$$

where $C_{x(y)}$ is a sort of scaling constant that depends on which particle we see. It has been shown that C_x is equivalent to the momentum deviation $\delta p / p$ of each particle. For reference, we plot, in Fig. 2(a), the orbits of three particles arbitrarily selected from the 3D crystalline beam in Fig. 1. These periodic oscillations driven by the alternating gradient lattice make the crystal temperature finite. In the present case, the average kinetic energy of the breathing motion is a few Kelvin much higher than the Doppler cooling limit. This value becomes greater and greater as the number of shells increases at higher line density. Scaling the three orbits properly, we obtain Fig. 2(b) that clearly demonstrates the validity of Eq. (4).

In order to predict the universal orbit, let us demand that $\langle C_x^2 \rangle = \langle C_y^2 \rangle$. We can then derive the coupled differential equations as follows [14]:

$$\begin{aligned} &\frac{d^2 D_x}{ds^2} + K_x(s) D_x - \frac{\alpha}{D_x + D_y} = \frac{1}{\rho}, \\ &\frac{d^2 D_y}{ds^2} + K_y(s) D_y - \frac{\alpha}{D_x + D_y} = 0, \end{aligned} \tag{5}$$

where K_x and K_y are the beam focusing functions determined by the lattice design, the α-parameter is defined by $\alpha = K_{sc} / \langle (\delta p / p)^2 \rangle$ with K_{sc} being the beam perveance, ρ is the curvature of the reference orbit, and the independent variable *s* is the path length. We have verified that the periodic solutions to Eqs. (5) completely agree with the scaled orbits in Fig. 2(b). It is worthy to recognize that Eqs. (5) approach the well-known envelope equations of a "zero-emittance" beam in a linear channel when $\rho \to \infty$.

The horizontal (vertical) emittance is usually defined as the area occupied by the beam in x-x' (y-y') phase

space, where $x' \equiv dx/ds$ ($y' \equiv dy/ds$). Equations (4) indicate that all particles have the same x'/x and y'/y at any location of the ring, which implies that the phase-space distribution is always a straight line as shown in Fig. 3. The transverse emittance of a crystalline beam is, therefore, zero regardless of its configuration in real space. This is true no matter whether the crystal is continuous or bunched. We can thus state that crystalline beams have the highest quality physically realizable.

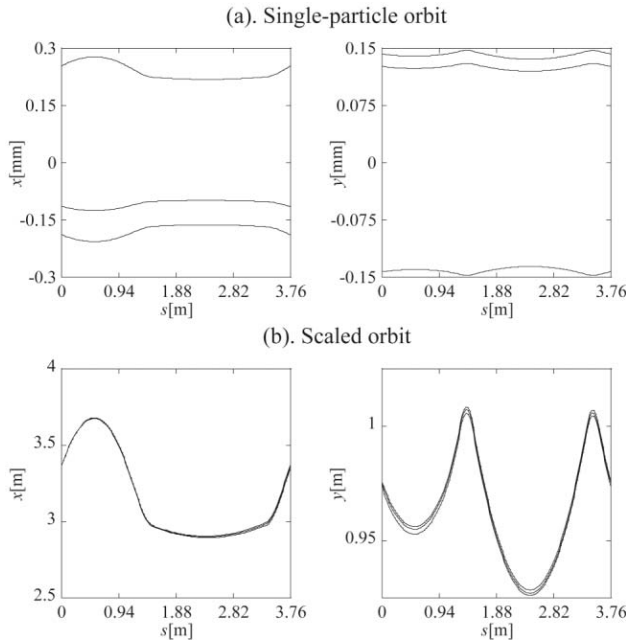

Figure 2: Transverse orbits of particles in a crystalline state at S-LSR. The trajectories over only a single lattice period have been plotted. Since the S-LSR ring has six-fold symmetry, the identical pattern repeats six times every turn. (a): Actual orbits of three $^{24}Mg^{+}$ ions extracted from the shell crystalline beam in Fig. 1. (b): Scaled orbits obtained from the data in the upper pictures. All scaled curves have almost completely overlapped.

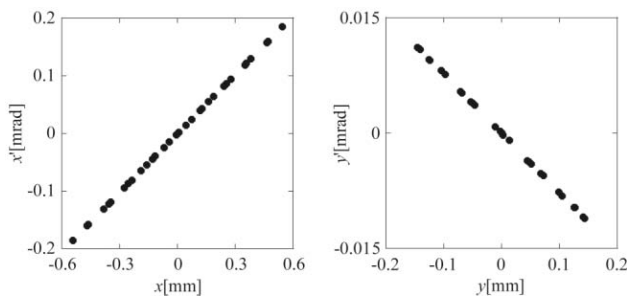

Figure 3: Phase space configuration of the double-shell crystalline beam in Fig. 1. The linear distribution is maintained all around the storage ring, although the tilt angle changes periodically according to the function $(dD_{x(y)}/ds)/D_{x(y)}$. If the focusing force is uniform along the orbit, all particles are aligned on the $x' = 0$ axis (thus, $T = 0$) and never move.

OBSTACLES TO BEAM CRYSTALIZATION

Heating Sources

Even if the two necessary conditions in Eqs. (2) and (3) are satisfied, crystallizing a beam is not always possible in practice. There are additional undesirable factors that can seriously affect a cooling process toward crystallization. First of all, *intrabeam scattering* starts to dominate the beam as the emittance is more diminished. If the cooling efficiency is too low, the beam will settle into a sort of equilibrium at rather high temperature determined by the balance between the internal heating and external damping forces. We expect that the heating rate comes to a peak in the liquid phase where the average Coulomb potential is on the same order as the average kinetic energy $k_B T$. Once the beam goes beyond the peak, the heating due to random Coulomb collisions becomes less dangerous and eventually disappears in a perfect crystalline state. The cooling force must, therefore, be strong enough to overcome the heating-rate mountain. Note that this effect does not take place in a spatially uniform lattice.

Another critical effect that may limit the achievable beam emittance is the so-called *coherent resonance*. Equation (3) can actually be interpreted as the sufficient condition to avoid the occurrence of the linear coherent resonance at the space-charge limit. This suggests that we should be careful in applying Eq. (3) because ordinary beams are far from space-charge limit before cooling. At high temperature, we find that the betatron phase advance per single focusing period must be below $\pi/2$ (rather than $\pi/\sqrt{2}$) to suppress the linear collective resonance [15-17]; whenever the phase advance per lattice period exceeds 90 degrees, the beam will encounter a severe resonance stopband before an ordered state is reached. Recent particle-in-cell simulations have revealed that high line-density beams cannot cross a linear resonance stopband even under the influence of a strong cooling force [13]. Once the lowest-order resonance is excited, the coherent tune of the beam is locked around a certain value and no further cooling becomes achievable. A similar phenomenon has been observed experimentally in a European storage ring [18]. We thus conclude that an ultralow-emittance state cannot generally be reached with a realistic cooling force unless the ring satisfies

$$\max(\nu_x, \nu_y) < \frac{N_{sp}}{4}. \qquad (6)$$

Similarly to the collisional heating, we do not have to care about this instability provided that the external driving force is uniform.

Notice that the condition (6) can be met only approximately. Strictly speaking, the superperiodicity of any storage ring is unity because of error fields and various insertion elements. An important question arises

then; namely, what degree of symmetry breakdown is acceptable in attaining a crystalline beam? Sensitivity studies have pointed out that magnetic imperfection at less than 0.1% level can be tolerated [19]. Resonances induced by error fields are, therefore, not so severe as long as dipole and quadrupole magnets are well constructed and well aligned. On the other hand, attention must be paid to the fact that we usually cool a beam in only one or two straight sections. This leads to additional symmetry breakdown of external forces felt by the beam. We have confirmed that 1D and 2D crystalline beams are relatively insensitive to how many times they go through cooling sections every turn. By contrast, it seems almost hopeless to ensure the stability of a multi-shell crystalline beam when the number of cooling sections is unequal to N_{sp}. This heating mechanism is in connection with the dispersive nature of a storage ring.

Dispersive Effects

So far, nobody has succeeded in accomplishing Coulomb crystallization of a fast ion beam in a storage ring. There are several primary reasons for that. Firstly, laser cooling is inefficient in damping the transverse betatron oscillations of fast stored ions. Secondly, a storage ring is much more complex than compact ion traps. Magnetic field imperfections and other noise sources are inevitable which may give rise to beam heating. Thirdly, the effects of momentum dispersion caused by bending magnets are present in a ring. This third factor is the most problematic and has made beam crystallization extremely difficult to achieve in practice.

The problem is, in one word, that dipole fields couple the transverse and longitudinal motions of particles. The path length of each particle depends on the horizontal coordinate x in a crystalline state whenever the beam orbit is closed; a radially outer particle at larger x has to travel a longer distance than inner particles every turn. In the meanwhile, all particles have an identical revolution frequency in order for the ordered configuration to be maintained. The average longitudinal velocities must, therefore, be different depending on their horizontal positions. A regular cooling force is designed to simply equalize the longitudinal momenta of stored particles, which is not appropriate for the dispersive character of a crystalline beam. In order to stabilize a crystalline structure, we need to develop a "tapered" force represented as [12, 20]

$$\Delta\left(\frac{\delta p}{p}\right) = -f_s\left(\frac{\delta p}{p} - \gamma C_{xz}\frac{x}{\rho_0}\right), \qquad (7)$$

where the left hand side denotes the change in $\delta p / p$ before and after the cooling section, ρ_0 is the curvature in the bending regions, f_s corresponds to the strength of the damping force, and C_{xz} is called the tapering factor determined by the lattice design. Tapered cooling is essential to form a circulating crystalline beam with a finite horizontal extent. Such a special force becomes unnecessary only in a dispersion-free system. A possible scheme to eliminate dispersive effects has recently been proposed in Ref. [21].

Once we switch on a radio-frequency (rf) cavity in a storage ring, a crystalline beam comes to show even more unique behavior that can never be reproduced in an ion trap [22]. Since the rf field accelerates or decelerates particles, the transverse motion of a crystalline beam is influenced by the energy modulation through the dispersive coupling from bending magnets. Then, even a string crystal can no longer stay on the reference orbit but horizontally oscillates about it [23]. Figure 4 is a typical bunched Coulomb chain in a storage ring. Any bunched crystalline beams are forced to execute analogous dispersive oscillations depending on several machine parameters. The stability property of crystalline beams in a storage ring is thus much more complicated than that of regular Coulomb crystals in a linear ion trap. The ring-shaped Paul trap system "PALLAS" developed by a German group may enable us to make a systematic study of these important subjects on beam crystallization [24].

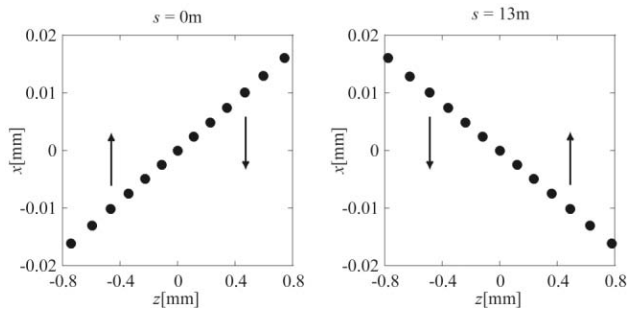

Figure 4: Example of a bunched string crystalline beam in the storage ring S-LSR. The top views observed at two different locations of the ring have been shown. The bare synchrotron tune assumed here is 0.07.

3D LASER COOLING

We are now in a position to discuss the experimental feasibility of beam crystallization. Since it is not difficult to construct a machine that simultaneously fulfills the conditions (2) and (6), the most essential issue is how to develop a proper 3D cooling force. For this purpose, we here consider the application of the *resonant coupling method* (RCM) to laser cooling [25,26]. The dissipative force provided by a laser light is known to operate only in the longitudinal direction. RCM is employed to extend this powerful 1D cooling force to the transverse degrees of freedom quite easily. All we have to do is simply the excitation of linear coupling resonances. In order to examine how much can be done with laser cooling, we carried out advanced MD simulations in which realistic photon-ion interactions can be incorporated [27]. Figure 5 shows MD results where two counter-propagating lasers have been used to cool a $^{24}Mg^{+}$ beam in S-LSR. As the transverse tunes have been set at $(\nu_x, \nu_y) = (2.067, 1.073)$, we can excite linear synchro-betatron resonances by

adjusting the longitudinal tune ν_s to around 0.07. The coupling sources are a solenoid magnet and momentum dispersion at the location of an rf cavity [26]. In Fig. 5(a) where the operating point is far from synchro-betatron resonances, no transverse cooling has occurred as expected. By contrast, a dramatic 3D cooling effect can be seen in Fig. 5(b) where $\nu_s = 0.07$. The effective cooling time is only about 200 msec and the normalized root-mean-squared (rms) emittances have finally reached to the order of $10^{-12}\,\mathrm{m}\cdot\mathrm{rad}$ in all three directions. In this example, we have assumed a very low line density at which the formation of a 1D string is anticipated. We have actually confirmed that, after a laser scanning completed at the 60000th turn, ions in each rf bucket are arranged into a linear chain analogous to that in Fig. 4. It is also possible to construct an approximate zigzag configuration in the same way. On the other hand, it seems unfeasible to generate a 3D crystalline beam with conventional cooling methods because of the reasons outlined in the last section. In fact, we have not observed the formation of a clear shell configuration in MD simulations as long as a realistic cooling procedure is taken into account [28].

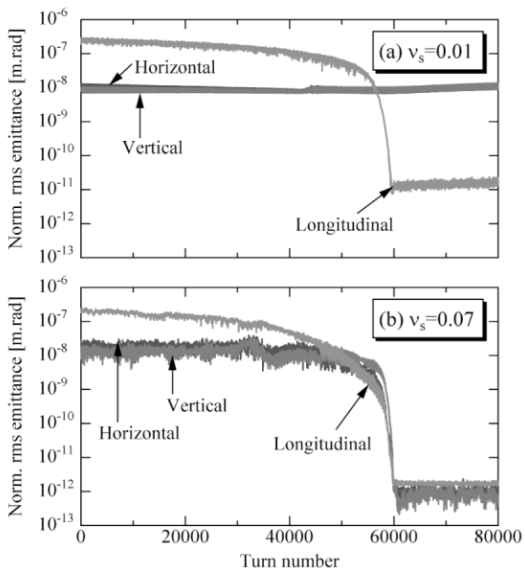

Figure 5: Time evolution of normalized rms emittances of a $^{24}Mg^{+}$ beam cooled with two-counter-propagating lasers in S-LSR. The kinetic energy of the beam is 35 keV. The upper picture shows an off-resonance case while, in the lower picture, linear coupling resonances have been excited among the three degrees of freedom. For more information about these MD simulations, see Ref. [27].

SUMMARY

In theory, it is possible to realize a Coulomb crystallized state where the beam is nearly frozen and the emittance is equal to zero. In practice, however, the production of a crystalline beam is extremely difficult due to some limitations in available accelerator technologies. Since cooling is generally executed in a circular machine, the existence of dipole fields yields a serious dynamical complication in the beam behavior. We have currently concluded, on the basis of a number of MD simulations, that only 1D and 2D crystalline beams may be attainable with existing technologies. In order to accomplish a 3D crystalline state, we must somehow counteract the dispersive heating.

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