MAIN REGULARITIES OF PARTICLE REDISTRIBUTION IN SPACE CHARGE DOMINATED BEAM TRANSPORT AND BUNCHING

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INTRODUCTION

Space charge effects are one of the main factors that can cause beam losses. When focusing and phasing forces are comparable to Coulomb ones summary beam field contains considerable much nonlinear component of proper field that leads to effective phase volume growth. During high-current beam motion a low-density extent beam part (so called "halo") appears. Two problems must be solved in order beam losses not exceed the given value:

1) input beam parameters must be chosen in such manner that beam size in the channel was minimal;

2) channel bore radius must be chosen with reserves to ensure full transmission of halo particles.

The main features of charge redistribution in high-current beam during bunching, accelerating and transporting must be revealed in order to solve above-mentioned problems. Experience leads us to use results obtained by many scientists. Below we summarize results of the work on this problem that were got by the authors.

BEAM CHARGE REDISTRIBUTION DURING HIGH-CURRENT BEAM TRANSPORTING

The main feature of the process is formation of core (area near axis with high charge density) and halo (peripheral domain with low charge density) with intense particle interchange between these two domains. It is common knowledge that core-halo formation takes place during high-current beam transporting. Nevertheless nobody gives strong mathematical definitions for core or halo sizes. We suppose to consider plot of Coulomb force versus radius and Coulomb force maximum $\mathbf{E_r}$ **(**see Fig. 1) can be defined as core boundary.

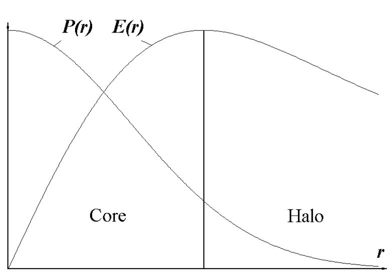

Fig.1.
Beam core definition.
P(r) – beam density, *E(r)* Coulomb force

If we use quadruple focusing, core has form of ellipse in *(x,y)*-plane. Ellipse semi-axes are defined as distances between origin and points of maximum for Coulomb force projections on each of transverse axes E_x, E_y.

The studies were conducted in order to establish that charge distribution inside core becomes uniform during the motion. For this fact confirmation charge distribution inside core was replaced by uniform one after some time. Result comparison shows that charge distributions core sizes, *rms*-emittances and so on have only a weak difference.

The conducted studies show that in steady state beam can be nominated as matched one if core oscillation frequency coincides with focusing period frequency. In such cases RMS and full emittances growth will be minimal.

Among infinity number of phase space distributions that give uniform projections on *(x,y)*-plane there is well-known KV-distribution. So we can use well-known analytical results for obtaining of matched beam initial parameters.

The main laws of mismatched SCD-beam charge redistribution during its transport can be stated on the base of performed investigations:

1) The main futures of charge redistribution both for matched and mismatched SCD-beams are the same in any focusing channel. A beam core-halo formation with active particle interchange is viewed as well as establishment of core uniform charge distribution.

The vast majorities of core particles are ex-halo ones or will be halo ones in coming.

2) SCD beam core oscillations are damping and halo oscillation amplitudes are growth.

3) limited oscillation amplitudes are observed for considered parameters (*rms*-emittance, core radius, beam boundary radius and so on).

4) As expected the relative *rms*-emittance growth was increased with mismatching increasing.

5) The relative *rms*-emittance growth was practically the same for Gauss and uniform input distributions.

6) In the case of mismatching more than 1.5 *rms*-emittance growth is bigger and faster for beam which core radius exceeds matching value in comparison with beam which core radius is the same times smaller.

7) In a transition regime *rms*-emittance growth takes place in points where core radius growth.

8) Comparison of results with uniform and Gauss input charge distributions in beam transporting in periodical channel shows that in the both cases final core

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mismatching is much smaller that input one. In the versions with Gauss input distributions only one frequency coincide with frequency of external focusing force stays clearly defined. It can be say that core automatically matched with periodical focusing channel. In the versions with uniform input distributions core oscillation frequency does not coincide with external force frequency. It can be concluded that when potential energy of input (Gauss) beam noticeable bigger than the same one of final beam charge redistribution leads to beam matching and accompanies by noticeable kinetic energy growth. In another case potential energies of output and input beams slightly differs and full matching does takes place but core oscillations in this case also noticeable reduced.

9) *rms*-emittance growth with the same input mismatching is less if input beam density is bigger.

10) Transition process study shows that a mechanism of core density equalization takes place during core oscillations. This process is connected with oscillations of charge amount inside core and is in anti-phase with core oscillations. When core size is reduced number of particles that lost core exceeds number of particles that enters core, i.e. core charge reduced and core density aligning. The opposite process takes place when core size increased. Kinematics effect explains the process partly: halo particles catch up with beam boundary when core is reduced and halo particles move towards beam boundary when core is increased.

Conclusion can be made that the state with matched core is energetically best suited to beam transporting in the focusing channel (it is a state with beam minimal internal energy).

THE BUNCHING PROCESS MAIN FEATURES FOR HIGH-CURRENT BEAM

In the previous part SCD-beam transporting was considered when beam density in equal manner was changed in each transverse crossection and it had remained intact in longitudinal direction. In present division we try to elucidate the main features of SCD-beam bunching. As usual we will consider the simplest model of the process. In the frame of the model we proposed that particle transverse motion is absent and we observe only Coulomb force reaction on bunching in longitudinal direction. In considered case charge distribution in transverse directions will be the same for all beam crossections (we suppose that it is uniform) and charge density modulation take place only in longitudinal direction. Synchronous phase -90^0 corresponds to beam center that moves with constant velocity. Because initial beam is continuous and non-modulated so modulation always will be periodical.

In given case we have a possibility to deduce and analyze several relations for proper field of beam flow with periodic charge distribution.

Let us considered axial symmetrical charge flow along Z-axis with velocity v and radius R_b inside cylindrical channel with radius R_a. Let us considered that beam current *I(r,z)* periodically varies along Z with period *L*.

If density $\rho(r,z)$ is periodical one it can be presented as Fourier series

$$\rho(r,z)=\sum_{k=0}^{\infty}\rho_k\cos\omega kz$$

Then we obtain potential *U* in the form of series

$$U(r,z)=-\sum\rho_k\left(\frac{1}{\omega kR_b}\right)^2\left(1-\frac{I_0(\omega krR_a)}{I_0(\omega kR_a)}\cdot C(\omega kR_a,\omega kR_b)\right)\cos\omega kz,$$

where $\omega=2\pi/L$, function *C(x,y)* defines as

$$C(x,y)=\frac{I_1(y)K_0(x)+K_1(y)I_0(x)}{I_1(y)K_0(y)+K_1(y)I_0(y)}=y\cdot\left(I_1(y)K_0(x)+K_1(y)I_0(x)\right)$$

Correspondingly

$$E_z=-\frac{\partial U}{\partial z}=\frac{2I}{\varepsilon_0 vL}\sum_{k=0}^{\infty}q_kF_k(\omega kr,\omega kR_a,\omega kR_b)\sin\omega kz,$$

where q_k are decomposition harmonics for $\frac{\partial}{\partial z}\rho(r,z)$.

Let us consider the axis field and. suppose that $x=\omega kR_a,\gamma=R_b/R_a$. Then

$$F_k(0,x,\gamma x)=F_{k0}(x,\gamma)=\frac{1}{(\gamma x)^2}\cdot\left(1-\frac{C(x,\gamma x)}{I_0(x)}\right)$$

Herewith

$$\lim_{x\to\infty}F_{k0}(x,\gamma)=\frac{1}{4}\left(1-\ln\gamma^2\right)=D_0,$$

Below $F_{k0}(x,\gamma)$-functions will be called as harmonic factors.

Let us consider ion bunching in the field of traveling wave with amplitude E_m and frequency ω. We suppose that particles are moving along channel axis and particle velocities are non-relativistic

$$m\frac{dV}{dT}=-eE_m\sin(kz-\omega t)+eE_c$$

where $\omega=\frac{2\pi c}{\lambda},\ k=\frac{2\pi}{L},\ L=\beta_c\lambda,\ \beta_c=v_c/c,\ E_c$ is Coulomb field intensity. Jumping from variable *t* to variable $v_st=z_s$ we obtain

$$m\frac{dV}{dz_s}=-\frac{eE_m}{v_s}\sin\left(k\left(z-z_s\right)\right)+eE_c$$

Choosing $\Psi=k(z-z_s)$ as non-independent variable and jumping to bunching phase $\tau=\Omega z_s,\ \ \Omega^2=\frac{keE_m}{mv_s^2}$

we obtain the main equation of bunching

$$\frac{d^2\Psi}{d\tau^2}=-\sin\Psi+\frac{2ID_0}{\varepsilon_0c\beta_s^2\lambda E_m}\sum_{n=0}^{\infty}q_n\frac{F_{n0}(x,\gamma)}{D_0}\sin n\Psi$$

Bunching equation general view shows that with given relations between geometrical sizes R_a/L and R_b/R_a bunching process is defined by unique parameter

$$\alpha = 2ID_0 / \varepsilon_0 c \beta^2 E_m \lambda$$

It means that in the frame of the same geometrical relations bunching process does not changed if $I/(WE_m\lambda)$-value is kept, where W is particle energy. If maximal allowable value of α_{max}, is defined then ultimate beam current is directly proportional to particle energy and external RF field amplitude and is in inverse proportion to operating frequency f

Remark 1. It can be shown oneself that ultimate permissible beam current will be growth with increasing of R_a/L because all Coulomb field harmonics are reduced. However external field intensity at the axis E_m also reduced as $1/I_0(2\pi R_a/L)$, that gives more upstanding effect towards reducing the beam current limit.

Remark 2. The above research differs slowly from universally accepted. It is usual to suppose that ratio of phase force to Coulomb one does not depend on beam energy and Coulomb force reduced with beam energy growth due geometrical factor R_a/L reducing. In the present work coulomb parameter α reduces with opposite proportionality when beam energy growth though Coulomb field harmonics increase when L growth. We can calculate that first harmonic coefficients are growth only 2-2.5 times during unlimited growth of L. As beam mean velocity β increases effect of α reducing is prevailed. If in the relation for Coulomb field intensity factor $1/(2\pi R_b/L)^2$ will be included into α, an universally accepted result will be achieved.

It is evident that particles positioned near bunching center during their moving towards center form potential barrier for following particles. During the first quarter of period the barrier is growth and each next particle feels bigger resistance from Coulomb field then previous one. So during the first quarter of period a hard core was generated in the central domain.

Starting from some value of α particle energy loss needed the barrier get over is comparable with particle kinetic energy derivable from external RF field. Starting from this value of α beam can be nominated as SCD-beam.

With further α increase additional pulse derived by particle after exit from core leads to essential increase of its phase oscillation amplitude. Starting from some value of α fraction of particles that overflow period boundary (which amplitude exceeds π) sharply increased. We will call such particles as "lost" ones.

Increasing α more and more we will come to situation when particles are fully decelerated by core field and they cannot pass throughout bunch center. However bunching will be possible in this case also. It is necessary to match rate of potential barrier growth with particle kinetic energy. The rate of external RF field growth can be used as matching instrument.

Computer investigation results give a possibility to make several conclusions about SCD-beam bunching:

1. During bunching in the field with increment amplitude beam can be considered as SCD-beam for $\alpha > 0.07$. For $\alpha > 1.2$ beam losses are appeared and beam bunching without losses is impossible in principle.

2. For effective SCD-beam bunching it is principal to have beginning part with increment field and with length 1 -1.5 periods of phase oscillations.

3. With beam current growth ($\alpha > 0.8$) Coulomb field harmonic spectrum defined by the first harmonic that essentially exceeds the other ones.

Specification and main result of analysis for charge redistribution inside SCD-beam are contained in reports [1–6].

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