

# A MODIFIED QUADSCAN TECHNIQUE FOR EMITTANCE MEASUREMENT OF SPACE CHARGE DOMINATED BEAMS \*

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## Abstract

The quad-scan technique is one of the most commonly used methods for measuring emittances in photo-injectors at medium range energy in which the beam is not primarily space charge dominated. The space charge can nevertheless contribute strongly to the beam size evolution along the drift from the scanning quadrupole to the measurement screen. To take the space charge into account, we fit the beam sizes while solving the envelope equation for each quadrupole value. This algorithm is first applied to round beams and then extended to asymmetric beams. The benchmarking of this solver with the envelope equation tracker with PARMELA simulations is presented. The quadrupole scan algorithm including space charge correction for the round and not round beams is presented.

## 1 MOTIVATION

The beam trace emittance in a single plane is given by  $(\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2)^{1/2}$  where the average  $\langle \rangle$  is taken over the whole beam distribution of particles with position  $x$  and angle  $x'$ . It corresponds to the square root of the determinant of the beam matrix. To measure the emittance, one of the techniques commonly used is the quadrupole scan. We consider a beam with beam matrix  $\sigma_{p_0}$  at point  $P_0$ .

$$\sigma_{p_0} = \begin{bmatrix} \langle x_\sigma^2 \rangle & \langle x_\sigma x'_\sigma \rangle \\ \langle x_\sigma x'_\sigma \rangle & \langle x'^2_\sigma \rangle \end{bmatrix} \quad (1)$$

This beam is transported through a quadrupole of strength  $kL$  and a drift of length  $l$ , represented by the transport matrix  $M$  and striking a screen at  $P$ . The beam matrix at  $P$  is given by  $\sigma_p$ . The term  $\sigma_p(1,1)$  writes

$$M_{11}^2 \sigma_{p_0}(1,1) + M_{12} M_{11} \sigma_{p_0}(1,2) + M_{22}^2 \sigma_{p_0}(2,2) \quad (2)$$

In the quadrupole scan technique, the strength  $k$  of the quadrupole is varied. With three  $k$  values, one can deduce the three unknowns and compute the beam emittance. To get rid of the systematic errors one usually uses many more points. This over-determined system also contains information about the two other emittances.

With this linear analysis, one ignores the strong space charge effects. For this reason, at low energy, it is standard to use the pepper pot emittance measurement

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technique instead of the quadrupole scan.

In this paper, we describe a method to take space charge into account in any quadrupole scan measurement. Also we extend the method to deduce the two transverse emittances and the longitudinal emittance.

## 2 ENVELOPE EQUATION

The envelope equation [1] describes the evolution of a beam in the presence of space charge forces. With the assumption that the space charge force can be linearized the envelope equation can be written as three coupled equations describing the evolution of an ellipsoidal beam as described very thoroughly in [2].

$$\begin{aligned} X'' + k_x^2(s)X - \frac{3K}{2} \frac{1 - \xi(Z/\sqrt{XY})}{Z(X+Y)} - \frac{\epsilon_x^2}{X^3} &= 0 \\ Y'' + k_y^2(s)Y - \frac{3K}{2} \frac{1 - \xi(Z/\sqrt{XY})}{Z(X+Y)} - \frac{\epsilon_y^2}{Y^3} &= 0 \\ Z'' + k_z^2(s)Z - \frac{3K}{2} \frac{1 - \xi(Z/\sqrt{XY})}{XY} - \frac{\epsilon_z^2}{Z^3} &= 0 \end{aligned} \quad (3)$$

$$X^2 = 5 \langle x^2 \rangle, Y^2 = 5 \langle y^2 \rangle, Z^2 = 5 \langle z^2 \rangle$$

With the beam perveance  $K$

$$K = \frac{qN}{2\pi\epsilon_0} \frac{1}{\gamma^3\beta^2} \frac{q}{mc^2} \text{ and } qN \text{ is the charge} \quad (4)$$

$$\xi(s) = \frac{s}{2} \int_0^\infty \frac{dt}{(t+1)(t+s^2)^{3/2}} \quad (5)$$

## 3 COMPARISON WITH PARMELA

### 1D result

We compared the evolution of beam sizes  $R$  and its conjugate variable  $R'$  for a round beam from PARMELA with those obtained when transporting the 1D envelope equation (first equation of (3) with  $X = Y = R$ ). We assumed a constant peak current and a constant emittance. For a bunch with Gaussian profile, one usually uses 2.355 times the rms bunch length to compute the peak current. It appears that using  $\sqrt{12}$  time the rms bunch length to compute the peak current was giving better agreement as shown in figure 1. The 3D envelope equation tracker includes the variation in bunch length along the drift and thus automatically makes the appropriate correction.

Those differences come from the fact that individual slices have in fact a different peak current. When applying the same algorithm to the individual slices using the correct slice peak current and initial  $R$  and  $R'$ , we obtained a satisfying agreement as shown in figure 2.

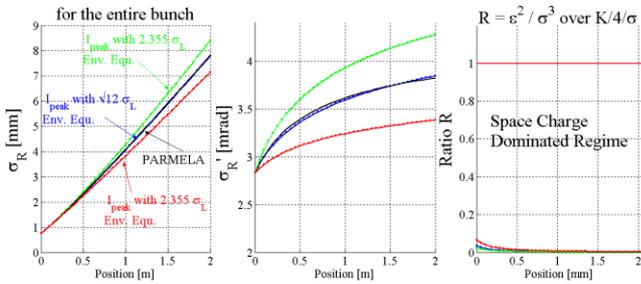


Figure 1- Evolution of beam size and divergence for a 5.7 MeV beam along a drift; the peak current has been computed 3 different ways (1) with 2.355  $\sigma_1$  (2) with  $\sqrt{12} \sigma_1$  and (3) using the total bunch length

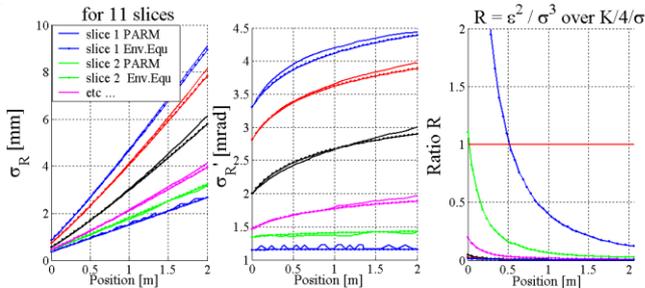


Figure 2 – Evolution of beam size and divergence for a 5.7 MeV beam along a drift, for the 6 first slices out of 11; Comparison between PARMELA (PARM.) data and Envelope Equation (Env. Equ.) solver

A square pulse was also studied. In this case, the number of particles varies strongly inside the slices close to the head and tail since the longitudinal space charge force is stronger on the steep edges. So particles drift from slice to slice and the peak current varies. Accordingly, the agreement with PARMELA is not as good. This type of behavior also occurs for bunches with longitudinal asymmetric profiles.

### 2D result

We then solved numerically the 2 first equations of (3) for non-round beams. Those two equations are coupled through the rms horizontal and vertical beam sizes. The agreement is satisfying as shown in figure 3. The calculation was done here with PARMELA using the 3D algorithm. But it does not differ much from the 2D result as the beam aspect ratio does not exceed 2 to 1. The discrepancies are justified in the next paragraph.

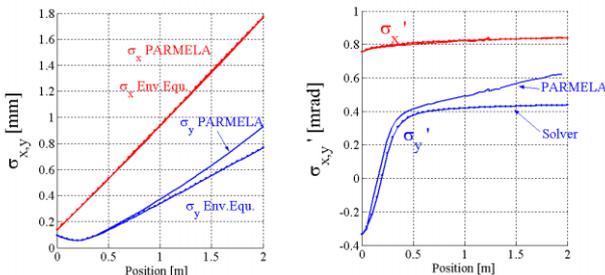


Figure 3- Evolution of beam size and divergence along a drift for a non-round beam. The solver agrees well with

the simulation over 1.5 m. The beam was here at 30 MeV but in the space charge dominated regime.

### 3D result

We solve numerically the 3 equations of (3). The peak current is then automatically computed correctly. Figure 4 shows that in case of a strong bunch lengthening along the drift, the 3D solver gives some very good results.

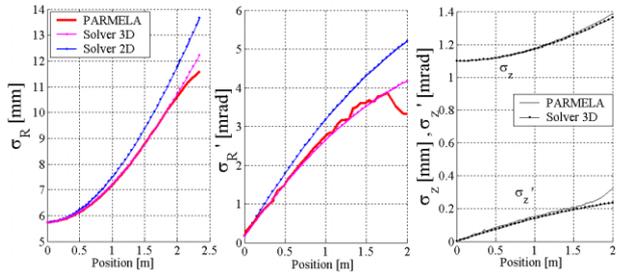


Figure 4- Evolution of beam transverse size (1), divergence (2) and bunch length and its derivative (energy spread) (3) along a drift for a 8.5 MeV beam;

The 3D envelope equation tracker was applied for various beam of low energy 5MeV and 8.5 MeV and to higher energy beams such as 30 MeV. It gave some very satisfying agreement with PARMELA. Beams with asymmetric longitudinal profiles cannot correctly be represented by the rms quantities.

## 4 QUADRUPOLE SCANS

For typical emittance measurements based on quadrupole scans, two quadrupoles Q1 and Q2, are used. The scan is done by varying the strength of the second quadrupole. The first one is chosen such that the beam on the screen has good dimensions and provides a fair intensity. One can choose the settings such that both the horizontal and vertical beam sizes will go through a waist during the scan.

The three coupled equations of (3) can be transformed into six first order differential equations. The transfer matrix approach can then be used. It is equivalent to that of the envelope equation but easier to manipulate. This formalism has been described thoroughly in [2] and it is also that developed for Trace 3D [3,4].

$$\sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle & \langle xz \rangle & \langle xz' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle & \langle x'z \rangle & \langle x'z' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle & \langle yz \rangle & \langle yz' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle & \langle y'z \rangle & \langle y'z' \rangle \\ \langle xz \rangle & \langle x'z \rangle & \langle yz \rangle & \langle y'z \rangle & \langle z^2 \rangle & \langle z z' \rangle \\ \langle xz' \rangle & \langle x'z' \rangle & \langle yz' \rangle & \langle y'z' \rangle & \langle z z' \rangle & \langle z'^2 \rangle \end{pmatrix}$$

$$\sigma_{i+1} = M\sigma_i M \text{ and } X^2 = \langle x^2 \rangle, Y^2 = \langle y^2 \rangle, Z^2 = \langle z^2 \rangle$$

$$M_{sc} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \Delta s/f_{sc,x} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \Delta s/f_{sc,y} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \Delta s/f_{sc,z} & 1 \end{pmatrix}$$

$$1/f_{sc,x} = K/2 \left[ \frac{1}{5} \frac{1}{X^2} \right]^{3/2} R_d \left[ \frac{Y^2}{X^2}, \frac{Z^2}{X^2}, 1 \right]$$

$$1/f_{sc,y} = K/2 \left[ \frac{1}{5} \frac{1}{Y^2} \right]^{3/2} R_d \left[ \frac{Z^2}{Y^2}, \frac{X^2}{Y^2}, 1 \right]$$

$$1/f_{sc,z} = K/2 \left[ \frac{1}{5} \frac{1}{Z^2} \right]^{3/2} R_d \left[ \frac{X^2}{Z^2}, \frac{Y^2}{Z^2}, 1 \right]$$

The transport matrix used for drifts and quadrupoles are standard. But they are fragmented in N pieces. For instance, the drift of length L is fragmented into N pieces to apply the space charge kicks every N/L.

$$M_{d,L} = M_{sc} M_{d,L/N} M_{sc} M_{d,L/N} M_{sc} M_{d,L/N} \text{ etc ...}$$

We checked that the coupling terms in the  $\sigma$  matrix can be neglected. We thus only kept the terms  $\sigma_{i,i}$  and  $\sigma_{i,i+1}$  for  $i = 1$  to 6. They constitute the 9 unknowns we want to determine.

### Round beam result

To reduce the number of unknowns, we first solve for the round beam at the entrance of Q1. That reduces the number of unknowns to 6. The transformations are

For a point with the 6D coordinates

$$X_o = (x, x', y, y', z, z')$$

$$X = M_{Q2} M_{d,1} M_{Q1} M_{d,2} X_o$$

The beam matrix is then given by

$$\sigma_p = M_{Q2} M_{d,1} M_{Q1} M_{d,2} \sigma_{po} M_{d,2}^t M_{Q1}^t M_{d,1}^t M_{Q2}^t$$

with  $M_{Q1}(k)$

and  $M_d$  is function of  $(\sigma_{i,local})$

The data to fit are

$\sigma_p(1,1)$  as a function of k

and  $\sigma_p(3,3)$  as a function of k

As shown in figure 5, the solver without space charge matches well the horizontal beam size but does not fit the vertical one correctly. It gives a 71.4 mm.mrad horizontally and 69 mm.mrad vertically. The solver which includes the space charge fits well both horizontal and vertical beam sizes and gives the correct emittance of 60 mm.mrad. The longitudinal emittance, not represented here, given by the solver with space charge is also determined accurately to be 6.4 mm.mrad.

To solve this problem numerically, an internal MATLAB solver is used[5]. It consists of a constrained non-linear least-square fit algorithm based on the Levenberg-Marquardt method. This algorithm is also described in [6].

The robustness of the algorithm needs to be assured. In particular, this algorithm should work for noisy data, such as experimental data.

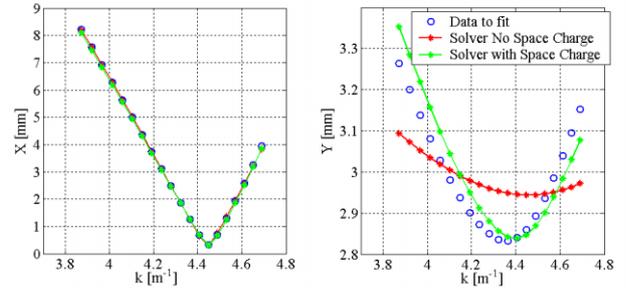


Figure5 – Beam Size (1) horizontal and (2) vertical on measurement screen. The beam has a 8.5 MeV and is space charge dominated

### Consideration for Elliptic BEAMs

To fit the beam at Q2 entrance, where it is elliptical, we can use the results of the round beam at the entrance of Q1 and transport them to Q2. However, a general algorithm which solves for the 9 unknowns has also been developed as it is useful for flat beam injectors. This algorithm fails for the case described in figure 5 when we look for a solution at Q2 entrance at which the beam is elliptical. The solution found corresponds to a local minimum of the least square function and does not lead to the solution.

## 5 CONCLUSION

We have demonstrated by comparing PARMELA simulations with the tracking of the envelope equation that the 3D envelope equation tracking can be applied very accurately for low energy beams. Even if the envelope equation represents the evolution of rms quantities, it also applies to non-Gaussian beams as long as they have ellipsoidal symmetry. If the longitudinal profile is asymmetric, the envelope equation is not appropriate. The space charge correction to a quadrupole scan fitting algorithm works fine for a round beam. The longitudinal emittance can be successfully deduced. Improvement of the solver for the 9 parameters of a non-round beam requires more sophisticated algorithms than the internal Matlab solvers.

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