REVERTED RESONANT "WAKEFIELD"

E. S. Masunov, MEPhI, Moscow, 115409, Russian Federation
A. V. Smirnov, DULY Research Inc., CA 90275, USA

Abstract

A new kind of coherent radiation induced by uniformly moving charge is introduced. It appears as a wavepacket forerunning ahead of the charge. It was derived as a complementary case of existing theory of resonant wakefield extended to include the exotic "overturned wake" affected by the leading front distortions due to dispersion. Corresponding conditions, causality and potential realizations in special types of slow-wave systems are analyzed and discussed. Those can include moving media and meta-materials, plasma and some of periodic structures.

INTRODUCTION

Resonant long-range wakefields have common nature with the Cherenkov effect and are very well studied for about half a century. Classic wakefileds are generated behind the short bunch (or charge) at the following conditions: i) synchronism between the charge velocity and modal phase velocity: $v_{ph} = v$; ii) normal dispersion of the slow-wave system (or medium): $v_{gr} \le v_{ph}$.

In this paper we consider the situation when $v_{gr} > v = v_{ph}$. So far it was widely assumed that no any resonant propagating field could be generated in this case. From the other hand, some TWTs are operating successfully in the vicinity of the $v_{gr} \approx v_{ph}$ [1]. Theoretical analysis [2,3] predicts the resonant wave to be emitted in front of the bunch at $v_{gr} > v$ similarly as it takes place in a super-radiant single-pass FEL. We shall distinguish such a reversed "wakefiled" considered here from "reversed Vavilov-Cherenkov radiation". This term was introduced originally by Veselago in 1968 [4] and refers to more trivial case of negative group velocity in a medium.

RESONANT FIELDS INDUCED IN A SLOW-WAVE SYSTEM

To find the fields induced by a charge in an arbitrary slow-wave guide it is convenient to apply the method of eigenmodes $\vec{E}_{\pm s}, \vec{H}_{\pm s}$ of variable amplitudes [5]. Along with Fourier-transformation it allows taking into account charge distribution in the bunch as well as dispersion effect for the fields in the time-domain [6,2]. Assuming point charge q of limited lifetime $\tau_0 = L/v$ propagating along a semi-infinite waveguide with transverse coordinate $\vec{\rho}_\perp(z)$ one can derive the following expression for the resonant modal fields induced:

$$\vec{E}_s(\vec{r},t) = -2q\,\mathrm{Re}\,A_s^L(z,t)\frac{\vec{E}_s^{\,0}(z,\vec{r}_\perp)E_{-sz}^0(z_1,\vec{\rho}_\perp(z_1))}{|N_s|\cdot|v^{-1}-v_{gr\,s}^{-1}|}, \quad (1)$$

where $E_{\pm s}^0(z,r_\perp) = E_{\pm s}/\exp(\pm ih_s z)$, $h_s = h_s' + ih_s''$, $h_s' > 0$;

$\tilde{h}_s = h_s' - ih_s'' v_{gr\,s}/(v - v_{gr\,s})$;

$A_s^L(z,t) = \exp(i\tilde{h}_s(z-vt))\left[A_s(z_1,\tau) - A_s(z_1 - L,\tau)\right]\times$
$\Pi\left((vt-z)/(v-v_{gr\,s})\right)$; $\tau = t - z/v$,

$\Pi(x)$ is the symmetric Heaviside function: $\Pi(0) = 1/2$;

$N_s = \int_\perp dS\{\vec{E}_s \times \vec{H}_{-s} - \vec{E}_{-s} \times \vec{H}_s\} \approx -4P_s$; $\quad z_1 = \dfrac{z - v_{gr\,s}t}{1 - v_{gr\,s}/v}$;

$t_1 = z_1/v$, and, for the 2nd - order dispersion [2,7]:

$$A_s(z_1,\tau) = \frac{1}{2}erfc\left(\frac{z_1(1-v_{grs}/v)}{-(1+i)\sqrt{|v'_{grs}|\tau}}\right),\ v'_{gr\,s} = \frac{d^2\omega}{dh_s'^2}.$$

In the limit of linear dispersion (i.e., $dv_{grs}/dh \to 0$) the field amplitude $|A_s^L(z,t)|$ at $v \ne v_{gr\,s}$ is proportional to the following propagating factor:

$$\left[\Pi(t_1) - \Pi(t_1 - \tau_0)\right]\Pi\left((vt-z)/(v-v_{gr\,s})\right). \quad (2)$$

One can see from (1) that coherent propagating waves are emitted under the following conditions:

$$2Q_s|v - v_{gr\,s}| >> c,\ T_1 = L|v_{gr\,s}^{-1} - v^{-1}| >> 2\pi/\omega_{1s}. \quad (3)$$

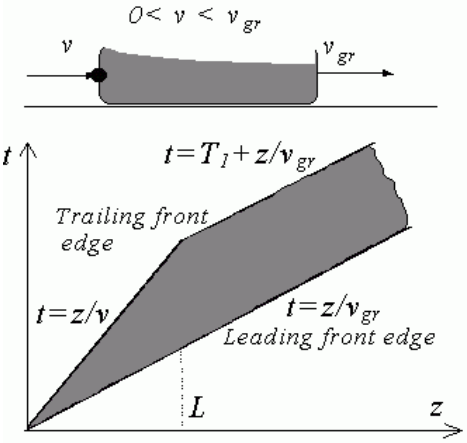

Figure 1: Space-time diagram for the resonant field radiated at anomalous dispersion.

The fields (1,2) were derived without any limitation imposed on the relationship between the group and charge velocities and have to be valid for both cases: $v_{gr} < v$ and $v_{gr} > v$. The leading (group) front of the radiated

field is subject to diffusion (1) because of non-linearity of anomalous dispersion.

Along with the conventional situation when $v_{grs} < v$, the case $v_{grs} > v$ obeys the causality principle as well. At $v_{grs} > v$ the field surpasses the charge (see Fig. 1) and is no longer a normal wake. One can see from (1,2), in both cases the waveform is defined through the retarded argument $t_1 > 0$, which is always less than the current time *t*: $t - t_1 \equiv \frac{vt - z}{v - v_{grs}} > 0$. In other words, the fields are defined by the perturbations occurring in past time.

SYSTEMS WITH ANOMALOUS DISPERSION

Dispersive and moving medium

The condition of anomalous dispersion in an unbounded motionless medium having refraction index n_{med} is trivial:

$$1 - n_{med} < \omega\, dn_{med}/d\omega < 0. \qquad (4)$$

Usually anomalous dispersion is accompanied by resonant absorption of the molecules that breaks corresponding constraint in (3). However, anisotropic "metamaterials" [8,9,10] with unusual properties due to negative dielectric macroscopic permitivity (or negative magnetic permeability) can have $dn_{med}/d\omega < 0$ [11] and, maybe, moderate losses at some frequencies.

One can obtain an anomalous dispersion in a normal medium moving with the velocity $\vec{v}_{med} = \vec{\beta}_{med} c$. If the medium flow is non-relativistic, collinear to the phase velocity, and higher order dispersion is negligible one can find directly from [12]:

$$\beta_{gr} - \beta_{ph} \approx \frac{2\beta_{med} + n_{med}(\beta_{med} n_{med} - 1)}{n_{med}^2} \frac{\omega}{n_{med}} \frac{dn_{med}}{d\omega}. \qquad (5)$$

For a cold plasma formed by (relativistic) electrons moving with velocity β_{med} through a motionless neutralizing background (ions) the solution of the dispersion equation is well known:

$$\beta_{gr} - \beta_{ph} = \pm \beta_{med} / (\omega/\omega_p \pm 1), \qquad (6)$$

where $\beta_{gr} = \beta_{med}$, the signs ± correspond to slow and fast waves respectively, $\omega_p = q\sqrt{n_0/(m_0 \varepsilon_0 \gamma_{med}^3)}$ is the plasma frequency, and $\gamma_{med} = 1/\sqrt{1 - \beta_{med}^2}$ is the relativistic factor for the moving medium.

Thus one can provide $\beta_{gr} > \beta_{ph}$ for slow-wave in drifting plasma or in a passive normal medium at $\beta_{med} > n_{med}/(n_{med}^2 + 2)$, provided high-order dispersion and dissipation effects are not significant.

Dispersive or moving medium in a conducting pipe

For a pipe filled with motionless dispersive medium ($dn_{med}/d\omega \neq 0$) one can easily obtain the following condition of anomalous dispersion ($v_{gr} > v_{ph} > 0$):

$$\frac{1}{\beta_{phs}^2 n_{med}} - 1 < \frac{\omega}{n_{med}} \frac{dn_{med}}{d\omega} < \frac{1}{\beta_{phs}^2 n_{med}^2} - 1. \qquad (7)$$

Let us consider now a pipe with moving medium. Using Lorentz' transformations one can find the dispersion and the following condition of anomalous dispersion:

$$\frac{\gamma_{med} g_s}{|n_{med}| h} > \frac{1}{\sqrt{\beta_{med}^2 - n_{med}^2}}, \quad where \quad |n_{med}| < |\beta_{med}| \qquad (8)$$

where g_s is the modal transverse wavenumber ($g_s = j_{0s}/b$ for a cylindrical pipe of radius *b*), j_{0s} is the s-th root of the Bessel function (of the first kind of the zero-th order for monopole mode).

Relativistic plasma can be a good candidate for anomalous system to satisfy (8). But self-consistent behavior of the plasma makes the analysis more complicated than it is presented here. Along with the active character of the system it can diminish the simplified concept of group velocity.

Periodic slow-wave systems

In a long periodic slow-wave structure the group velocity does not depend on the harmonic number for the given frequency. Consequently, one can always find some n^{th} spatial harmonic for which $v_{gr} > v_{phn} > 0$ in the vicinity of the passband middle provided $N_{per} > \max\{2N_f, n\}$. Moreover, one can anticipate that ohmic losses will satisfy the condition (3) even at room temperatures.

Metal helix was the first classical metal slow-wave structure and continues to be employed successfully in numerous TWTs and BWOs for more than fifty years. Its specific dispersive properties can be applied directly to our case of interest. For instance, helical structures designed to operate at non-relativistic beam energies can have $dv_{ph}/d\omega > 0$ along with $v_{gr} > 0$ at some frequencies for the first and the second space harmonics [1,13,14].

One of such structures is four-thread helix in a pipe depicted in Figure 2. The lowest fundamental mode is quadrupole mode (f≈7.3 GHz, Q≈4000) having β_{gr} =-0.61 for the phase advance per cell $\vartheta = \pi/2$. Correspondingly, for the space harmonic at $\vartheta = 3\pi/2$ at the same frequency we have β_{gr} =+0.61 and β_{ph} =0.21. So, ~12keV electron (or ~7MeV proton) bunch having substantial quadrupole component will induce this mode as a forerunning wave. Monopole mode has β_{gr} =+0.57 group velocity at $\vartheta = \pi/2$ (f≈17GHz, Q≈9000).

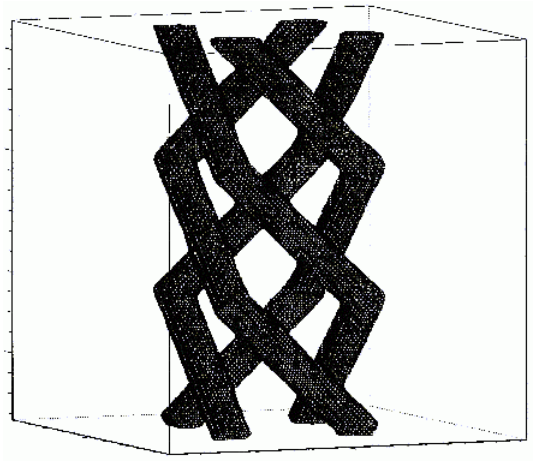

Figure 2: Example of helical slow-wave structure (metal pipe is not shown).

For the 1st spatial harmonic (at $\vartheta=5\pi/2$, monopole mode) one obtains $\beta_{ph}=0.29$ at the same group velocity and frequency. It means that ~25keV electron (or ~26MeV proton) bunch can radiate the reverted "wakefield".

In open waveguides [15,16,17] the group velocity is naturally high and approaches to the speed of light. However, at shorter wavelengths (infrared and optical range) we usually have $\vartheta >> \pi$, oversized mode of operation $N_f >> 1$, no slow waves in the system [17] or the number of slow and fast harmonics interacting with the bunch is too big [18] resulting in degradation of efficiency and length of interaction.

DISCUSSION

At longer wavelengths and low-energy bunches the simplest proof-of-principle system would be some of metal-dominated periodical structures.

Resonant wakefields can affect performance of some high intensity ion linacs (especially RFQ).

BBU effect can occur at $v_{gr} > v_{ph} > 0$. In the absence of reflections, beam break-up would appear as diffusion and shortening of the leading front of the beam pulse (instead of trailing edge shortening for conventional BBU).

Hypothetical laser acceleration scheme in the media having $v_{gr} > v_{ph} > 0$ requires the bunch to be injected *before* the driving electromagnetic (laser) pulse front edge. Analogously, in the case of collinear acceleration driven by a charged bunch in such a medium, its radiation will "push" ahead the bunch to be accelerated in front of the driving bunch.

The treatment given here is not applicable directly to the Cherenkov radiation. However, the group velocity concept could be very useful in studying this effect in more general case when $v_{gr} \neq v_{ph}$. For example, the pre-threshold short-pulse radiation [19,20,21] can be explained qualitatively as a normal wake at $v_{ph} - v_{gr} << v \approx v_{ph}$.

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