APPLICATION OF ENVELOPE INSTABILITY TO HIGH-INTENSITY RINGS *

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Abstract

The space-charge limit is imposed by beam response to low-order machine resonances. Here, the coherent response of the beam to such resonances is discussed, including the parametric resonance of collective beam modes with the periodic lattice, also known as the "envelope instability", when the second-order beam modes are considered. The relation of this parametric resonance to the coherent resonance condition of an integer type is explained. Practical application of such resonant responses to both structural and imperfection driven harmonics is addressed.

INTRODUCTION

When choosing the operating point in the tune space, one carefully avoids resonances driven by the lattice periodicity (structure resonances). However, unavoidable presence of errors in the magnetic field sets restrictions associated with the imperfection resonances. As a result, the condition that the individual particle tune should not be depressed by the space charge to integer or half-integer values is known as the space-charge limit. We note that such a definition of the space charge limit is different from the one used in a special class of circular machines (for example, in cooler rings) where additional efforts are undertaken to compensate for emittance growth. The maximum achievable intensity associated with crossing of the integer or half-integer tunes was first formulated using the single-particle approach. Subsequently, a more accurate treatment of collective beam dynamics gave better understanding of the beam response to such resonances [1]-[2]. Such a coherent resonance condition, corresponding to the half-integer single-particle resonance ($n/2 = \nu$, where $\nu = \nu_0 - \Delta\nu_{sc}$ and ν_0 is the zero-current tune), is

$$n = \Omega_2 = 2\nu_0 - \Delta\Omega_{2,sc}, \qquad (1)$$

where Ω_2 is the frequency of the 2nd order coherent oscillation mode of the beam. The coherent resonance condition in Eq. 1 was first derived using an approximation of smooth focusing. Subsequently, it was shown that AG focusing can lead to an additional subset of collective instabilities [3]. Such an instability due to the lattice periodicity, corresponds to a coherent resonance of the parametric (half-integer) type:

$$n/2 = \Omega_2, \qquad (2)$$

* Work supported by the SNS through UT-Battelle, LLC, under contract DE-AC05-00OR22725 for the U.S. Department of Energy.

also known as the "envelope instability" [4]. As one can see from Eq. 2, the envelope instability limits the allowable tune space to only 0.25 and thus may have an additional impact on the performance of high-intensity machines. It is therefore extremely important to understand when such envelope instability should be taken into account, and whether it can alter the space-charge limit governed by Eq. 1. The primary goal of this paper is to explain the difference between the coherent integer and half-integer envelope responses and provide practical guidelines for consideration of the envelope instability in rings. For completeness, we also discuss coherent resonances for high-order beam modes.

GENERAL ANALYSIS

We start with the $m = 2$ modes, allowing us to employ the envelope equation:

$$a'' + K(s)a - \frac{\epsilon^2}{a^3} - \frac{\kappa}{a} = 0, \qquad (3)$$

where $K(s)$ is the periodic focusing function, ϵ is the beam emittance, κ is the space charge parameter, and a is the radius of a round beam. For simplicity, we replace the periodic focusing by a time-dependent perturbation. The linearized envelope equation for small oscillations $[a \rightarrow a_0(1+u)]$ is then

$$u'' + \Omega_2^2 u + (nt) = (1+u)\nu_0^2 \sum_n \alpha_n \cos(n\theta) + (nt), \qquad (4)$$

where nt stands for "nonlinear terms". If the coefficient α_n becomes large, as in the case of periodic focusing, a more accurate treatment of the stationary state is required [5]. In Eq. 4, the term which drives matched periodic oscillation of the beam envelope and the term which describes the oscillations around such a periodic solution are both kept on the r.h.s., for comparison. In principle, these two terms should be treated separately, since the first term is just the closed orbit solution (matched solution in the presence of a time dependent perturbation). However, when only the second term is kept, as typically done to consider beam stability as a result of periodic focusing, an important implication of the first term may be overlooked. In particular, the resonant growth of such stationary solutions near the half-integer tunes, due to the first term, becomes an important effect, known as Smith/Sacherer space-charge induced coherent beam response to the imperfection resonances. Here, the resonant response is considered both for the case when the integer and half-integer coherent response occurs at different tunes, as well as when the driving harmonics provide a resonance condition for both resonances at the same tune.

The first term on the r.h.s. of Eq. 4 corresponds to an integer $\Omega_2 = n_1$ coherent resonance of the beam envelope with harmonic n_1, which occurs near the half-integer single-particle tunes. The second term on the r.h.s. of Eq. 4 gives the coherent resonance of the half-integer (parametric) type: $\Omega_2 = n_2/2$, where n_2 is now another harmonic different from n_1. The limitation due to this linear parametric resonance of the envelope modes with the periodic focusing structure suggests a design with the zero-current single-particle phase advance σ_0 per focusing period below 90^0. In rings, such a condition corresponds to the structure resonance which occurs near the single-particle tunes $\nu = N/4$ with N being the structure harmonic. However, the structure resonances are typically avoided by the choice of the operating tune-box resulting in the limitation due the imperfection resonances. Note that in a special class of rings (such as cooler rings), where additional measures are undertaken to compensate emittance growth due to the crossing of the imperfection resonances, one recovers the situation as in linear transport systems.

Resonance strength

The width of the linear parametric resonance is [5]:

$$\Delta\epsilon \approx \frac{\nu_0^2}{2\Omega}\alpha_n = \frac{\nu_0^2}{2\sqrt{2}\sqrt{1+\eta^2}}\alpha_n, \qquad (5)$$

where, as an example, we substituted for Ω the frequency of the in-phase mode, with η being the tune depression defined as $\eta = \nu/\nu_0$. It depends linearly on the strength of the imperfection error α_n (to first order). Such a strength is very small for typical imperfection errors (much less than the 1% level). As a result, the corresponding resonance is expected to be very narrow and the envelope growth is detuned at a very low level due to the non-linear terms in Eq. 4, which was confirmed by numerical simulation [6]. This is, in fact, the reason why the effect of the envelope instability in rings is negligible, provided that the tune-box is chosen free of the structure resonances and only imperfection harmonics are of a concern.

On the contrary, when the source of the parametric driving term in Eq. 4 is due to the periodicity of the lattice, the width of the resonance may become significant. Strictly speaking, a perturbation approach is not applicable for very large α_n, and one needs to solve the exact equation with periodic focusing numerically. This defines the stopbands of the structure resonances which should be avoided.

Combined resonance response

If the zero-current tune is chosen in the tune-box free from the structure resonances then the effect of the parametric envelope resonance due to the imperfection harmonics at $1/4$ tunes is negligible. Also, there is no integer-type

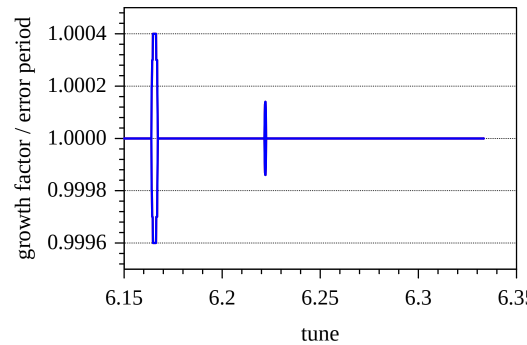

Figure 1: Growth factors for imperfection driven envelope instability with working point $\nu_{0x,y} = 6.333$ and a large error of 4% in harmonic $n = 25$.

envelope growth at such tune values with the stationary solution for periodic oscillations of the beam envelopes being well defined. When one approaches the integer or half-integer tunes, this results in a periodic growth of the beam envelopes which is described by the coherent integer response in Eq. 1. At such tunes there is also the possibility of parametric growth of the envelopes due to higher harmonics. For the parametric resonance to take place at such tunes, the stopband of the parametric resonance (due to the α_{n_2}) should be much larger than the integer stopband due to the α_{n_1}. For example, in the PSR LANL lattice with the zero-current betatron tune above $\nu = 2.5$, the parametric resonance of the beam envelope would take place at high-intensity operation. This is because the strength of the $n = 10$ harmonic (α_{10}) is much larger than that of the $n = 5$ (α_5) harmonic, since $n = 5$ is a weak imperfection harmonic while $n = 10$ is the strongest harmonic with the lattice super-periodicity $P = 10$ [8].

Extension to non-linear modes

For the case of the non-linear imperfection errors one has to consider tune values near the corresponding imperfection resonances. Similar to the $m = 2$ modes, the high-order modes can have resonant growth near

$$n = \Omega_m, \qquad (6)$$

which is the coherent resonance condition for any order beam mode Ω_m derived by Sacherer [2]. To derive such a resonance condition for $m > 2$ modes one needs to use either high-order beam moment equations or the Vlasov equation. In addition, the effect of the periodic focusing adds the possibility of Ω_m resonating at the half-integers, which corresponds the parametric resonance of beam modes [7]:

$$n/2 = \Omega_m. \qquad (7)$$

The practical discussion for the typical strength of an imperfection error is now similar to the discussion for the $m = 2$ modes [5].

For completeness, we note that in the absence of non-linear imperfections, the periodic oscillation of high-order beam modes is now well defined so that the condition $n = \Omega_m$ no longer applies, and stability is now determined solely by the parametric condition $n/2 = \Omega_m$ [7]. In fact, this becomes the dominant effect in the high-current transport channel or cooler rings. With harmonic n now being the structure harmonic, the beam encounters a whole set of instabilities during the space-charge tune depression. Such instabilities were first numerically explored in connection with transport channels [3], and recently were analytically described using the terminology of resonances with an application to cooler rings [7].

When the beam has a large mismatch, the nonlinear terms ignored in the linearized approach can play an important role. In such a very general case, the condition for the non-linear parametric resonance is [5]:

$$n/k = \Omega_m, \tag{8}$$

where k now stands for the exponent of the non-linear term in the driving potential. This is similar to the non-linear envelope resonances $n/k = \Omega_2$ when the beam envelopes are mismatched [9]. Also note that in such a form, the resonance condition applies also for the coupling resonances, since the subscript m only indicates the order of the mode.

NUMERICAL ANALYSIS

Imperfection envelope instability

We used the KVXYG [4] code, which determines the growth factors of the envelope perturbations. A parametric resonance of the beam envelope may be expected in a lattice with a working point above $1/4$ tunes. We have taken a constant focusing lattice with $\nu_{0x,y} = 6.333$ and a 25-th harmonic gradient error, which implies $\sigma_0 = 91.2^0$ per error harmonic. For a large error of 4%, only very narrow stopbands of the out-of-phase and in-phase modes are found at $\sigma = 89.59^0$ and $\sigma = 88.78^0$ (corresponding to $\nu_{x,y} = 6.222$ and $\nu_{x,y} = 6.165$), respectively. We also confirmed that for errors of 2% and 1%, the width of these stopbands decreases linearly with the error strength, in agreement with the stopband of the parametric resonance given in Eq. 5. As a result, the instability gets detuned at a very low level. This allows the conclusion [6] that the imperfection driven envelope instability for working points above the fractional quarter-integer tunes can be ignored.

Structure envelope instability

The case of the parametric resonance of the beam envelope with the structure lattice harmonics is similar to the one studied with application to the transport channels or cooler rings. As an example, for a working point above $\nu_{0x,y} = 6$, as in the SNS, and the lattice which consists of 24 basic cells, we calculated the instability stopband for a cell with $\sigma_0 = 96^0$ corresponding to $\nu_{0x,y} = 6.4$ (here $\nu_0 = N\sigma_0/(2\pi)$, where N is the number of cells in the lattice). We noted that a pronounced instability stopband

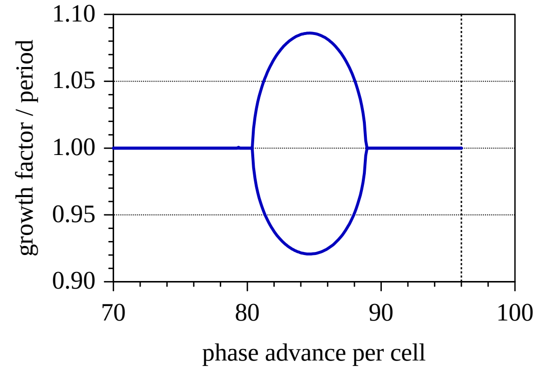

Figure 2: Growth factors for structure driven envelope instability with zero-current phase advance of $\sigma_0 = 96^0$, corresponding to $\nu_{0x,y} = 6.4$.

(Fig. 2) with a growth factor above unity starts for full-current phase advance $\sigma < 88.84^0$ (corresponding to a tune $\nu_{x,y} = 5.92$). The strong flutter of the matched FODO envelope couples the in-phase and out-of-phase eigenmodes and leads to a single stopband. In a realistic lattice one tries to avoid structure resonances by choosing the working point correspondingly.

SUMMARY

We examined the impact of integer and half-integer resonances of the collective beam modes on intensity limitation in the high-intensity rings. The imperfection driven resonance of the beam envelopes was found to be negligible. As a result, it is not expected to impose an additional restriction in the tune space.

ACKNOWLEDGMENTS

One of the authors (A.F.) is indebted to J. Holmes, S. Danilov, S. Cousineau and G. Parzen for many useful discussions on this subject. He also wishes to thank the Accelerator Physics group of the SNS project.

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