COMPENSATION OF NONLINEAR RESONANCES IN THE PRESENCE OF SPACE CHARGE *

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Abstract

Imperfection non-linear resonances can lead to undesirable beam loss and thus limit the high-intensity operation. In the presence of space charge, beam response to such resonances is strongly influenced by collective beam dynamics. It is thus crucial to understand the effectiveness of the non-linear resonance compensation. We explore various procedures in the resonance correction and suggest some practical applications. Effectiveness of resonance correction for the high-intensity operation is discussed.

INTRODUCTION

A single-particle theory of nonlinear resonances and their correction is well developed. Nonlinear fields cause tune dependence on amplitude and islands to appear in the phase-space plots. When crossing the resonance, the large amplitude particles can be locked into the islands, moved to large amplitudes, and possibly be lost. Space charge introduces several aspects into resonance crossing. When the space-charge tune depression is increased, particles with the smallest amplitudes reach the resonance first as they have the largest tune shift. In addition, the space charge introduces the largest source of nonlinearity which changes the response to a resonance. For dynamic resonant response of a beam, the static description via the single-particle tune shift becomes inaccurate. A dynamic solution which includes the space-charge force and density redistribution is required [1] . All these aspects of the space charge raises very important questions: What is the role of the space charge in resonance crossing? How to account for the space-charge nonlinearity in resonance correction? Can one have a good correction of the resonances for the high-intensity operation? In an attempt to answer these question, we performed a systematic study of resonance correction issues for the high-intensity operation of the SNS ring, using the DYNA [2] and UAL [3] codes.

RESONANCE CORRECTION

The single-particle resonance condition is

$$n_xQ_x + n_yQ_y = p, \tag{1}$$

where n_x, n_y, p are the integers, with p being the driving harmonic. In the presence of the space charge it is slightly modified [1]. In this paper we use the single-particle terminology but also make an analogy with the collective dynamics. A linear stopband can be written as

$$\Delta\epsilon =\mid \kappa \mid J_x^{N/2} J_y^{N/2} \left[\frac{n_x^2}{J_x} + \frac{n_y^2}{J_y}\right], \tag{2}$$

where J_x, J_y are actions, and $N =\mid n_x \mid + \mid n_y \mid$ is the resonance order. The excitation strength κ is

$$\kappa = \frac{1}{2\pi \mid n_x \mid! \mid n_y \mid!} \int_0^{2\pi} \beta_x^{\frac{|n_x|}{2}} \beta_y^{\frac{|n_y|}{2}} e^{i\psi} K(\theta) d\theta, \tag{3}$$

where $\psi = n_x(\mu_x - Q_x\theta) + n_y(\mu_y - Q_y\theta) + p\theta$, and $K(\theta)$ is related to the error field multipoles.

One way of correction is to minimize κ, using Eq. 3. Although this can be done in simulation when the field errors are known, in real life problems arise from the source of errors which are unknown. The measurement of the stopband has some difficulties. First, by crossing the resonance, one can measure accurately only a symmetric stopband. If there is a source of the nonlinearity, the stopband becomes asymmetric, and the loss is different depending on the direction of the resonance crossing. More importantly, the loss observed, for example, on a current monitor depends not just on the resonance strength but on the beam-pipe characteristics as well. In that sense, the measurement via the stopband is indirect and does not provide a desired accuracy.

Another way of correction is to measure the islands of the nonlinear resonance. The island width, to first order, is

$$\Delta J \sim \sqrt{\frac{\kappa}{\alpha}} J_0^{N/4}, \tag{4}$$

where α describes tune change at the island center, J_0 is action at the island center. Measurement of the island width gives a more direct information about the resonance strength since it is independent from losses on the beam pipe. Of course, one needs to answer the question what is the role of the nonlinearity α in such a measurement, especially when it will dynamically change and become large due to the space charge in the process of accumulation. A systematic study of this question was performed and finding are summarized below.

An accurate way to obtain information about the island width, is to perform tune (Q) vs amplitude (A) scan near the resonance of interest. The flat region in tune values on the Q vs A diagram gives information about the width of the island [4]. The resonance is corrected by minimizing this flat region to zero. Such a method requires a significant

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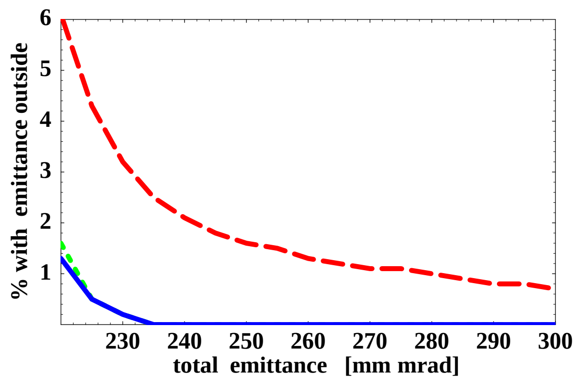

Figure 1: w.p.(6.36,6.22), $N = 0.6 \cdot 10^{14}$: sextupole errors, no correction-red (long-dash) line; correction of $3Q_x = 19$-green (short-dash) line; no errors-blue (solid) line.

amount of time due to a required amplitude scanning. An alternative way to get similar information quickly is to kick the beam into the islands and measure the corresponding frequency spectrum. When some portion of the beam is locked into the island, one obtains the corresponding peak with the tune measurement device. The correction knobs are used to adjust the amplitude of the peak in the spectrum to zero. Such a method was recently applied for correction of nonlinear resonances at RHIC [5]. In simulations, we use the Q vs A method to find the best strength for the correctors and compare it with the stopband compensation.

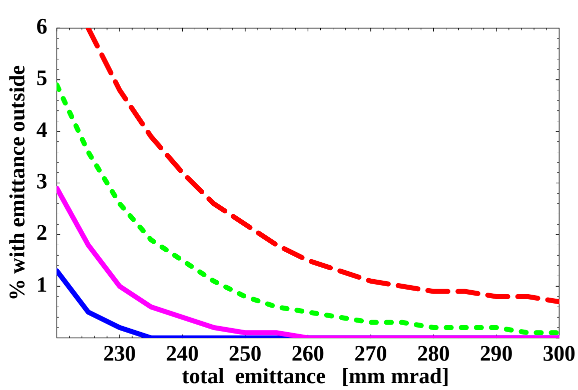

Figure 2: w.p. (6.36,6.22), $N = 2 \cdot 10^{14}$: sextupole errors without correction - red (long-dash) line; correction of $3Q_x = 19$-green (short-dash); correction of $3Q_x = 19$, $2Q_y - Q_x = 6$ resonances - pink; no errors - blue (solid).

APPLICATION TO THE SNS

First, the strength of correctors are found for a zero space charge using the DYNA code. The nonlinearity due to the quadrupole fringe fields generate islands of a finite width. It was also found that the magnitude of the nonlinearity is not important, and that correction works nicely even for dynamic increase of the strength of the nonlinearity due to the space charge. The corresponding correctors were then used in the UAL to simulate a dynamic accumulation process with and without the resonance correction. Various aspects of correction were studied using the sextupole resonances. The studies were then extended to include the 4th order resonances. Here, we present the studies based on a lumped source of errors, where randomly distributed errors were lumped into a single magnet. Also, for the studies of the sextupole resonances, the strength of errors was taken 5 times bigger than currently measured while for the octupole resonances - 10 times bigger. A larger strength of errors was required in order to see an appreciable emittance growth during the accumulation cycle.

Correction of 3rd order resonances

With only the sextupole errors being introduced, Fig. 1 shows the beam halo at the end of accumulation for the w.p. $(6.36, 6.22)$ and intensity $N = 0.6 \cdot 10^{14}$ protons. Without correction, the loss due to the $3Q_x = 19$ resonances is about 2% at the acceptance of $240\ \pi$ [mm mrad]. When the resonance is corrected (with two correctors at large β_x), the loss goes to zero at this acceptance. The intensity is then further increased to $2 \cdot 10^{14}$ and the loss increases to 1.6% (shown in Fig. 2). This is due to the next sextupole resonance $2Q_y - Q_x = 6$, crossed at higher intensity. We now use two other correctors at large β_y to correct this resonance. The corresponding loss with correction of both resonances decreases to 0.5%. Note, that although a good correction of $p = 6$ harmonic was possible in simulation, it will not be that good in practice, since the present location of the sextupole correctors in the SNS provides good correction only for the odd harmonics. As a result, use of other magnets with a proper phase advance (like some of chromatic sextupoles) will be needed for compensation of the resonances due to the sixth harmonic.

The fact that present location of correctors in the SNS allows a good control over the odd harmonics tempted us to explore how far in intensity we can actually go with the resonance correction, choosing, for example, the w.p. (6.4,6.3). All the working points used here, of course, should be understood as an approximate. In real life, one would be unlikely to tune the lattice exactly to a sum resonance. However, the space charge pushes the tunes away from this resonance, and a good correction is possible, as shown in simulations. Figure 3 demonstrates correction of $Q_x + 2Q_y = 19$ and $3Q_x = 19$ resonances, using two different sets of correctors. The pink (dotted) curve corresponds to $N = 3 \cdot 10^{14}$ and black (upper solid) curve corresponds to $4 \cdot 10^{14}$ with both corrections being applied. Adjusting the location of primary scraper, we can attempt to push intensity to almost $4 \cdot 10^{14}$ with relatively low losses, which was speculated before by constructing the loss curves without resonance correction [6].

Simultaneous correction of various resonances

The studies were extended to include the normal and skew octupole resonances, although the SNS does not have

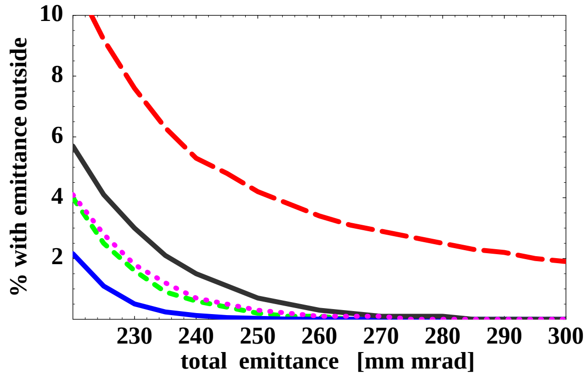

Figure 3: w.p. (6.4,6.3): $N = 2 \cdot 10^{14}$, no correction - red (long-dash line); correction of $3Q_x = 19$ and $Q_x + 2Q_y = 19$ resonances: $2 \cdot 10^{14}$-green (short-dash), $3 \cdot 10^{14}$-pink (dotted line), $4 \cdot 10^{14}$-black (upper solid line).

skew-octupole correctors at the present moment. In all the cases, we were able to achieve a good correction results. For example, Fig. 4 shows high-intensity loss for the w.p. (6.36,6.22), with both sextupole and octupole errors, and correction of 3rd and 4th order resonances.

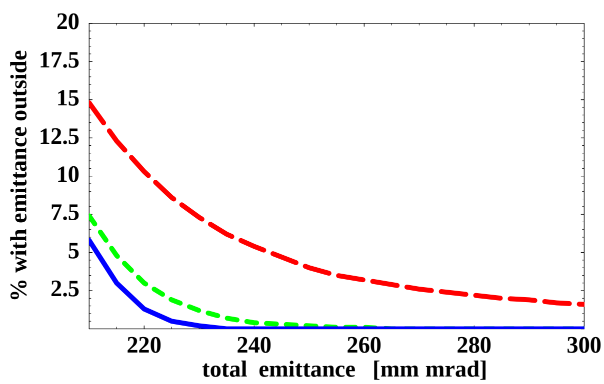

Figure 4: w.p. (6.36,6.22), $N = 2 \cdot 10^{14}$: no errors-blue (solid line); octupole and sextupole errors, no correction-red (long-dash line); correction of $3Q_x = 19$, $2Q_y - Q_x = 6$, and $2Q_x + 2Q_y = 25$ resonances-green (short-dash).

Figure 5 shows the tune space with all the 3rd and 4th order resonances which require correction for the two working points being considered. We were able to achieve very good correction with a loss at the primary scraper of about $5 \cdot 10^{-3}$. Even though, the loss is still higher than the one without resonance excitation, it should be possible to reach the uncontrolled beam loss of desired level by adjusting the aperture of the primary scraper.

SELF CONSISTENT DESCRIPTION

Although we used the terminology of a single-particle dynamics, the same description can be given using a self-consistent approach of collective dynamics. It provides a physical picture of dynamics process, including the growth

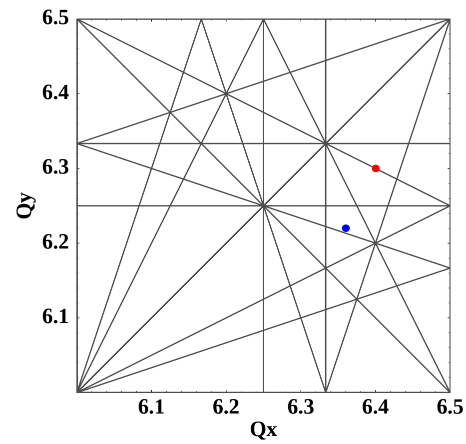

Figure 5: Tune-space with 3rd and 4th order resonances need for correction of w.p. considered.

of emittance and adiabatic increase of losses when intensity is slowly increased [1]. It also explains why the strength of the nonlinearity is not important in the resonance correction. This is because the nonlinearity determines the maximum amplitude of period-p oscillations of the corresponding collective mode (fixed point of an island), while the width of the resonance, in first order, is determined just by the resonance strength κ. As a result, if one manages to measure and correct the stopband of the resonance with a good accuracy experimentally, such a correction will give results as good as the island correction. This was confirmed in present simulations with good correction being achieved, using both the stopband and island correction methods.

SUMMARY

Correction of the nonlinear resonances with application to high-intensity operation was studied. Our findings suggest that it should be possible to correct resonances with a good accuracy and thus control beam loss at a low level.

ACKNOWLEDGMENTS

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