INVESTIGATION AND SIMULATION OF MUON COOLING RING WITH TILTED SOLENOIDS

V. Balbekov, FNAL, Batavia, IL 60510, USA *

Abstract

Alternating solenoid focused muon cooling ring without special bending magnets is considered and investigate in detail. Both fringe field between solenoid coils with opposite directed current, and an inclination of the coils in vertical plane are used to provide a bending and closing of the particle trajectories. Realistic (Maxwellian) magnetic field is calculated and used for a simulation. Methodic is developed and applied to find closed orbit at any energy, dispersion, region of stability, and other conventional accelerator characteristics. Earlier proposed RFOFO cooling ring with 200 MHz RF system and liquid hydrogen absorbers is investigated in detail. After an optimization, normalized 6D emittance about 20 mm^3 and transmission 57% are obtained.

INTRODUCTION

So called RFOFO ring for an ionization cooling of muon beams was proposed and simulated for the first time in Ref. [1]. Only solenoids was used in this design to get magnetic field, presuming that the vertical (bending) component can be obtained by tilting of the solenoid coils. Actually bending field in Ref. [1] was generated by a truncated Fourier decomposition of the field from a bent solenoid. The real coils to generate the axial field, in the presence of bending field, was not considered then.

Simulation with realistic field produced by vertically tilted coils was performed for the first time in Ref. [2]. Both fringe field between solenoids with opposite directed current, and an inclination of the coils in vertical plane are used to bend the particle trajectories and to get closed orbits in a wide energy interval. This report is a further development and optimization of this cooler. [1]

LATTICE

Following Ref. [1], the ring consisting of 12 periods is considered, each providing turn angle 30°. The period shown in Fig.1 includes 2 solenoid coils with opposite direction of currents, 5 cavities, and liquid hydrogen wedge absorber. Period length is 275 cm along the centerline of the coils that is a circle of radius $R_0 = 275 \times 12/(2\pi) \simeq 525.21$ cm. Therefore circumference of the ring is 33 m along this line. Space between the centers of an absorber and nearest solenoid coils is 55 cm. Another parameters of the equipment are taken from Ref. [1] and listed in Table 1.

* Work supported by the Universities Research Association, Inc., under contract DE-AC02-76CH03000 with the U. S. Department of Energy.

[1] Note at edition: Recently analogous investigation is done and similar results are derived in Ref. [3].

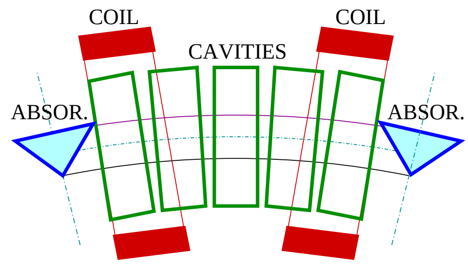

Figure 1: Layout of the cooling ring (one period, schematically).

Table 1: Parameters of the Cooler

Inner radius of coils	77 cm
Outer radius of coils	88 cm
Coil length	50 cm
Current density	± 95.27 A/mm^2
Tilting angle of the coil	± 52 mrad
Accelerating frequency	199.2 MHz
Accelerating gradient	16 MeV/m
Synchronous phase	33°
Absorber thickness at the center	28.5 cm
Energy loss at the center	12.52 MeV
Gradient ox energy loss dE/dy	0.7 MeV/cm

MAGNETIC FIELD

We will work in a cylindrical frame $r\theta y$; however, more conventional variables $x = r - R_0$ and $z = R_0\theta$ are actually used ($R_0 = 525.21$ cm). Magnetic field in this frame is obtained in several stages:

• Flat field map of single coil is generated in its natural frame $r_{coil}z_{coil}$ for the region $0 \le z_{coil} \le 540$ cm, $r_{coil} \le 480$ cm.

• Grid is prepared in the cylindrical xyz frame for the area: -36 cm $\le x \le 14$ cm, -23 cm $\le y \le 27$ cm, -770 cm $\le z \le 770$ cm, at the step of 1 cm in any direction.

• Position of each node of the grid is transformed to the frame of a positive coil with its inclination taken into account. Field in this point is calculated in $r_{coil}z_{coil}$ frame using the coil field map and linear interpolation.

• Obtained field components are transformed to the cylindrical frame xyz resulting field map of single tilted coil with positive current centered at $x = y = z = 0$. Field of the coil with opposite current and inclination is generated using properties of symmetry.

• Centers of positive and negative coils are moved to $z = \mp 82.5$ cm correspondingly, and their fields are added resulting field of the coil pair.

• Fields of central pair and 2 pairs shifted to the left and right with a step of 275 cm are added resulting field map of the period at -137.5 cm $\le z \le 137.5$ cm.

Components of periodical magnetic field are plotted against longitudinal coordinate in Figs. 2 and 3 at different transverse coordinates. The longitudinal field almost does not depend on transverse coordinates, radial field depends mostly on x (and z), and vertical field – on y (and z).

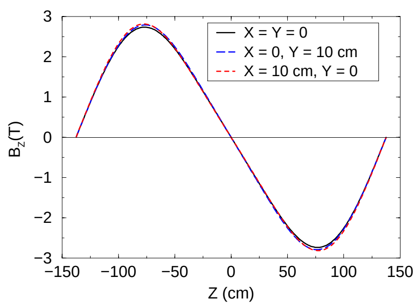

Figure 2: Longitudinal magnetic field at different transverse coordinates.

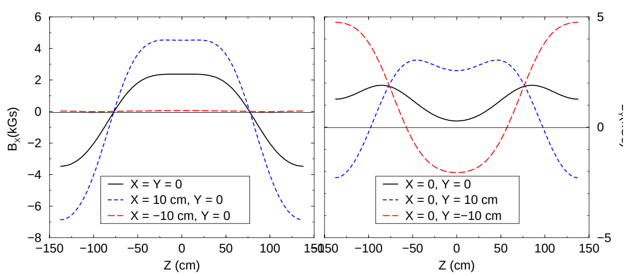

Figure 3: Transverse magnetic field at different transverse coordinates (left – radial, right – vertical).

CLOSED ORBIT AND DISPERSION

A search of closed orbit is not a trivial problem in such complicated magnetic field. Symmetry of the field facilitates it, because deviations of closed orbit from centerline should be even function of z at any energy. The results are presented in Fig.4 at total energy from 190 MeV to 270 MeV with step 10 MeV. Swing of the orbit is minimal at reference energy $E_{ref} = 220$ MeV at chosen inclination angle 52 mrad. The orbit is shifted on about 11 cm to the ring center. where the particle is kept both by transverse and longitudinal fields. Magnetic field on this orbit is plotted in Fig.5.

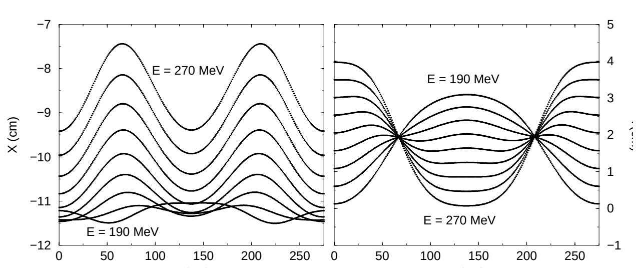

Figure 4: Deviation of closed orbit from centerline *vs* distance at different energy. (left – radial, right – vertical).

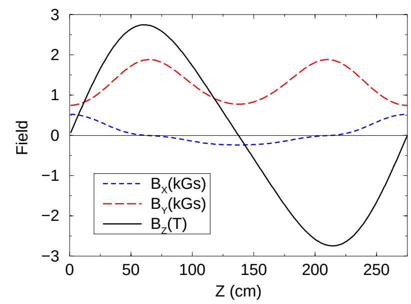

Figure 5: Magnetic field vs longitudinal coordinate on 220 MeV closed orbit.

Dispersion, that is deviation of closed orbit from reference one, is shown in Figs.6. Left plot shows dependence of dispersion at the absorber center on total energy. The vertical dispersion is surprisingly linear, however radial one is nonlinear at all. Therefore only vertical wedge absorbers will be used in the ring for emittance exchange. Right Fig.6 represents dispersion function *vs* distance at E = 220 MeV. Vertical dispersion is about 8 cm at the absorber center.

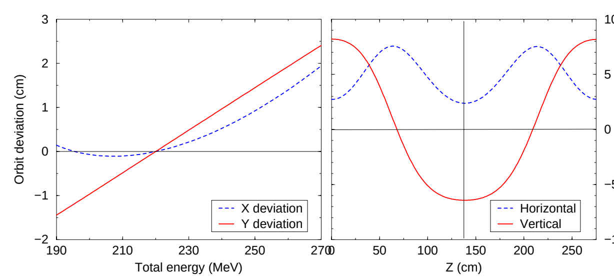

Figure 6: Left: dispersion at the absorber center *vs* total energy. Right: dispersion function at 220 MeV *vs* distance.

REGION OF STABILITY

4D transfer matrix was calculated for small deviations of a particle with arbitrary energy from corresponding closed orbit. Its eigenvalues are plotted in Fig.7. It is seen that region of stability extends from 185 to 265 MeV, end eigenvalues are almost equal in pairs (there is conjugated pair). Long-dashed line on this plot represents beta-function the same rectilinear channel, which has almost the same region of stability. It means, probably, that this 2D beta-function can be used for estimations of equilibrium emittance, etc. By such a consideration, equilibrium transverse emittance of a cooled beam should be about 1.5 mm without an emittance exchange.

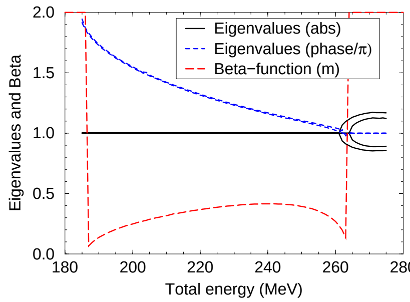

Figure 7: Eigenvalues of the transfer matrix and beta-function at the absorber center *vs* total energy.

COOLING SIMULATION

2000 particles are used for the cooling simulation of Gaussian beam with the parameters listed in Table 2 (3rd column is commented later). After random generation, the

Table 2: Parameters of injected and cooled beams

	Inject	10 turns
Horizontal emittance (cm)	1.2	.26
Vertical emittance (cm)	1.2	.23
Longitudinal emittance (cm)	1.6	.31
Horizontal r.m.s. size (cm)	5.0	2.3
Vertical r.m.s. size (cm)	5.0	2.1
Horiz. r.m.s. momentum (MeV/c)	25	12
Vertical r.m.s. momentum (MeV/c)	25	12
R.m.s. bunch length (*ct*) (cm)	11	4.7
R.m.s. energy spread (MeV)	15	6.9
Transmission without decay	1	.75
Transmission with decay	1	.57
Merit factor	1	68

following correlation has been applied to take into account linear dispersion

$$x = x_{random} + D_x(\Delta p/p)_{random},$$

and similar expression for y where $D_x = 2.69$ cm and $D_y = 8.15$ cm (see right Fig.6). Then a correlation due to dependence of revolution frequency on transverse momentum is included by the formula:

$$E_{total} = \Delta E_{random} + E_{ref}\sqrt{1 + (p_{t,random}/mc)^2}.$$

Note that all the correlations are excluded by inverse transformations at the emittance calculation of the cooled beam. The beam is injected in the absorber center at transverse coordinates: $x_c = -11.36$ cm, $y_c = 1.57$ cm corresponding to closed orbit at reference energy 220 MeV.

Evolution of the beam emittance and transmission at the cooling is shown in Fig.8. Parameters of the beam after 10

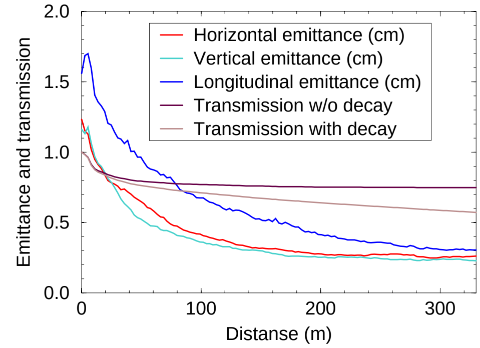

Figure 8: Evolution of the beam parameters at the cooling.

turns (330 m) are presented in 3rd column of Table 2. Merit factor is defined as

$$M = T\frac{\epsilon_{6,initial}}{\epsilon_{6,final}}$$

where T is transmission with decay, $\epsilon_6 = \epsilon_x\epsilon_y\epsilon_z$ is 6D beam emittance.

Transverse and longitudinal phase space of the beam before and after 10 turns cooling are representer in Figs. 9 and 10.

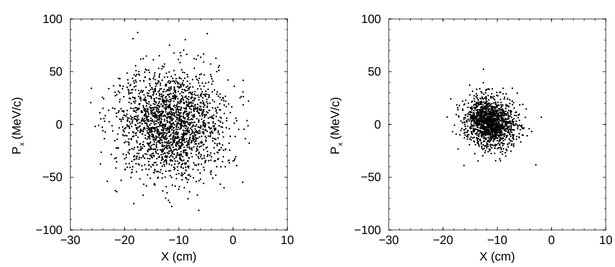

Figure 9: Projection of phase ellipsoid on $x - p_x$ plane before (left) and after the cooling (right)

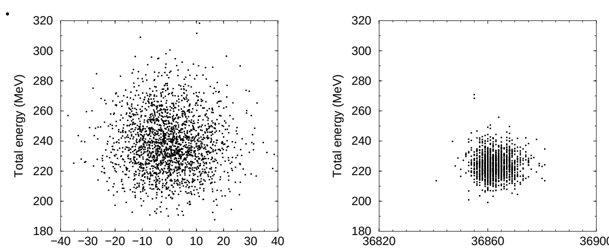

Figure 10: Projection of phase ellipsoid on $ct - E$ plane before (left) and after the cooling (right)

REFERENCES

[1] J.S.Berg, R.C.Fernow, R.B.Palmer. ”An Alternating Solenoid Focused Ionization Cooling Ring”. FNAL McNote 239, March 2002.

[2] V.Balbekov. ”Simulation of RFOFO Ring Cooler with Tilted Solenoids”, FNAL McNote 264, November 2002.

[3] R.C.Fernow, J.S.Berg, J.C.Gallardo, R.B.Palmer, “Muon Cooling in the RFOFO Ring”, FNAL McNote 273, April 2003.