GLOBAL BEAM-BASED ALIGNMENT METHOD

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Abstract

This paper presents an algorithm for orbit centering in the quadrupole magnets developed specifically for the early commissioning stage of SPEAR3 light source. During commissioning many factors can reduce the effectiveness of simpler beam-based alignment techniques. Our method is more tolerant towards inaccurate optics model, undetected large alignment errors, systematic errors and faults in the BPM system.

INTRODUCTION

Beam Based Alignment (BBA) techniques estimate the beam offsets with respect to the centers of magnets, most commonly quadrupoles, in an accelerator lattice. BBA uses measured changes in the beam position in response to static changes or periodic modulation of magnet strength. The two well-known approaches to BBA are: model-based and empirical zero-response search.

Model-based techniques are prone to error due to the difference in the optics of the used model and the real machine. This difference is caused in part by the unknown orbit offsets in sextupole magnets, which creates normal and skew quadrupole terms not accounted for in the model. The method proposed here greatly improves the model-based approach by extracting additional information about the optics distortion from the same orbit response data.

METHOD

The method is based on finding an equivalent (computer) model of a storage ring that produces the same orbit shifts in response to physical quadrupole shift and tilt as the machine in response to a change in the quadrupole strength.

Analytical solution

Using the analytical results and notation of [1] the shift in the closed orbit around the ring $\delta\bar{x}_s = \left(\delta x \;\; \delta x' \;\; \delta y \;\; \delta y'\right)^T$ as a result of changing the strength Δk of one quadrupole is:

$$\delta\bar{x}_s = M_{s\leftarrow m}\left[I - M_m\right]^{-1}\underline{\delta}\Delta\bar{x}_m \qquad (1)$$

Where M_m is the one turn transfer matrix. Subscript m denotes quantities at the midpoint of the quadrupole. $M_{s\leftarrow m}$ is the transfer matrix to a location s in the ring. Vector $\Delta\bar{x}_m = \left(\Delta x_m \;\; \Delta x'_m \;\; \Delta y_m \;\; \Delta y'_m\right)^T$ describes the transverse position and angle orbit offset with respect to the quadrupole center.

$$\underline{\delta} = \Delta kl\begin{bmatrix} 0 & \dfrac{\sigma^-}{k} & 0 & 0 \\ -\sigma^+ & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{\sigma_h{}^-}{-k} \\ 0 & 0 & -\sigma_h{}^+ & 0 \end{bmatrix} \qquad (2)$$

Assuming a horizontally focusing quadrupole, positive k

$$\sigma^\pm = \frac{l\sqrt{k} \pm \sin\left(l\sqrt{k}\right)}{2l\sqrt{k}}\ , \ \sigma_h{}^\pm = \frac{l\sqrt{k} \pm \sinh\left(l\sqrt{k}\right)}{2l\sqrt{k}} \qquad (3)$$

The interpretation of (1) is particularly simple when there is no coupling between x and y planes. In the uncoupled case for the horizontal shift

$$\delta x_s = \delta_s^\theta \theta_m + \delta_s^\Delta \Delta_m \qquad (4)$$

Transverse offset Δx gives rise to the first term, which is a familiar orbit perturbation due to a dipole error:

$$\delta_s^\theta = \sqrt{\beta_m\beta_s}\,\frac{\cos\left(\left|\Delta\phi_{s\leftarrow m}\right| - \pi\nu\right)}{2\sin(\pi\nu)} \qquad (5)$$

Angular offset $\Delta x'$ generates the second term:

$$\delta_s^\Delta = \sqrt{\frac{\beta_s}{\beta_m}}\,\frac{\alpha_m\cos\left(\left|\Delta\phi_{s\leftarrow m}\right| - \pi\nu\right) - \sin\left(\left|\Delta\phi_{s\leftarrow m}\right| - \pi\nu\right)}{2\sin(\pi\nu)} \qquad (6)$$

$$\theta_m = -\Delta kl\sigma^+\Delta x_m \qquad \Delta_m = \frac{\Delta k}{k} l\sigma^-\Delta x'_m \qquad (7)$$

Equivalent model

It is possible to approximate the Δk response of a real storage ring as an orbit change in an *equivalent model*. In order to show this, we examine the differential orbit changes in response to two types of model perturbations that can be easily implemented in most accelerator optics codes.

When a quadrupole is shifted by ξ in x, the propagation of particle is given by

$$\begin{pmatrix} x \\ x' \end{pmatrix}_e = G_x\begin{pmatrix} x \\ x' \end{pmatrix} + \left[I - G_x\right]\begin{pmatrix} \xi \\ 0 \end{pmatrix} = G_x\begin{pmatrix} x \\ x' \end{pmatrix} + \xi D_\xi \qquad (8)$$

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$$D_\xi = \begin{pmatrix} 1-\cos\left(l\sqrt{k}\right) \\ \sqrt{k}\sin\left(l\sqrt{k}\right) \end{pmatrix} \tag{9}$$

$$G_x = \begin{bmatrix} \cos\left(l\sqrt{k}\right) & \frac{1}{\sqrt{k}}\sin\left(l\sqrt{k}\right) \\ -\sqrt{k}\sin\left(l\sqrt{k}\right) & \cos\left(l\sqrt{k}\right) \end{bmatrix} \tag{10}$$

The other type of model perturbation involves derivative kicks of $\pm\vartheta$ at the quadrupole ends. This leads to

$$\begin{pmatrix} x \\ x' \end{pmatrix}_e = G_x \begin{pmatrix} x \\ x' \end{pmatrix} + \vartheta D_\vartheta \tag{11}$$

$$D_\vartheta = \begin{pmatrix} -\frac{1}{\sqrt{k}}\sin\left(l\sqrt{k}\right) \\ 1-\cos\left(l\sqrt{k}\right) \end{pmatrix} \tag{12}$$

Note that the second type of perturbation does not exactly correspond to a physical tilt of the quadrupole, only in the thin lens approximation.

Comparing (1) with the closed orbit solution for the model

$$\begin{pmatrix} \delta x_s \\ \delta x'_s \end{pmatrix} = M_{s\leftarrow m}\left[I - M_m\right] g^{-1} D \tag{13}$$

where g is the transfer matrix for one-half of the quadrupole, and observing that

$$g^{-1}D_\xi = \begin{pmatrix} 0 \\ 2\sqrt{k}\sin\left(\frac{l\sqrt{k}}{2}\right) \end{pmatrix}$$

$$g^{-1}D_\vartheta = \begin{pmatrix} -\frac{2}{\sqrt{k}}\sin\left(\frac{l\sqrt{k}}{2}\right) \\ 0 \end{pmatrix} \tag{14}$$

it is easy to see that orbit shifts from perturbations D_ξ and D_ϑ are the same as (5) and (6) when

$$\xi = -\frac{1}{4}\left(\frac{\Delta k}{k}\right)\frac{l\sqrt{k}+\sin\left(l\sqrt{k}\right)}{\sin\left(\frac{l\sqrt{k}}{2}\right)}\Delta x_m$$

$$\vartheta = -\frac{1}{4}\left(\frac{\Delta k}{k}\right)\frac{l\sqrt{k}-\sin\left(l\sqrt{k}\right)}{\sin\left(\frac{l\sqrt{k}}{2}\right)}\Delta x'_m \tag{15}$$

Determination of offsets

We write the orbit shift observed at M points caused by changing a *single quadrupole q* as a column vector

$$\delta\overline{Z}_q = \left(\delta x_1 \quad \cdots \quad \delta x_M \quad \delta y_1 \quad \cdots \quad \delta y_M\right)^T{}_q \tag{16}$$

We can minimize the difference $\left\|\delta\overline{Z} - \delta\tilde{\overline{Z}}\right\|$ between the measured $\delta\overline{Z}$ and the model $\delta\tilde{\overline{Z}}$ computed as

$$\delta\tilde{\overline{Z}}_q = \frac{\partial\tilde{\overline{Z}}_q}{\partial\xi_x}\xi_x + \frac{\partial\tilde{\overline{Z}}_q}{\partial\xi_y}\xi_y + \frac{\partial\tilde{\overline{Z}}_q}{\partial\vartheta_x}\vartheta_x + \frac{\partial\tilde{\overline{Z}}_q}{\partial\vartheta_y}\vartheta_y + c_q\overline{Z}_\eta \tag{17}$$

where $\frac{\partial\tilde{\overline{Z}}}{\partial\xi_{x,y}}$ and $\frac{\partial\tilde{\overline{Z}}}{\partial\vartheta_{x,y}}$ are computed differential shifts from model perturbations ξD_ξ (8) and ϑD_ϑ (11),

ξ, ϑ, and c are free parameters,

$\delta\overline{Z}_\eta$ is the measured dispersion to account for the energy shifts associated with kicking the orbit transversely.

In SPEAR3 optics the effect of angular offset (6) is small and the systematic error in determining the position optics by using

$$\delta\tilde{\overline{Z}}_q = \frac{\partial\tilde{\overline{Z}}_q}{\partial\xi_x}\xi_x + \frac{\partial\tilde{\overline{Z}}_q}{\partial\xi_y}\xi_y + c_q\overline{Z}_\eta \tag{18}$$

instead of (17). In this case the error can estimated as $\frac{\alpha_m}{\beta_m k}\frac{\sigma^-}{\sigma^+}\frac{\Delta x'}{\Delta x}$ is <1% for all quadrupoles

When the above procedure is applied to each quadrupole *independently* it will introduce systematic errors in the determined offsets due to the optics differences between the real storage ring and the model. It is possible to avoid this systematic error by turning the problem into a least squares minimization of $\delta\overline{Z} - \delta\tilde{\overline{Z}}$ for all quadrupoles simultaneously at the same time varying the model parameters, most importantly the quadrupole and skew-quadrupole term in model sextupoles.

We form a vector of orbit shifts in response to the changes of Δk in all quadrupoles:

$$\delta\overline{Z} = \begin{pmatrix} \delta\overline{Z}_1 \\ \vdots \\ \delta\overline{Z}_N \end{pmatrix} \tag{19}$$

and minimize the difference between the measured and model values:

$$\delta\overline{Z} - \delta\tilde{\tilde{Z}}^{(0)} = A\begin{pmatrix} \Delta\overline{\xi}_x \\ \Delta\overline{\xi}_y \\ \Delta\overline{c} \\ \Delta\overline{k}_{Norm} \\ \Delta\overline{k}_{Skew} \end{pmatrix} \qquad (19)$$

where $\tilde{\tilde{Z}}^{(0)}$ is a vector formed from the zero-iteration solutions of (18) for individual quadrupoles. A is the design matrix of the least squares problem, whose columns are numerical derivatives of $\delta\tilde{\tilde{Z}}$ with respect to varied parameters:

$$\overline{A}_i = \left.\frac{\partial\tilde{\tilde{Z}}}{\partial p_i}\right|_{(0)} \qquad (20)$$

SIMULATION RESULTS

Simulated BBA using the proposed method was carried out for SPEAR3. Realistic alignment errors of 200 μm r.m.s. transverse shift and 500 microradian r.m.s. roll in all magnets were introduced in addition to 1 μm BPM noise.

Figure 1 shows the true orbit position offsets in quadrupoles compared with the offset predicted by a zero-iteration fit (18) and the least-squares solution (19)

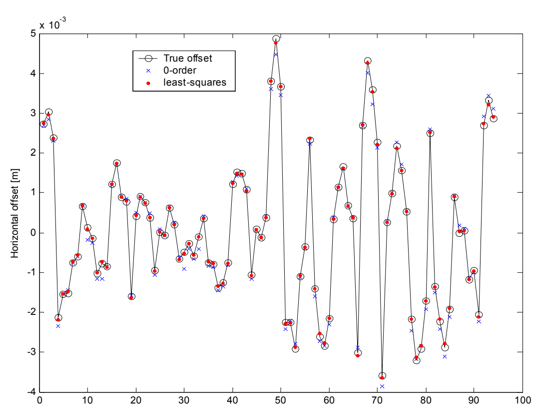

Figure 1: Real and predicted orbit offsets.

Figure 2 compares errors in the offset determination of the zero-iteration solution and the least squares solution.

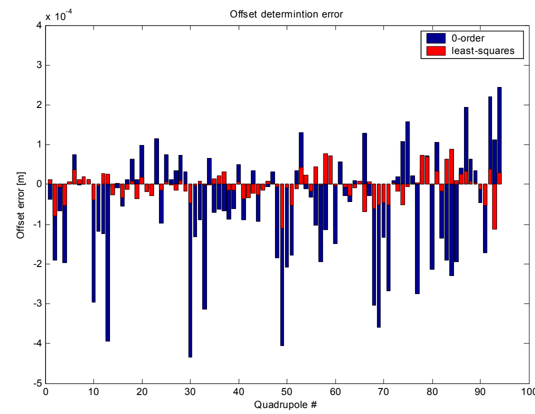

Figure 2: Error of the offset determination

DISCUSSION

The proposed method is practically useful during commissioning because the time consuming calculation of design matrix A is done *prior* to the measurement of all quadrupole Δk-response. The only computer-intensive operation to be performed during BBA procedure is finding the least squares solution of (19) through the singular value decomposition of A.

The proposed technique is an extension of the idea behind the powerful response matrix analysis method [2]. When this idea is applied to a BBA problem, changes in quadrupole strength Δk act as corrector magnets.

REFERENCES

[1] G. H. Hoffstaetter, F. Willeke, Phys. Rev. ST Accel. Beams Vol. 5, 102801 (2002)

[2] J. Safranek, "Experimental determination of storage ring optics using orbit response measurements", Nucl. Inst. and eth. A388, 27 (1997)