# COMPLETION OF THE SEXTUPOLE DRIVING TERMS MEASUREMENT AT THE SPS 

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## Abstract

This paper represents the completion of the series of sextupole driving terms measurements in the SPS which started in June 1998. The following two items have been missing from earlier reports on these studies: measuring two dimensional resonances and the resonance phase. The possible dependence of these terms on collective effects was studied. Lastly, the experiment was performed at two different energies of 26 and 80 GeV , to suppress energy dependencies. Comparisons to the tracking model show excellent agreement, proving that this technique is ready for other machines.

## INTRODUCTION

In previous experiments [1] and [2] a beam based method to measure resonance driving terms has been established. These terms are measured from the Fourier analysis of beam data after applying a transverse kick. This technique has its origin in the pioneering study described in [3] and the improvement of the Fourier transform algorithm [4]. A complete description of the relation between the Fourier spectrum of the single particle motion and the resonances was provided in [5].

In a real machine the beam is not a single particle but a particle distribution and processes like the beam decoherence change the Fourier spectrum of the turn-by-turn motion. The effect of the decoherence due to amplitude detuning has been described in [1]. The relevant conclusion is that the spectral line $(\mathrm{m}, 0)$ of a decohered signal is reduced by a factor of $|\mathrm{m}|$ compared to the single particle case. In order to compare the results from the experiment to single particle simulations the corresponding factor is applied to the experimental results.

During this experiment measurements were performed at different intensities and different energies to investigate any dependence on these parameters.

## SEXTUPOLAR RESONANCE TERMS

In order to find an appropriate observable related to the phase of the resonances an insight into the properties of the phases of the spectral lines follows. From the equation of motion given in [5] the amplitude and phase of the spectral line $(1-j+k, m-l)$ are given by the following complex quantity,
$-2 i j f_{j k l m}\left(2 I_{x}\right)^{\frac{j+k-1}{2}}\left(2 I_{y}\right)^{\frac{l+m}{2}} e^{i\left[(1-j+k) \psi_{x_{0}}+(m-l) \psi_{y_{0}}\right]}$,
where $I_{x, y}$ are the transverse actions. The term $f_{j k l m}$ is defined as the sum over all the non-linear elements of the same type, i.e.

$$
\begin{equation*}
\sum_{j} k_{j} e^{i\left[(-j+k) \psi_{x j}+(m-l) \psi_{y j}\right]} \tag{2}
\end{equation*}
$$



Figure 1: Phase of the line $(-2,0)$ minus the phase of the line $(1,0)$ versus longitudinal position from experiment and model for the baseline machine at 26 GeV .
where $k_{j}$ contains all the factors as beta functions and strengths. The change in the phase of the spectral line ( $1-j+k, m-l$ ) over a region free of sources is now computed. Let the betatron phases change by $\Delta \psi_{x}$ and $\Delta \psi_{y}$ over this region. Then all the $\psi_{x j}$ and $\psi_{y j}$ change by $-\Delta \psi_{x}$ and $-\Delta \psi_{y}$ respectively. The change in the phase of the spectral line is given by the sum of the change of the phase of the term $f_{j k l m}$ plus the change of the phase of the exponential part of eq. 1 , which yields

$$
\begin{align*}
& (j-k) \Delta \psi_{x}+(l-m) \Delta \psi_{y}+(1-j+k) \Delta \psi_{x} \\
& +(m-l) \Delta \psi_{y}=\Delta \psi_{x} \tag{3}
\end{align*}
$$

This means that the phase of any spectral line from the horizontal motion changes by the same amount, $\Delta \psi_{x}$, over a region free of non-linear sources. In particular this is obvious for the horizontal tune line. Therefore, the following observable remains constant along sections free of non-linear sources for any m and n ,

$$
\begin{equation*}
\operatorname{Phase}(m, n)-\operatorname{Phase}(1,0), \tag{4}
\end{equation*}
$$

where $\operatorname{Phase}(m, n)$ represents the phase of the spectral line $(\mathrm{m}, \mathrm{n})$. This observable changes abruptly at the location of the non-linear sources as can be seen from [2]. The use of this observable in conjunction with the amplitude of the spectral line allows the unambiguous localisation of nonlinear fields. In figure 1 this observable as measured from one single file is plotted together with the model prediction versus the longitudinal location for the baseline machine at 26 GeV . It has to be mentioned that a constant quantity had to be added to the experiment values to meet the model prediction, due to a different origin of the phases. The amplitude of the corresponding driving term $f_{3000}$ is shown in figure 2 together with the prediction from the model. Both, phase and amplitude, show a good agreement.

As in previous years measurements were also done with the eight extraction sextupoles powered to $(++++--$


Figure 2: Amplitude of $f_{3000}$ versus longitudinal position from experiment and model for the baseline machine at 26 GeV . The blue line is used to connect the experimental points.


Figure 3: Phase of the spectral line $(-2,0)$ minus the phase of the spectral line $(1,0)$ versus longitudinal position from experiment and model for SPS with the extraction sextupoles powered to $(++++----) 30 \mathrm{~A}$ at 26 GeV .
$--) 30 \mathrm{~A}$. In figure 3 the phase of the spectral line $(-2,0)$ minus the phase of the spectral line $(1,0)$ is plotted versus the longitudinal position for experiment and model. The overall agreement is again good but the curve from the measurement is clearly more rough than the one from the model.

Sextupoles also introduce non-linear coupling, the horizontal component of the magnetic field is proportional to the product of the transverse coordinates $x y$. This monomial introduces, among others, the term $h_{0120}$ in the Hamiltonian, which drives the resonance $(1,-2)$ and contributes to the vertical motion, $h_{y}^{-}(N)$, with the following quantity,

$$
\begin{equation*}
-4 i f_{0120}\left(2 I_{x}\right)^{\frac{1}{2}}\left(2 I_{y}\right)^{\frac{1}{2}} e^{i\left[-\left(2 \pi \nu_{x} N+\psi_{x_{0}}\right)-\left(2 \pi \nu_{y} N+\psi_{y_{0}}\right)\right]} \tag{5}
\end{equation*}
$$

By virtue of this equation the term $h_{0120}$ introduces the spectral line $(-1,-1)$ in the vertical turn-by-turn motion. After normalising to the amplitude of the vertical tune the amplitude of this spectral line is $4\left|f_{0120}\right| \sqrt{2 I_{x}}$. This quantity is linear in the horizontal kick. For this reason $\left|f_{0120}\right|$ is measured by performing a linear fit. The data acquired for the baseline machine was not decohered therefore it can be directly compared to the single particle model. In figure 4 the amplitude of the generating function term $f_{0120}$


Figure 4: Amplitude of $f_{0120}$ versus longitudinal position from experiment and model for the baseline machine at 26 GeV .


Figure 5: Amplitude of $f_{3000}$ versus longitudinal position from experiment and model for the baseline machine at 80 GeV . The blue line is used to connect the experimental points.
is plotted versus the longitudinal location together with the model. The agreement seems to be better than that obtained for the term $f_{3000}$, figure 2 .

Next are the measurements done at the SPS at 80 GeV . In figure 5 the amplitude of the term $f_{3000}$ is plotted versus the longitudinal position from experiment and tracking model for the baseline machine. To asses any energy effect this is to be compared to the case at 26 GeV shown in figure 2 . The level of agreement between model and experiment is similar at the two different energies. For this reason energy effects seem to be not relevant.

In figure 6 the phase of the spectral line $(-2,0)$ minus the phase of the spectral line $(1,0)$ is plotted versus the longitudinal position for experiment and model. The level of agreement is similar when compared to the 26 GeV case, figure 1.

Measurements were also done with the extraction sextupoles powered to $(++++----) 100 \mathrm{~A}$. Since the energy is around 3 times larger than the previous 26 GeV , the current in the sextupoles has to be increased by a similar factor to obtain a similar strength. In figure 7 (Top) the amplitude of the generating function term $f_{3000}$ is plotted versus the longitudinal position together with the model. A factor two is applied to the data in order to restore the effect of decoherence. Model and measurements show large


Figure 6: Phase of the spectral line $(-2,0)$ minus the phase of the spectral line $(1,0)$ versus longitudinal position from experiment and tracking model for the baseline machine at 80 GeV .


Figure 7: Amplitude of $f_{3000}$ versus longitudinal position from experiment and model at 80 GeV . The vertical lines denote the position of the extraction sextupoles. Top: model with 8 sextupoles. Bottom: model with 7 sextupoles
discrepancies. Looking at the location of the first extraction sextupole a clear discrepancy is observed. Whereas the model predicts a large change of the amplitude of this term, the measurement gives a constant value in that region. We therefore suspected that this particular extraction sextupole was not connected. A comparison of the data with this new model without the first sextupole shows an excellent agreement, figure 7 (bottom). This hypothesis of a disconnected sextupole has been positively confirmed via the SPS alarm system [6], which reported a failure of this sextupole. The successful detection of a mispowered sextupole is an illustration of how this method will help in the commissioning of the LHC.

The effect of the beam intensity on the measurement of


Figure 8: Normalised amplitude of the spectral line (-2,0) versus kick amplitude for the four intensity settings.
the resonance driving terms is now discussed. These terms might depend on the beam intensity due to space charge and impedance effects [7] . During the experiment the beam intensity was changed to different values in the range from $0.5 \times 10^{10}$ to $6 \times 10^{10}$ protons. In figure 8 the normalised amplitude of this spectral line averaged over all pick-ups is plotted versus kick amplitude for the four different intensities. For the majority of the kicks the differences remain inside the error bars. No local discrepancies in the resonance terms were observed around the ring.

## CONCLUSION

For the first time the phase of the resonance terms has been measured around the ring. A very useful phaseobservable has been defined which allows the unambiguous localisation of non-linear sources when used in conjunction with the amplitude of the resonance. The resonance $(1,-2)$ has been also measured for the first time. By using this technique a sextupole that by accident was not powered was identified. Lastly, we have demonstrated that neither a change of energy nor of intensity has significant influence in measuring the resonance driving terms.

## ACKNOWLEDGMENTS

We thank G. Arduini, F. Calderini and N. Gonthier for providing the information on the SPS alarm system used in this report. We also thank R. Jones for the careful adjustment of the BPM system.

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