DESIGN OF FFAGS BASED ON A FODO LATTICE

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Abstract

An FFAG is a lattice with fixed magnetic fields that has an extremely wide energy acceptance. One particularly simple type of FFAG is based on a FODO lattice, where both quads can be combined-function bending/quadrupole magnets. The spaces between the combined-function magnets are left open for RF cavities and other hardware. This paper descibes a general method for creating lattice designs for this type of lattice which gives the lattice optimal properties for an FFAG accelerator. The properties of this lattice as a function of input parameters are explored. The use of sextupoles to improve lattice properties is also explored.

INTRODUCTION

There has been great interest in recent years in a neutrino factory or muon collider which would accelerate muons to energies in the 20–50 GeV range. Since muons decay, acceleration must be rapid, with an average gradient of at least 1 MV/m. Accelerating systems are a major component of the cost of these machines. A linac accelerating to the full energy would be extremely costly. Cost savings can be achieved by using some form of recirculating acceleration, in which the muons pass through the cavities multiple times. It is at best difficult to design a fast-ramping synchrotron for these lower energies, since it would be challenging to increase the magnetic fields at the rate at which one would like to accelerate the beam. This is especially true because of the large beam emittances that one typically deals with in muon-based machines, since larger magnet apertures lead to larger stored energies, increasing the power that must be delivered to ramp the magnets.

Most studies to this point have proposed CEBAF-style recirculating accelerators for muon acceleration. The accelerator has a racetrack shape, with two linacs connected by a series of arcs. The beam enters a different arc on each pass, depending on its energy. The primary difficulty with these machines is related to the multiple arcs. The switchyard connecting the linac to the several arcs becomes very complex, and it therefore becomes difficult to have more than a few arcs (4 in typical designs). This prevents a further reduction in cost by having more turns in the accelerator and therefore a smaller amount of RF in the linacs.

These considerations have led to the proposal of performing acceleration using a fixed-field alternating gradient (FFAG) machine. Instead of having a separate arc for each energy, these machines avoid the switchyard by having a single arc which accepts the entire range of energies over which one wishes to accelerate.

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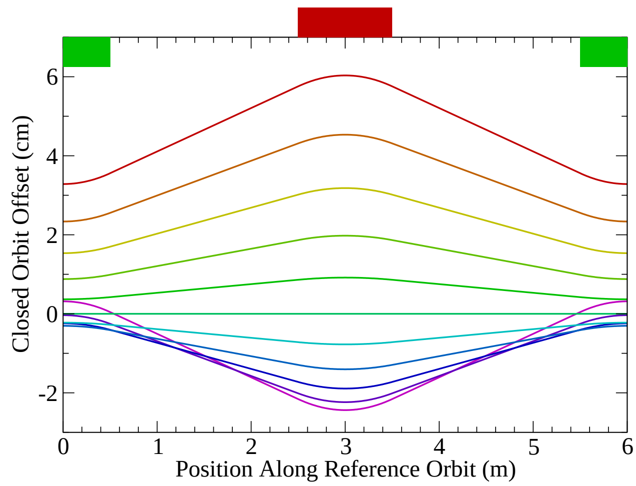

Figure 1: Closed orbits at various energies in a FODO FFAG lattice. Energies are from 10 GeV (bottom center, magenta) to 20 GeV (top center, red). Position of magnets are also shown at the top. The tune at 10 GeV is 0.3.

The traditional type of FFAG is the scaling FFAG. In this machine, the tunes and momentum compaction are independent of energy, and the closed orbits at different energies are geometrically similar to each other. In recent years non-scaling FFAGs have been proposed [1], which don't meet these criteria, yet still accept a wide range of energies. This paper describes some advanteages of "non-scaling" FFAGs over scaling FFAGs.

This paper describes how to design a FFAG lattice based on a FODO cell. The FODO cell in this case consists of two combined-function (gradient bend) magnets. Figure 1 shows the closed orbits at various energies in such a lattice.

OPTIMIZATION

A FODO lattice with two combined-function magnets has 7 free parameters: the drift space between magnets (same on both sides), two magnet lengths, two dipole field components, and two quadrupole field components. One adjusts these parameters to achieve a design which is optimum in some sense.

Tunes

One must not allow the tune of a single cell to reach a half-integer or integer. In a scaling FFAG, this is achieved

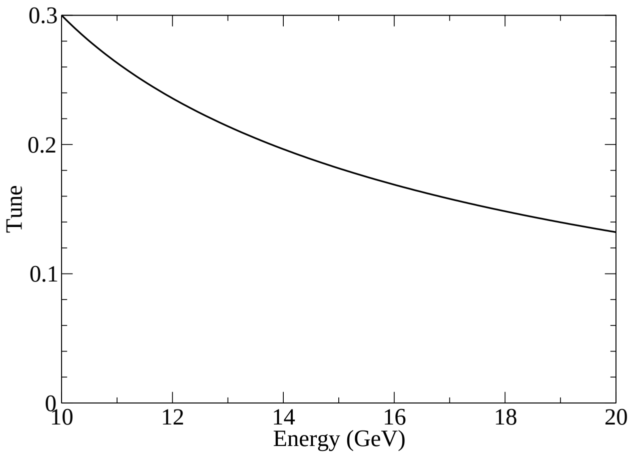

Figure 2: Tune as a function of energy in a FODO FFAG lattice.

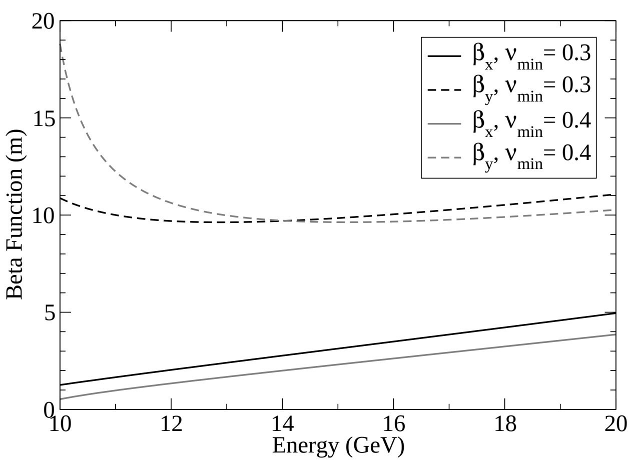

Figure 3: Beta functions as a function of energy at the center of the defocusing quadrupole, for two different tunes at the minimum energy.

by having a constant tune. In a FODO cell, if the lattice is stable, the tune is below 0.5. The tune increases with decreasing energy, as seen in Fig. 2, until one reaches the half-integer resonance; the lattice is unstable at energies below that point. Thus, at the lowest desired energy, the tune is set to a value safely below 0.5, and this is generally sufficient to have tunes below 0.5 over the entire energy range. Since we are accelerating very rapidly, one is less concerned with nonlinear resonances, since the tune will not remain near any particular value for a very long time.

A good choice for the tunes at the lowest energy is 0.3. This may seem to be quite far from 0.5, but in practice the tune rises very rapidly as the energy decreases. To get a finite energy acceptance at injection, to insure that one's design holds up once end fields are other nonlinearities are included, and to keep the beta functions at injection reasonable (see Fig. 3), it is essential to keep sufficiently far from the half integer resonance at injection. The miniumum required voltage (see next subsection and Fig. 4) and

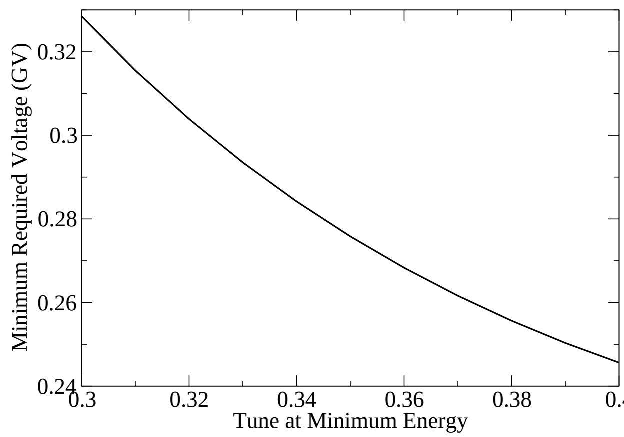

Figure 4: Minimum required voltage as a function of the tune at the lowest energy.

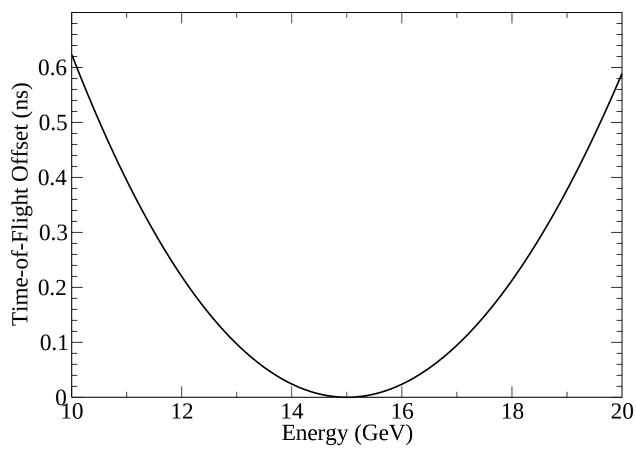

Figure 5: Time-of-flight as a function of energy in a FODO FFAG lattice.

the closed orbit swing improve when that tune is raised, so one does not want to make that tune too low.

Time-of-Flight Variation

Secondly, one wants to make optimal use of the installed RF. It can be demonstrated that there is a minimum amount of RF voltage that must be installed in an FFAG accelerator to achieve a desired energy gain [2]. This minimum requirement comes about because the time-of-flight depends strongly on energy. Because of the rapid rate of acceleration, the relatively high frequencies we desire to use, and the large amount of installed voltage, it is impractical to shift the RF phase to keep the RF synchronized with this time-of-flight variation. The time-of-flight variation therefore prevents one from staying at the peak of the RF crest, and this leads to the minimum RF voltage requirement. One advantage of the non-scaling FODO lattice over a scaling lattice is the parabolic shape of the time-of-flight as a function of energy (see Fig. 5), compared to the roughly linear behavior in the scaling lattice. This leads to a lower minimum required RF voltage for a given lattice scale.

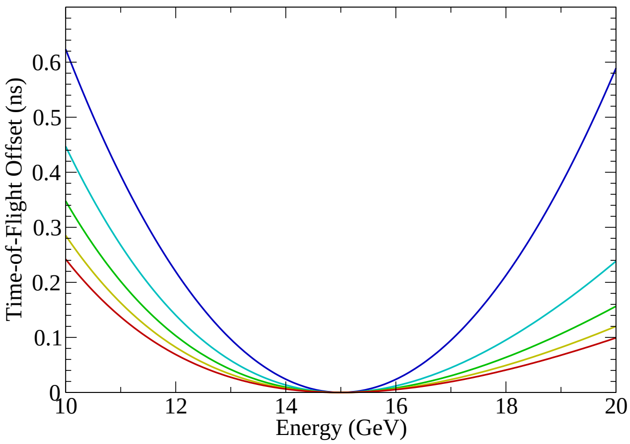

Figure 6: Time-of-flight variation with energy for increasing (top to bottom) sextupole strength.

To minimize the total time-of-flight range if the variation is parabolic in energy, it is clearly optimal to place the minimum of the parabola at the central energy. Thus, this is a constraint in the optimization process.

The time-of-flight range for a single cell is approximately proportional to the square of the bending angle in the cell. Since the minimum required RF voltage is proportional to the time-of-flight range for the entire ring [2], that voltage must be approximately inversely proportional to the number of cells in the ring. Thus, there is a cost optimum if the RF voltage that is actually installed is related to this minimum requirement, since the arc cost will be proportional to the number of cells.

Furthermore, the time-of-flight range is approximately proportional to the cell length. Thus, reducing the cell length would presumably lead to a reduced RF cost.

Since the time-of-flight is very quadratic as a function of energy, one would expect to be able to reduce it by adding sextupole components to the magnets. Figure 6 demonstrates that this is in fact the case. However, these sextupole components may have a negative impact on the dynamic aperture; this has not been examined as yet.

Magnet and RF Considerations

The cost of magnets and the cost of the RF systems are the dominant costs in an FFAG accelerator. While one would like to keep the magnets short to keep the cell lengths short, magnet costs increase as their pole tip fields increase, and there are technological limitations as to how high the fields can be. Furthermore, very short magnets become dominated by end effects and the associated nonlinearities. These field constraints and/or cost optimizations effectively provide two more constraints in the optimization.

The drifts in the lattice must be at least long enough to accommodate an RF cavity. Due to the high peak power requirements for room-temperature RF, one would prefer to use superconducting RF. The field from the magnets at superconducting RF cavities must not cause the cavities to quench. This requires an additional separation between the cavities and the magnets to allow the field to fall off. If the cavities are cooled before the magnets are powered, then the field at the cavities can be as high as 0.1 T [3]. However, before the cavities are cooled, the field must be around 10^{-5} T. Thus, one must insure that there is no residual magnetization remaining when the magnets are powered off. These considerations determine how much space must be left between the cavities and the magnets, and therefore what the minimum drift length in the FODO cell can be.

Since beta functions (and therefore magnet apertures) will be smallest when the cell length is the least, and since the time-of-flight variation is less when the cell length is least, one generally chooses the drift length to be the minimum allowed for the purposes of installing RF cavities.

Other Considerations

A cost-optimal design may not turn out to be what one wants to use due to muon decays. Thus, if an optimized design turns out to have too many decays, one may instead optimize the system to have the maximum tolerable decay.

A final concern is beam loading in the RF cavities. The beam generally extracts energy from the RF cavities far too quickly to be replaced. Thus, a design requiring an extremely large number of passes will not work well since too much stored energy will be extracted from the cavities. Thus, it may turn out that the desired solution is to have a certain maximum number of turns. Furthermore, this consideration precludes running cavities at very low voltages to try to minimize peak power requirements.

CONCLUSIONS

It is straightforward to design a FODO-based nonscaling FFAG to specified constraints using nonlinear fitting and optimization techniques. Design constraints that are required to insure that the FFAG operates properly have been described. Parametric dependence of some performance parameters on input constraints have been described. The plots in this paper required the creation of several dozen lattices, all of which were done in less than an hour of CPU time on a three-year old PC, using code that was written specifically for this purpose. Thus, one can easily conceive of performing a cost optimization of an FFAG lattice design, based on some model of magnet and RF costs.

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