CHARGED PARTICLE INTERACTION WITH A CHIRPED ELECTROMAGNETIC PULSE*

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Abstract

It is found that a charged particle can get a net energy gain from the interaction with an electromagnetic chirped pulse. Theoretically, the energy gain increases with the pulse amplitude and with the relative frequency variation in the pulse.

INTRODUCTION

Modern high-intensity lasers can generate electromagnetic (EM) pulses with the electric field amplitude E_0[TV/m]$\cong 3.2a_0/\lambda_L$[μm] in order of 10 TV/m (here $a_0=E_0/(m_ec\omega/e)$ is the normalized peak amplitude and λ_L is the laser wavelength). However, it is not easy for a charged particle to get a net energy gain of even 10 MeV after interaction with such laser pulses. In the laser pulse field, a free charged particle experiences the ponderomotive force. In the one-dimensional (1D) case the acceleration in front of the pulse is followed by deceleration in the descending part of the pulse, so that the net energy gain is zero. However, in a laser pulse of finite transverse extension an electron can leave the pulse before the decelerating field will compensate the acquired energy (see, e. g., recent articles [1] and references therein). Acceleration of free electrons to MeV energy, after interaction with a high-intensity ($I_0\approx 10^{19}$ W/cm^2, $a_0\approx 3$) laser pulse in vacuum, has been observed experimentally [2]. Free electrons can also acquire energy from the laser field if they are "born" inside the pulse (where $a\neq 0$) due to tunnelling ionisation [3].

In this article we show that a charged particle can get a net energy gain after interaction with an electromagnetic pulse with the carrier frequency changing from head to tail (chirped pulse) even in the one-dimensional case. Presently, high-intensity ($I_0\sim 10^{19}$ W/cm^2) short chirped laser pulses are available [4]. A chirped pulse can be generated as a result of reflection from a relativistic mirror when the gamma factor, γ_m, of the mirror changes during reflection. Computer simulations showed that γ_m of an electron mirror produced by a high-intensity femtosecond laser pulse focused on a thin solid target can increase from $\gamma_m\sim 1$ to the value of $\sim 10^3$. So, the carrier frequency of the reflected EM pulse $\omega_r\approx 4\gamma_m^2\omega_{in}$ (where ω_{in} is the incidence frequency) will increase considerably. The plasma electron density spike in a nonlinear laser wakefield can also serve as a relativistic mirror with changing velocity if the group velocity of the laser pulse, which is equal to the wake phase velocity, changes. This can take place, for example, in a non-uniform plasma. Below we consider the interaction of a charged particle (here–an electron) with a one-dimensional chirped EM pulse. This approach is valid when the particle remains close enough to the pulse axis, so that we can neglect the change of the EM field in the transverse direction.

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DYNAMICS OF AN ELECTRON IN AN ELECTROMAGNETIC FIELD

Suppose that the EM pulse propagates in the Z direction and is linearly polarized in the x direction. For a one-dimensional chirped pulse we can write $E_x=E_0(\zeta)\cos[\omega(\zeta)\zeta]$, $\zeta\equiv Z-ct$, $\mathbf{B}=\mathbf{e}_yB_y=\mathbf{e}_yE_x$, where $\omega(\zeta)$ is the carrier frequency and $\mathbf{B}$ is the magnetic field of the pulse. In the pulse field ($\mathbf{E}(E_x,0,0)$, $\mathbf{B}(0,B_y=E_x,0)$), we have for the electron's momentum components:

$$dp_x/d\tau = -(1-\beta_z)E_x, \tag{1}$$

$$dp_y/d\tau = 0, \tag{2}$$

$$dp_z/d\tau = -\beta_x E_x. \tag{3}$$

Here $\mathbf{p}=\mathbf{P}/m_ec$ and $\boldsymbol{\beta}=\mathbf{v}/c$ are the dimensionless momentum and velocity, $\tau=\omega_0 t$ is the dimensionless time, $\omega_0=\omega(0)$, the spatial variables are normalized to c/ω_0 and the electric and magnetic fields are, as usual, normalized to $m_ec\omega_0/e$. According to Eq. (2), the y-component of the momentum is conserved: $p_y(\tau)=p_y(\tau=0)\equiv p_{y0}$=const. From Eqs. (1)-(3) we find the well-known integral of motion [1]

$$\gamma(\tau)-p_z(\tau)=\gamma_0-p_{z0}\equiv C=const, \tag{4}$$

where $\gamma=(1+\mathbf{p}^2)^{1/2}$ is the relativistic factor. Equation (1) gives us the expression for the transverse momentum:

$$p_x = p_x(\xi_0)+\int_{\xi_0}^{\xi} E_x(\xi)d\xi \equiv p_{x0}+A, \tag{5}$$

where $\xi=(\omega_0/c)\zeta\equiv z-\tau$, $\xi_0=\xi(\tau=0)$. With known p_x and $p_y=p_{y0}$ one can find the longitudinal momentum p_z and γ from Eq. (4):

$$\begin{pmatrix} p_z \\ \gamma \end{pmatrix} = \begin{pmatrix} p_{z0} \\ \gamma_0 \end{pmatrix} + f(A,\vec{p}_0), \tag{6}$$

where $f=A(A+2p_{x0})/2C$. When an electron is initially non-relativistic (γ_0~1), $C\approx1$ and $f=A^2/2+Ap_{x0}$. For an initially relativistic electron with the longitudinal momentum prevailing, $(p_{x0})^2,(p_{y0})^2<<(p_{z0})^2>>1$, we have from (6):

$$\begin{pmatrix} p_z \\ \gamma \end{pmatrix} \approx \begin{pmatrix} p_{z0} \\ \gamma_0 \end{pmatrix} \left[1 + A(A+2p_{x0}) \begin{cases} \gamma_{\perp 0}^{-2},\ p_{z0} > 0, \\ \mp 1/4\gamma_0^2,\ p_{z0} < 0 \end{cases} \right], \tag{7}$$

where $\gamma_{\perp0}=[1+(p_{x0})^2+(p_{y0})^2]^{1/2}$. When initially the transverse motion dominates $((p_{y0})^2,(p_{z0})^2<<(p_{x0})^2>>1$, $|p_{x0}|\approx\gamma_0)$ then:

$$\begin{pmatrix} p_z \\ \gamma \end{pmatrix} \approx \begin{pmatrix} p_{z0} \\ \gamma_0 \end{pmatrix} + \frac{A(A+2p_{x0})}{2\gamma_0}. \tag{8}$$

LINEARLY CHIRPED PULSE

Thus, the dynamics of an electron are determined by the initial momentum and the value of $A(\xi_0,\xi)$. Let us consider the interaction of an electron with a chirped EM pulse over an infinite interaction region $\xi_0=+\infty$, $\xi=-\infty$. For simplicity we choose a linearly chirped Gaussian pulse, $E_x=a_0\exp(-\xi^2/\sigma^2)\cos(\Omega\xi)$, where $\Omega(\xi)=\omega/\omega_0=1+\Delta\Omega\xi/2\sigma$ is the normalized carrier frequency, $\Delta\Omega=\Omega(\sigma)-\Omega(-\sigma)$. So, for the linear chirp we obtain:

$$A = \int_{+\infty}^{-\infty} E_x d\xi = -\frac{\pi^{1/2}a_0\sigma}{(1+\nu^2)^{1/4}}\exp\left[-\frac{\sigma^2}{4(1+\nu^2)}\right]\times \cos\left[\frac{arctg(\nu)}{2}-\frac{\sigma^2\nu}{4(1+\nu^2)}\right], \tag{9}$$

where $\nu\equiv\Delta\Omega\sigma/2$. According to expression (9), when $\sigma^2/4(1+\nu^2)>>1$, the momentum (energy) acquired by the electron is negligibly small. For a non-chirped pulse ($\nu=0$) the value of A is maximum for $\sigma=2^{1/2}$: $A=-(2\pi)^{1/2}\exp(-1/2)a_0$. This case of a sub-cycle EM pulse was studied in Ref. [6]. For a multiple-cycle pulse considering here, momentum (energy) transferred to an electron after interaction with such a pulse can be considerable when $\nu^2>>1$. In this case:

$$A \approx -a_0\left(\frac{2\pi\sigma}{\Delta\Omega}\right)^{1/2}\exp\left(-\frac{1}{(\Delta\Omega)^2}\right)\cos\left(\frac{\sigma}{2\Delta\Omega}-\frac{\pi}{4}\right). \tag{10}$$

We see that for a non-chirped pulse $A\rightarrow0$ (known result) and that $A\neq0$ for a chirped pulse (new result) due to the change in the carrier frequency. One can see also that A is a periodic function with an amplitude and "frequency" depending on the chirped pulse parameters. The value of A is equal to zero when $\sigma/\Delta\Omega=\pi(1/2\pm m)$, m=1,3,5,... Fig. 1 shows the dependence $A(\Delta\Omega)$ for different lengths of the pulse. Thus, the energy of an electron can be changed

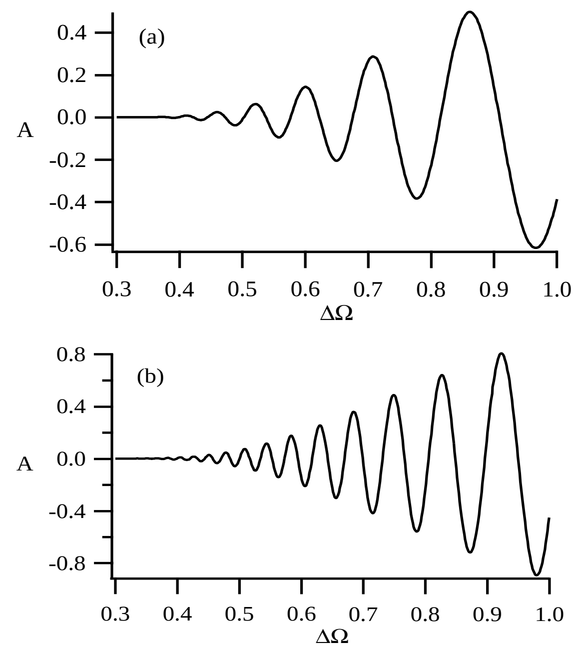

Figure 1: Electron's transverse momentum gain A after interaction with a chirped EM pulse, a_0=0.1, σ=50 (a) and σ=100 (b).

considerably, after interaction with a chirped EM pulse, by proper choice of the pulse parameters. For example, $A\approx2$ for $\Delta\Omega$=0.5, a_0=3, and σ=100. The latter two values approximately correspond to the experimental conditions of Ref. [2].

ELECTRON ACCELERATION

The effect found can be used to accelerate charged particles. For non-relativistic electrons the energy gain is $\Delta\gamma=\gamma-\gamma_0=A(A+2p_{x0})/2$ and the absolute energy spread in an electron bunch after interaction with the chirped pulse will be $\delta\gamma=\delta\gamma_0+\delta f=\delta\gamma_0+A\delta p_{x0}$, where $\delta\gamma_0$ (δp_{x0}) is the energy (transverse momentum) spread before interaction. In the relativistic case, according to (7), when $p_{z0}>0$, energy gain is proportional to the initial energy and can be considerable even for small value of A.

Assume initially $p_{x0}=p_{y0}=0$ and that the electron co-propagates with the pulse (for which $A=A_1$) with relativistic velocity. After interaction with the pulse, according to (5) and (7), $p_{z1}=p_{z0}(1+A_1^2)$ and $p_{x1}=A_1$. To compensate the transverse momentum, p_{x1}, acquired, a second pulse can be sent along p_{x1}, so that p_{x1} (p_{z1}) will be the longitudinal (transverse) momentum with regard to the second pulse. Then, after interaction with second pulse, according to (7) and (8), $p_{z2}=p_{z1}+A_2$ and $p_{x2}=p_{x1}+A_2(A_2+2p_{z1})/2\gamma_1$; here the signs of p_{x1} and p_{z1} in the frame of second pulse should be taken into account. So, by proper choice of the parameters of the pulses one can make p_{x2} equal to zero. This occurs, for example, when $A_1=A_2>0$, $A_2<<2p_{z0}$, and $p_{x1}<0$ and $p_{z1}>0$ in the frame of the second pulse. This two-step acceleration process can be repeated. The interaction time with the first pulse is $\Delta\tau\sim\tau_L/(1-\beta_{z0})$, where τ_L is the duration of the pulse. So, for large enough γ_0 the diffraction broadening of the pulse, which takes place on a time scale $\tau_d\sim Z_R/c$ ($Z_R=\pi r_0/\lambda_L$ is the Rayleigh length, r_0 is the focal spot size), can restrict the interaction time.

When a relativistic electron moves across a chirped pulse, the transverse momentum (energy) can be increased, $p_x=p_{x0}+A$. A second pulse propagating in parallel with the first one can accelerate the electron further, so that the longitudinal momentum acquired after interaction with the first pulse will be compensated.

Equations (1)-(3) were solved numerically. Figure 2 shows the dynamics of electron, which is initially at rest,

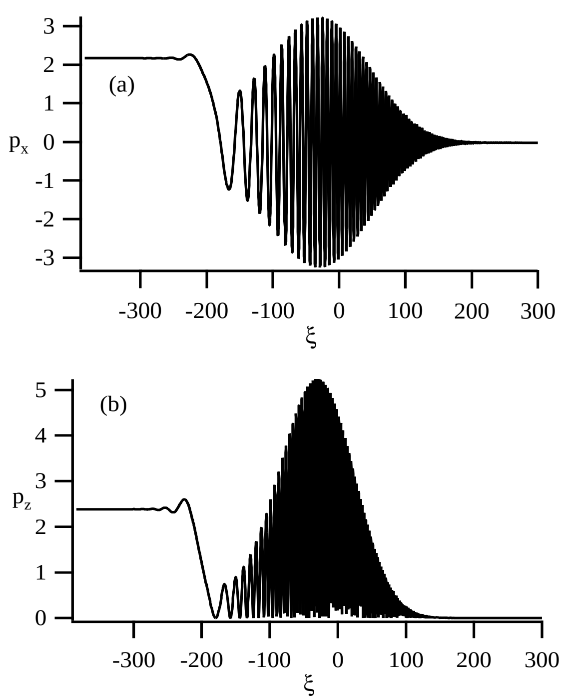

Figure 2 Dynamics of an electron interacting with a chirped pulse, $\mathbf{p}_0=0$, $a_0=3$, $\sigma=100$, and $\Delta\Omega=0.51$.

in the chirped pulse field. The electron momentums are $p_x=A\approx2.19$ and $p_z\approx2.39$ after interaction. In this case $\gamma=1+p_z$ (see equation (4)) and the interaction time is in the order of the pulse duration. When initially $p_{z0}=3$, the transverse momentum dynamics are the same, but the final longitudinal momentum is much higher, $p_z\approx17.71$ (see Fig. 3), in a good agreement with formula (9), $\gamma\approx p_z$. In this case the interaction takes place, in the laboratory frame, over a distance of about $4544\lambda_L\approx172l_{FWHM}$, where $l_{FWHM}=2(\ln2)^{1/2}\sigma$ is the full width at half maximum of the pulse. For comparison, the Rayleigh length is $\approx7854\lambda_L$ for a laser pulse with $r_0=30$ μm and $\lambda_L=1$ μm. For non-chirped pulses the electron momentum was found to be unchanged, so that $\mathbf{p}=\mathbf{p}_0$.

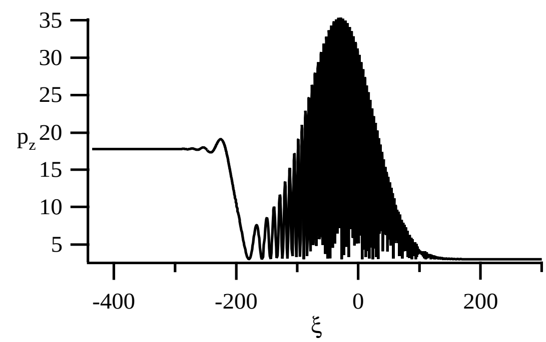

Figure 3: The calculated longitudinal momentum of an electron. All parameters are the same as those in Fig. 2 excepting initial longitudinal momentum, $p_{z0}=3$.

CONCLUSION

It has been shown that free charged particles can undergo a net energy gain after interaction with a chirped electromagnetic pulse. This new effect can be applied for particle acceleration as well as for diagnostic purposes – to measure the chirp in a pulse. The phenomena found can play an important role in chirped laser pulse–plasma interactions which are currently under intensive theoretical and experimental investigations. (see, e. g., [4,7]).

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