INSIGHTS IN THE PHYSICS OF THE DYNAMIC DETUNING IN SRF CAVITIES AND ITS ACTIVE COMPENSATION*

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Abstract

Elliptical SRF cavities operated in pulsed mode experience dynamic Lorentz detuning because of the time varying radiation pressure. Due to the narrow electromagnetic bandwidth of these resonators, the induced detuning can severely affect their matching conditions to the RF feeding system. The active compensation scheme using piezoelectric tuners has proven to be a viable and attractive method to minimize the effects of the Lorentz detuning. To optimize this compensation, mechanical parameters are extracted for the action of the Lorentz forces and the piezoelectric tuner from both measurement and simulation. This process gives insights into the physics of the dynamic detuning, such as the nature of the coupling differences of both detuning sources, the possible settings for the measurement of the dynamic detuning with the methods to deduce mechanical parameters from them, and the origin of possible parasitic effects in the measurements. These issues are reported and discussed along with some ideas for the optimization of the detuning compensation.

DETERMINATION OF THE CAVITY MECHANICAL PROPERTIES

Qualitative modeling by a vibrating string

The modeling of the cavity wall vibrations by a vibrating string is interesting because of its relative simplicity and because it contains various aspects of the physics. Firstly, the amplitude of the vibrations are small which is usually assumed in the vibrating string problem. Secondly, a string of finite length with fixed boundary conditions will produce a modal basis equivalent to the mechanical resonances of the cavity. Thirdly, some mode damping can be added to reproduce the attenuation in time of the cavity mechanical mode vibrations. Fourthly, distributed forcing, like the Lorentz forces, or local forcing, like the piezoelectric tuner action, are both possible. Fifthly, the detuning can be connected to the vibrations by integration of the transverse displacement over the longitudinal dimension of the string. Details of the modeling and of the calculus are presented in [3]. In the end, the model can be reduced to a system of second order ordinary differential equations (ODE) which support other analysis using such a modal approach [4]. In the following, the modal mechanical parameters are written $\{\Omega_m, Q_m, k_m\}$ where the index m refers to the m^{th} mechanical mode. Using the vibrating string model, it can be shown that the usually quoted parameter K for the Lorentz detuning, linking the detuning to the square of the field in the cavity as $\Delta f = -KE^2$, is the sum of all the modal k_m: $K = \sum_m k_m$. Also, it can be shown [3] that the coupling coefficients k_m are, as suggested in [4], the products of the projections of the acting force on the mode shapes by the projections of the frequency sensitivity on the mode shapes. As a direct consequence, it concludes that the coupling coefficients for the Lorentz force action are all of the same sign because the acting force is proportional to the radiation pressure $P_{rad} = \frac{1}{4}\{\mu_0|H|^2 - \epsilon_0|E|^2\}$ whereas, according to the Slater formulation, the frequency sensitivity is locally proportional to $\epsilon_0|E|^2 - \mu_0|H|^2$. This particularity does not hold for other sources of dynamic detuning such as piezoelectric tuners or microphonics because the acting forces do not depend on the cavity fields. In those cases, the coupling coefficients can therefore be of both signs. This fact seems confirmed by measurements as shown in the next section. The system of ODE for the Lorentz forces action and piezoelctric tuner action can therefore be written as

$$\Delta\ddot{\omega}_m + \frac{\Omega_m}{Q_m}\Delta\dot{\omega}_m + \Omega_m^2\Delta\omega_m = -\Omega_m^2 k_{m,L} V^2 \qquad (1)$$

$$\Delta\ddot{\omega}_m + \frac{\Omega_m}{Q_m}\Delta\dot{\omega}_m + \Omega_m^2\Delta\omega_m = \Omega_m^2 k_{m,P} V_p \qquad (2)$$

where V_P is the input voltage driving the piezoelectric tuner. As mentioned, the coupling coeffients $k_{m,L}$ associated to the Lorentz forces are only expected positive from the model. From Eq. (1) and Eq. (2), it concludes that in CW operation the relevant parameter is K whereas in pulsed operation, the position of the modes with respect to the harmonics of the repetition rate and the width of the resonance peaks are the prime concerns. For this reason, a harmonical analysis of the actions of the Lorentz forces and of the piezoelectric tuner is of direct interest. This can be performed by measuring the transfer functions for both sources of dynamic detuning. In the next section, those measurements and their study using simulations are presented.

Mechanical parameters for the action of the piezoelectric tuner

The amplitude of the transfer function associated to the piezoelectric tuner action was presented in [1]. For the corresponding measurement settings, there is no initial detuning ($\Delta\omega_0 = 0$), the current source is constant through time ($\tilde{I}(t) = \tilde{I}_0$), and the dynamic detuning is sinusoidal ($\Delta\omega(t) = \Delta\omega_{osc}\sin\omega_{osc}t$). The voltage in steady state is periodic, using the parameterization $\theta = \omega_{osc}t$ gives for the normalized voltage $\tilde{v}(\theta) = \frac{\tilde{V}_{SST}(\theta)}{R_L\tilde{I}_0}$

$$\tilde{v}(\theta) = e^{-j\sigma\cos\theta}\sum_n \tilde{P}_n(\sigma)\cos\theta_n\cos(n\theta - \theta_n) \qquad (3)$$

where $\tan\theta_n = n\frac{\omega_{1/2}}{\omega_{osc}}$, $\tilde{P}_n$ are polynomials of $\sigma = \frac{\Delta\omega_{osc}}{\omega_{osc}}$ with complex coefficients explicitly given in [3]. Using Eq. (3) it is possible to efficiently reproduce the transfer function of the voltage phase ϕ_V by finding the appropriate mechanical parameters for the piezoelectric tuner action. Particularly, using coupling coefficients of both signs

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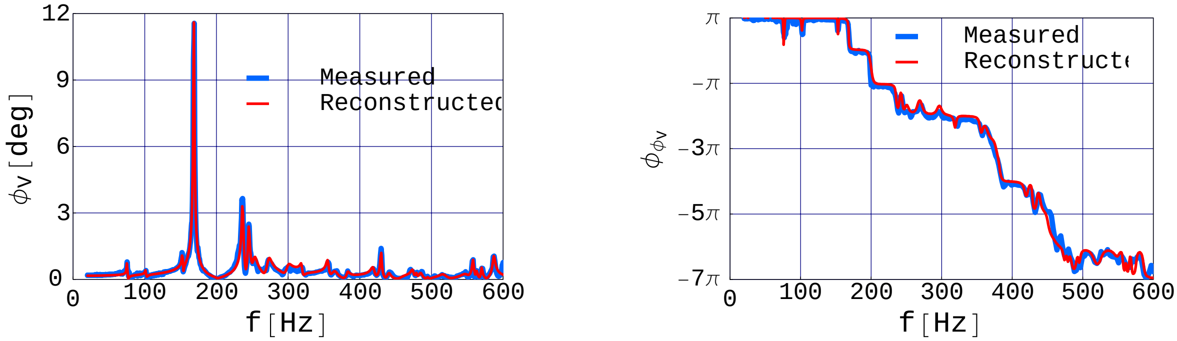


Figure 1: Measured and simulated transfer function (amplitude (left) and phase (right)) for the action of the piezoelectric tuner in the SNS medium beta cavity (Prototype cryomodule cavity #2). The input signal is the sinusoidal modulations of the piezoelectric input voltage $V_P(t)$, the output signal is the voltage phase $\phi_V(t)$.

allows to reproduce correctly the phase of the transfer function as displayed in Fig. 1.

Mechanical parameters for the action of the radiation pressure

A possible way to fill a cavity under dynamic detuning is the phase-lock loop configuration [6]. A phase-lock loop is a feedback loop designed to keep the phase between the forward RF power and the cavity voltage at a given constant value θ_l. As a consequence, the complexity of the coupled system of the voltage differential equations and the mechanical differential equations is greatly reduced because the voltage amplitude is independent of the dynamic detuning. Using this property, it is possible to obtain the transfer function associated to the action of the radiation pressure by exciting the cavity CW with small amplitude modulation on the forward power. The Lorentz transfer function was measured by JLAB in the SNS medium beta prototype cryomodule cavity #1 [5]. The modulated forward RF power $P_{RF} = P_0\{1 + \epsilon \sin \omega_{mod} t\}$, where ϵ is a small parameter, generates a detuning in the m^{th} mechanical mode

$$\Delta\omega_m(t) \approx -k_m V_l^2 \{1 + \epsilon \cos\phi_{mod} \cos\psi_m \\ Q_m \frac{\Omega_m}{\omega_{mod}} \sin(\omega_{mod} t + \phi_{mod} + \psi_m - \frac{\pi}{2})\} \quad (4)$$

where $V_l = \sqrt{8R_L P_0}\cos\theta_l$, $\tan\phi_{mod} = \frac{-\omega_{mod}}{\omega_{1/2}}$, and $\tan\psi_m = Q_m(\frac{\Omega_m}{\omega_{mod}} - \frac{\omega_{mod}}{\Omega_m})$. Using Eq. (4), it is possible to find adequate mechanical parameters to reproduce the Lorentz transfer function. Doing so, all the coupling coefficients are assumed positive as mentioned previously. The measured and the reconstructed transfer functions are presented in Fig. 2. The fact that all the coupling coefficients are of the same sign translates to a rather large response for the low frequency part of the amplitude of the Lorentz transfer function because the contributions of all the mechanical modes add constructively. In comparison, the low frequency part of the amplitude of the piezoelectric tuner transfer function is not as large because of the interference between modes with positive and negative coupling coefficients. Since all the $k_{m,L}$ are positive, the phase of the Lorentz transfer function remains bounded in $[0; \pi]$. More exactly, in $[-\frac{\pi}{2}; \pi]$ because of an additional $-\frac{\pi}{2}$ shift originating from the parameter ϕ_{mod} which evolves progressively from 0 to $-\frac{\pi}{2}$ as the frequency of the modulations ω_{mod} passes from small values to large values in comparison to the electromagnetic half-bandwidth $\omega_{1/2}$. This transition is the reason for the initial slope of the phase of the transfer function observed in Fig. 2. It is interesting to mention that the value of $\omega_{1/2}$ (and therefore of Q_{ex}) can be determined from this slope.

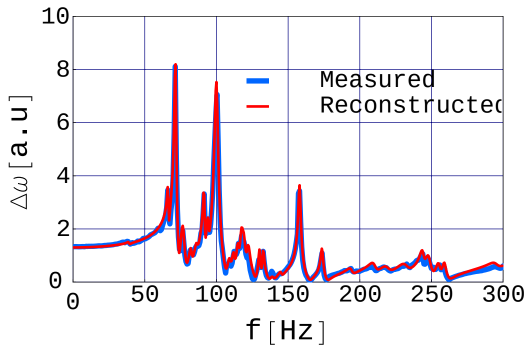
COMPENSATION SCHEME BASED ON A HARMONIC ANALYSIS OF THE LORENTZ DETUNING

Depending on the profile of the Lorentz detuning in nominal operation, a suited profile $V_P(t)$ has to be found to generate a detuning $\Delta\omega_P(t)$ equal and opposite to the Lorentz detuning $\Delta\omega_L(t)$ (at least during the beam pulse). Based on understanding from modeling, a harmonic analysis of the problem shows that an ideal compensation can be obtained even if the transfer functions for the piezoelectric tuner and for the radiation pressure are not identical in shape (i.e if the sets of coupling coefficients for both detuning sources are not proportional to each other). In the following, this harmonical approach is presented. For SRF cavities operated in pulsed mode, the Lorentz detuning in steady state contains only components of the repetition rate ω_{rep} and its harmonics. The Lorentz detuning can be written $\Delta\omega_L(t) = \sum_{n=0}^{\infty} \Delta\omega_{L,n} \cos(n\omega_{rep} t + \phi_{L,n})$. To obtain an ideal compensation, the detuning associated to the piezoelectric tuner should be such that $\Delta\omega_P(t) = \sum_{n=0}^{\infty} \Delta\omega_{P,n} \cos(n\omega_{rep} t + \phi_{P,n})$, with $\Delta\omega_{P,n} = \Delta\omega_{L,n}$ and $\phi_{P,n} = \phi_{L,n} + \pi$ for each harmonic n. In this prospective, the piezoelectric tuner input voltage must be of the form $V_P(t) = \sum_{n=0}^{\infty} V_{P,n} \cos(n\omega_{rep} t + \theta_{P,n})$. Supposing an infinite number of mechanical modes $m = 1, 2, ..., \infty$, it can be shown [3] that:

$$V_{P,n} = \frac{\Delta\omega_{L,n}}{\sqrt{a_n^2 + b_n^2}} \quad (5)$$

$$\theta_{P,n} = \phi_{L,n} + \pi - \varphi_n \quad (6)$$

where $a_n = \sum_{m=1}^{\infty} c_{n,m} \sin 2\psi_{n,m}$, $b_n = -\sum_{m=1}^{\infty} c_{n,m}\{1 + \cos 2\psi_{n,m}\}$, $c_{n,m} = \frac{k_{P,m} Q_m \Omega_m}{2n\omega_{rep}}$, $\tan\varphi_n = \frac{b_n}{a_n}$, and



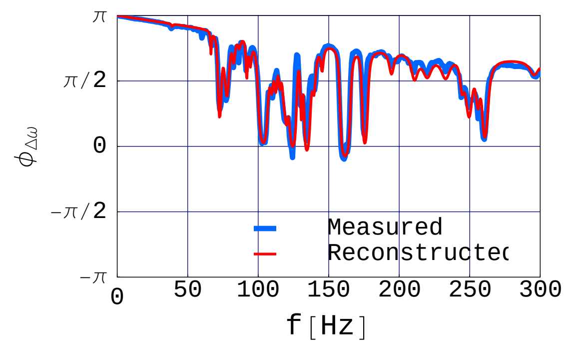


Figure 2: Amplitude and phase of the measured [5] and reconstucted transfer function associated to the Lorentz forces in the SNS medium beta cavity (Prototype cryomodule cavity #1). The input signal is the sinusoidal modulations of the forward RF power, the output signal is the dynamic detuning $\Delta\omega(t)$.

$\tan\psi_{n,m} = Q_m(\frac{\Omega_m}{n\omega_{rep}} - \frac{n\omega_{rep}}{\Omega_m})$. To illustrate this method, the case of the SNS medium beta cavity is considered. An example of a dynamic detuning profile for a cavity operated close to nominal condition was presented in [5] and is used for the illustration of the method. The initial part of the measured Lorentz detuning is caused by a parasitic signal [3] and is therefore suppressed in the analysis. The mechanical basis for the action of the piezoelectric tuner was reconstructed up to 600 Hz, as shown in Fig. 1. Since the repetition rate is 60 Hz, the Lorentz detuning function is truncated to the first ten harmonics as displayed in Fig. 3. The input voltage $V_P(t)$ is calculated using Eq. (5) and Eq. (6). The result is plotted in Fig. 4.

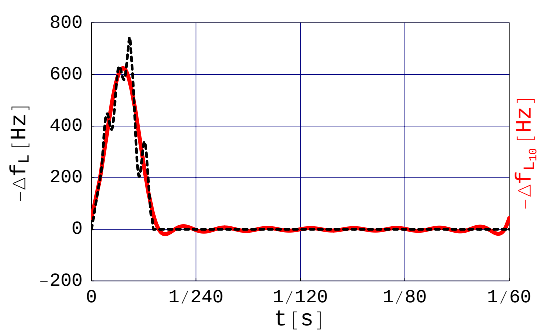


Figure 3: Measured Lorentz detuning function $\Delta f_L(t)$ [5] and its truncation to the 10^{th} harmonic of 60 Hz, $\Delta f_{L_{10}}(t)$. This truncated function constitutes the function compensated by the piezoelectric tuner input voltage $V_P(t)$ of Fig. 4.

CONCLUSION

The modeling effort was beneficial for a better understanding of the coupling of the Lorentz forces and of the piezoelectric tuner action to the cavity. From this understanding, the mechanical basis for both detuning sources could be reconstructed by the use of simulating tools. A harmonic analysis was presented to show the underlying basics of the compensation. The proposed scheme can be further simplified [3] for practical application.

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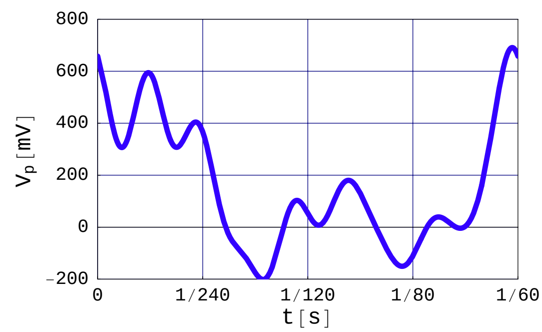

Figure 4: Input voltage function $V_P(t)$ for the piezoelectric tuner (repetitive 60Hz, before amplification) for the compensation of the truncated Lorentz detuning function $\Delta f_{L_{10}}(t)$.

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