THEORETICAL STUDIES OF BEAM-BEAM EFFECTS IN THE TEVATRON AT COLLISION ENERGY

T. Sen*†, B. Erdelyi and M. Xiao

Abstract

The dynamics due to the long-range beam-beam interactions depends on several beam parameters such as tunes, coupling, chromaticities, beam separations, intensities and emittances. We have developed analytical tools to calculate, for example, amplitude dependent tune shifts and chromaticities, beam-beam induced coupling, and betatron and synchro-betatron resonance widths. We report on these calculations and dynamic aperture calculations with long-term tracking. These theoretical results are compared with observations at collision energy and used to predict performance at design values of beam intensities and emittances.

1 INTRODUCTION

Beam-beam phenomena have limited the beam currents and the luminosity achievable in the Tevatron. Injected proton currents are about 10 times larger than the anti-proton currents so beam-beam effects have largely acted on the anti-protons at all stages of the operational cycles. At collision of Tevatron Run IIa , 36 anti-proton($\bar{p}$) bunches in three trains of twelve bunches collide with 36 proton(p) bunches. Each bunch will experience two head-on interactions at B0 and D0 and seventy long-range interactions. These long-range are distributed over the entire ring with differing beam separations and differing phase advances from one interaction to the next. Fig. 1 shows the beam separations (in unit of the $r.m.s.$ beam size) at all the seventy-two locations of beam-beam interactions for bunch 1, 6 and 12. The simulation of long-term tracking shows that long-range beam-beam interactions, in particular, these nearest interactions have the dominant effects on the dynamic aperture. It is well known that the long-range beam-beam force generates amplitude dependent tune shift, which in turn implies that the chromaticity shift is also amplitude dependent. These amplitude dependencies induce the familiar tune footprint, and also coupling and chromaticity spread within a bunch.

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2 BEAM-BEAM TUNE SHIFT

For details on the derivation of the formulae see [1]. The expression for the horizontal amplitude dependent tune shift is

$$\Delta\nu_x = \frac{4\pi C}{\varepsilon_x}\int_0^1 \frac{e^{-(p_x+p_y)}}{v\left[v\left(r^2-1\right)+1\right]^{1/2}}\sum\nolimits_x\sum\nolimits_y dv, \quad (1)$$

where

$$\sum\nolimits_x = \sum_{k=0}^{\infty}\frac{\left(\frac{a_x}{d_x}\right)^k}{k!}\Gamma\left(k+\frac{1}{2}\right) \times\left[I_k\left(s_x\right)\left(\frac{2k}{a_x^2}-v\right)+I_{k+1}\left(s_x\right)\frac{s_x}{a_x^2}\right], \quad (2)$$

$$\sum\nolimits_y = \sum_{l=0}^{\infty}\frac{\left(\frac{a_y}{d_y}\right)^l}{l!}\Gamma\left(l+\frac{1}{2}\right)I_l\left(s_y\right). \quad (3)$$

As shorthand notations we introduced the ratio of rms beam sizes $r=\sigma_y/\sigma_x$, and dimensionless variables for the amplitudes and separations according to $a_x=\sqrt{2\beta_x J_x}/\sigma_x$, $d_x=D_x/\sigma_x$ and similarly defined a_y and d_y. Using these notations, the following relationships have been used in (1): $p_x=v\left(a_x^2+d_x^2\right)/2$, $r_x=va_x^2/2$, $s_x=va_xd_x$, $p_y=fv\left(a_y^2+d_y^2\right)/2$, $r_y=fva_y^2/2$, $s_y=fva_yd_y$,where $f=\frac{r^2}{v(r^2-1)+1}$. The vertical amplitude dependent tune shift is derived analogously, due to symmetry in x and y.

For example, the tune footprint shown in Figure 2 includes all beam-beam interactions acting on $\bar{p}$ bunch 6, and the analytical results are superimposed on the footprint obtained by tracking.

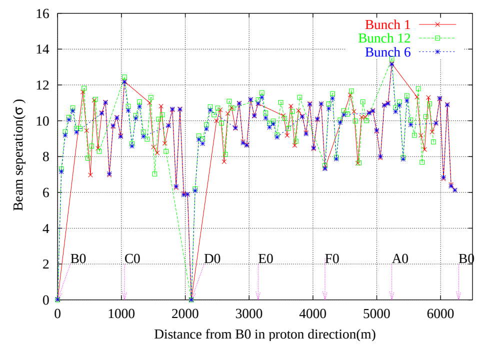

Figure 1: Separations at beam-beam encounters for pbar bunch 1, 6 and 12

3 BEAM-BEAM CHROMATICITY

To provide a formula for the computation of the chromaticities, we split the separation into two parts: one due to the closed orbits of on-momentum particles, the other due to dispersion for off-momentum particles. Denoting

* tsen@fnal.gov

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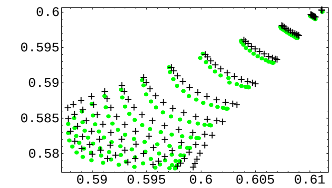

Figure 2: Tune footprints corresponding to all 72 interactions. Shown is a superposition of the analytical results with the tune footprints obtained by FFT of tracking data.

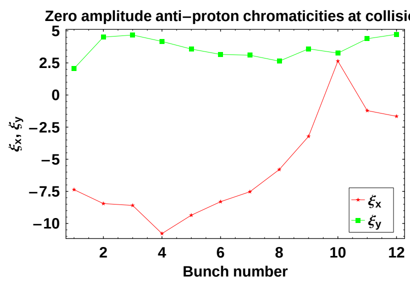

Figure 3: Zero amplitude chromaticities of a train of 12 anti-proton bunches.

the dispersion (in units of rms beam size) at the location of the interaction by η, first we make the following replacements in (1): $d_x \mapsto d_x + \eta_x\delta$, $d_y \mapsto d_y + \eta_y\delta$, where δ is the relative momentum or energy deviation. The horizontal zero amplitude chromaticity is given by

$$\xi \frac{4}{d^6}\left(d_x^3\eta_x + 3d_x^2 d_y \eta_y - 3d_x d_y^2 \eta_x - d_y^3\eta_y\right). \quad (4)$$

The vertical chromaticity can be calculated similarly.

Figure 3 shows the zero amplitude chromaticities of a train of 12 $\bar{p}$ bunches. Notice the large variations in the bunch-to-bunch horizontal chormaticities.

4 BEAM-BEAM INDUCED COUPLING

The minimum tune split is a measure of the global coupling and is given by the amplitude of the complex term driving the difference resonance $(\nu_x - \nu_y) = p$ [2]. This driving term due to beam-beam interaction at a location s in the ring is given by

$$F(a_x, a_y) = -\frac{N_p r_p \sqrt{\beta_x \beta_y}}{2\pi\gamma_p}\frac{r}{\sigma_x^2}(a_x + d_x)(a_y + d_y)$$
$$\times \exp\left\{i(\psi_x - \psi_y - (\nu_x - \nu_y - p)\frac{s}{R})\right\} \times$$
$$\int_0^1 dv \frac{v}{[v(r^2-1)+1]^{3/2}}$$
$$\cdot \exp\left\{-\frac{v}{2}[(a_x^2 + d_x^2) + f(a_y^2 + d_y^2)]\right\} \quad (5)$$

Fig. 4 shows the minimum tune splits of zero amplitude particles for 12 bunches in a train. Bunch to bunch difference in coupling can be identified.

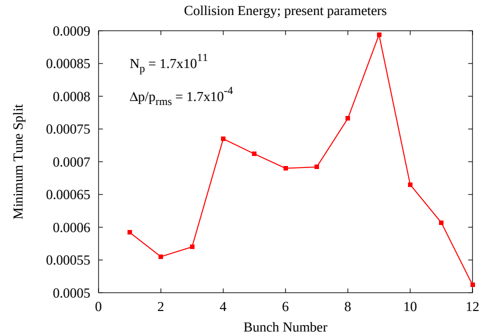

Figure 4: Small amplitude beam-beam coupling at collision

5 RESONANCE STRENGTHS

Resonance driving terms have been computed by the normal form method in the code Cosy Infinity. Their normalization along the diagonal at an amplitude of 2σ and summation over the subresonances are shown in Figure 5. The two pictures show the resonance strengths of the resonances driven by the lattice nonlinearities, and by the addition of the beam-beam interactions. The largest strength is always scaled to 1. Clearly, the lattice drives mostly third order resonances while the beam-beam effects drive the seventh and fifth order ones. There is no single dominating resonance. This makes active beam-beam compensation harder. We are looking into current wire correction of the resonance strengths, and as a first step we are investigating whether with a few appropriately placed wires the same type of resonance structure could be excited. In case of affirmative answer the beam-beam driven resonances could be significantly reduced.

6 FOOTPRINT COMPENSATION

Since at collision the nearest parasitics dominate the nonlinear dynamics, we attempted to minimize the footprints, by compensating for the aspect ratios or dispersions, as shown in [1]. Compensation of aspect ratios clearly reduces both the shift and the spread of the tunes, as can be seen in the left plot of Figure 6. On the other hand, compensation of aspect ratios does not have a dramatic effect on the chromaticity footprint. Perhaps more importantly, aspect ratio compensation does not harm the chromaticity footprint. The chromaticity footprint is mainly affected by compensation of the dispersions; there is a significant reduction in the size of the footprint. The result is contained in right plot of Figure 6. Unfortunately, by tracking with the conditions corresponding to the compressed footprints we concluded that the compensations have a marginal effect on the DA, increasing the average DA by up to ≈ 0.5 σ and the minimum by 1 σ, but fail to show the dramatic effects similar to the footprint size reduction.

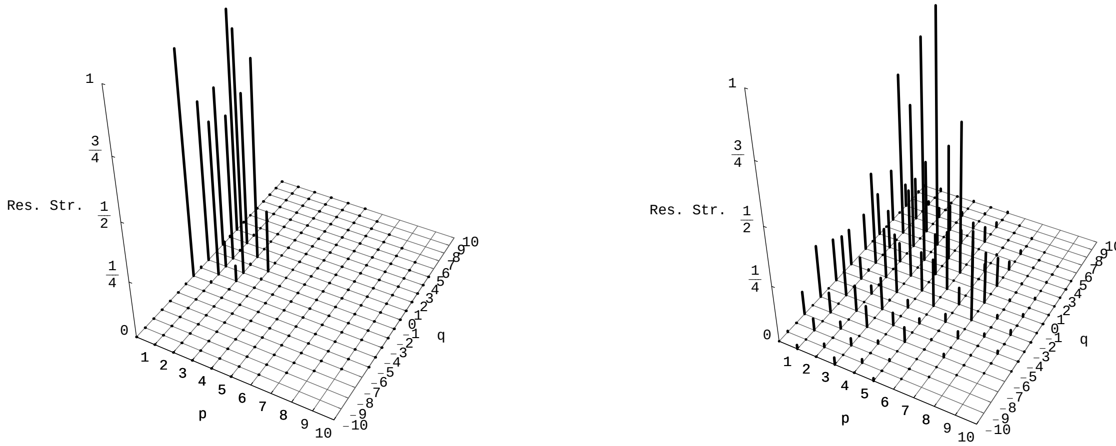

Figure 5: Resonance strengths at collision $a)$ without and $b)$ with beam-beam effects.

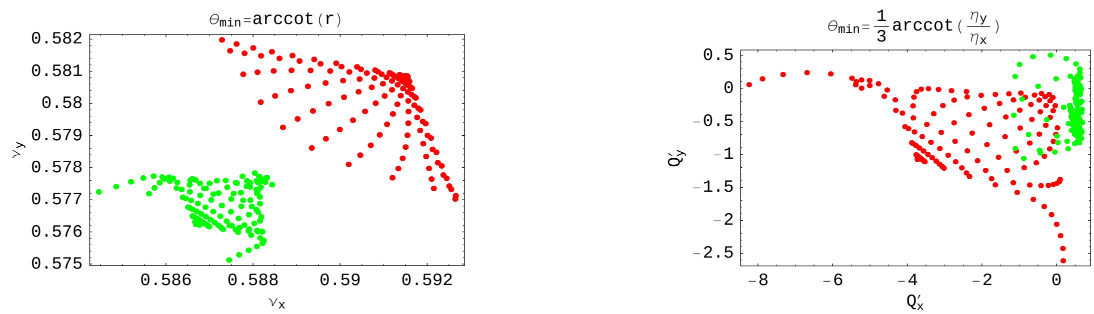

Figure 6: Left:Tune and Right: Chromaticity footprint compensation of the nearest parasitic beam-beam interactions.

7 NONLINEAR EFFECTS ON DYNAMIC APERTURE

We have calculated DA(Dynamic Aperture) due to the beam-beam effects by long-term tracking [3]. Due to the large proton bunch length(comparable to β^*=0.35m) and the rapidly varying betatron phase at the IPs, the anti-protons experience these kicks over a range of phases. For head-on interactions, we divided a proton bunch into 9 slices, taking the bunch length effects into account. These effects include hour glass effects (assuming a Gaussian beam distribution longitudinally) and phase variations (propagation between slices).The long-range interactions are modeled by delta function kicks. Fig. 7 gives the dynamic aperture in function of proton bunch intensities for $\overline{p}$ bunch 1, 6 and 12. Particles are tracked for 100,000 turns. We can see that DA of $\overline{p}$ bunch 1 is better than that of $\overline{p}$ bunch 6 except at one point. DA of $\overline{p}$ bunch 12 is also better than that of $\overline{p}$ bunch 6 at most of the intensities.

8 CONCLUSION

We have developed analytical tools for the computation of the amplitude dependent tune shifts, linear chromaticities and couplings due to beam-beam interactions. The expressions can be used for efficient numerical evaluation at any amplitude, separation, dispersion and aspect ratio.

At collision, the DA is largely dependent on proton intensity, and different from bunch to bunch. We found that the DAs are determined by the long-range interactions. The tune footprint is largely determined by the head-on collision, but head-on collision have very little effect on the DA.

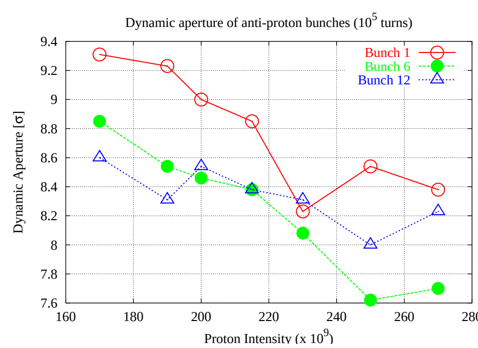

Figure 7: Dynamic aperture $vs.$ proton beam intensity

9 REFERENCES

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