NONLINEAR RESONANCE BENCH-MARKING EXPERIMENT AT THE CERN PROTON SYNCHROTRON

I. Hofmann*, G. Franchetti, GSI, 64291 Darmstadt, Germany
M. Giovannozzi, M. Martini and E. Metral, CERN, 1211 Geneva, Switzerland

Abstract

As a first step of a space charge - nonlinear resonance benchmarking experiment over a large number of turns, beam loss and emittance evolution were measured over 1 s on a 1.4 GeV kinetic energy flat-bottom in the presence of a single octupole. By lowering the working point towards the resonance a gradual transition from a loss-free core emittance blow-up to a regime dominated by continuous loss was found. Our 3D simulations with analytical space charge show that trapping on the resonance due to synchrotron oscillation causes the observed core emittance growth as well as halo formation, where the latter is explained as the source of the observed loss.

1 INTRODUCTION

The study of the combined effect of space charge and nonlinear resonances over as many as 10^5 - 10^6 turns is gaining importance with the need of optimizing the performance of synchrotrons employing high intensity or high phase space density. The latter is relevant for the PS as part of the injector for the LHC; furthermore for the SIS100 of the GSI future project [1], where it is necessary to hold the high-intensity bunches between injections over typically 1 s, and at a loss level not exceeding 1%. Up to now comparison of simulation with experimental work for second or higher order resonances has been successfully carried out in the millisecond time frame [2, 3]. In the realm of long-term behavior, instead, where self-consistent 3D simulation is beyond current computer capabilities, the question of adequate approximations in space charge calculation is a challenging one. Moreover, an explanation of the proper mechanisms describing the combined effect of nonlinearity, space charge and synchrotron oscillation - as suggested recently in a simplified model in Ref. [4] - is crucial.

2 MEASUREMENTS

The measurements where carried out as part of a high intensity machine development time at the PS in October 2002. The number of protons in the bunch was $1.1 \cdot 10^{12}$ – small enough to avoid overlap with other resonances. A vertical maximum space charge tune shift of 0.12, and a horizontal one of 0.075 (for minimum amplitude particles) were achieved with relatively small emittances of $\epsilon_x = 9$ mm mrad and $\epsilon_y = 4.5$ mm mrad (unnormalized at 2σ). The bunch profiles measured 10 ms after injection were found to be Gaussian in all directions in the absence of the octupole. The vertical machine tune was set to $Q_y = 6.12$, and the horizontal tune was varied in the interval $6.25 < Q_x < 6.32$. The chromaticity is close to the natural one, hence the small momentum spread of 10^{-3} allows ignoring chromatic effects. The kinetic energy was kept at the injection value of 1.4 GeV with a measurement window of 1 s ($4.4 \cdot 10^5$ turns) over which the bunch intensity was monitored with a current transformer. The calibrated octupole (here $k_3 = 1.215 \cdot I$ m^{-3}) was powered to 40 A at 110 ms after injection to excite the resonance $4Q_x = 25$. We used the transverse profiles measured with the flying wire (20 m/s), fitted a Gaussian profile to them, and determined their rms emittances. Initial and - in most cases - final profiles were actually found quite close to Gaussian.

In Fig. 1 results of final measurements 1 s after injection are plotted as a function of the machine working point. Our main finding is the existence of two regimes: an emittance growth dominated regime for Q_x sufficiently above the resonance - in our example $Q_x > 6.28$, and a loss dominated regime for $Q_x < 6.28$. It is noted that for the working point of maximum loss ($Q_x = 6.27$) the emittance also shrinks, since large amplitude particles are preferably extracted. The time evolution of the bunch intensity for $Q_x = 6.27$ is shown in Fig. 2. Note the continuous loss at a nearly constant rate after an initially enhanced loss (the intensity drop at 1200 ms is caused by the kicker event).

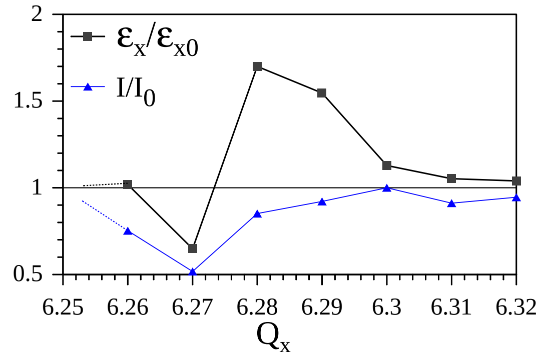

Figure 1: Experimental results on final rms emittance (of Gaussian fit) and beam current relative to initial values.

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[*] i.hofmann@gsi.de

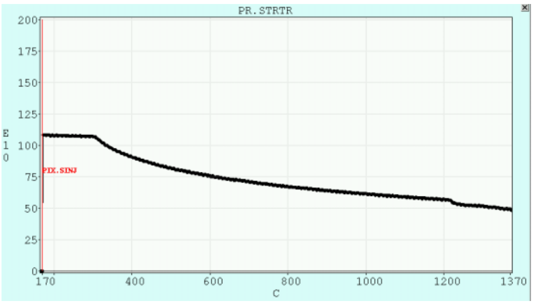

Figure 2: Measured bunch intensity as function of time at $Q_x = 6.27$ with octupole powered at 280 ms (110 ms after injection at time 170 ms).

3 SIMULATION

For an interpretation of these results we have carried out a series of simulation runs in 2D and 3D. We have replaced, for simplicity, the linear PS focusing lattice by constant focusing and ignored lattice nonlinearities besides the contribution from the well-defined octupole. We have also ignored the smaller vertical beam emittance of the experiment and assumed a circular cross section to match with the limitation in the analytical 3D space charge model, which is based on a rotational ellipsoid. The horizontal emittance has been chosen such as to reproduce accurately the maximum horizontal space charge tune shift extracted from the measurement, which we believe is the crucial issue since we are not dealing with a coupling resonance.

The loss observed in the experiment cannot be attributed to the shrinking of the dynamic aperture as a result of the octupole alone. To support this we have carried out a numerical study on the dynamic aperture by searching the maximum stable radius of test particles placed into 20 different directions in the upper half of the $x - y$ plane. We have found that the nominal octupole (40 A) leads to a dynamic aperture (10^5 turns) of about 5σ (with σ the horizontal rms beam size) near $Q_x = 6.25$, which may not be small enough to cause extraction of particles, and a more complete knowledge of machine nonlinearities may be required to explain the loss. Assuming 200 A octupole current we have calculated that the dynamic aperture shrinks to a radius of 2.5σ near $Q_x = 6.25$ for 10^3 turns, and about 2.2σ for 10^5 turns. Note that one expects a theoretical reduction proportional to the inverse square root of the octupole strength, which is roughly confirmed by our simulations.

We first attempt a comparison with the fully self-consistent 2D particle-in-cell (PIC) version of the MICROMAP code [5] with 10^5 simulation particles. We employ a Gaussian distribution function and a 64x64 grid filling a rectangular boundary of 70x70 mm size. We find no loss for 40 A: the rms emittance growth remains below 2%; for comparison, it is 15% - independent of octupole strenght - at $Q_x = 6.25$ in the absence of space charge (the time it takes is about inversely proportional to the octupole strength). We explain this strikingly low saturation level as a consequence of the large space charge de-tuning relative to the natural de-tuning effect of the octupole. A more pronounced effect, with a loss regime and an emittance growth regime analogous to the experiment, is obtained assuming a 5 times stronger octupole. This is shown in Fig. 3 after 100 turns, where the effect is practically saturated. The saturated maximum emittance growth is now about 8%, which reflects the reduced space charge de-tuning relative to the octupole strength. The loss regime confirms the shrinking of the dynamic aperture for $Q_x \to 6.25$.

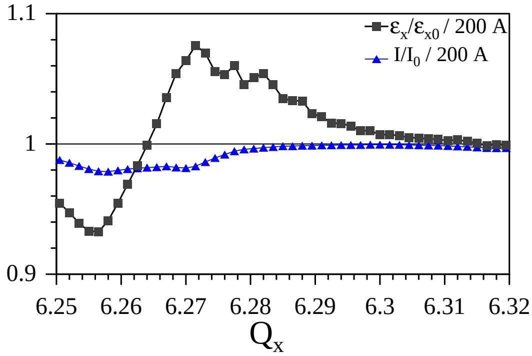

Figure 3: Result of fully self-consistent 2D simulation with 5x stronger octupole (200 A) after 1000 turns. Shown are rms emittances and total intensity in units of initial values.

In order to explore the evolution over significantly longer times we replace the fully self-consistent space charge calculation by an analytical approximation. While such analytical space charge models ignore the dynamically changing space charge force, they have the advantage of being much faster and avoiding the inherent noise of PIC-simulation. As a first example we extend the above 2D coasting beam study with the same transverse rms values, a Gaussian transverse distribution and an analytical space charge force consistent with the initial Gaussian distribution using a method generalized from Ref.[6]. The result after 10^3 turns is found to deviate by not more than ±10% from the self-consistent simulation, with practically no change between 10^3 and 10^5 turns. This suggests that self-consistency might not be an important factor as long as emittance growth or loss are small enough.

For the 3D simulation we employ a density distribution of the kind $(1-x^2/a^2)^3$ in all three directions. We can then perform an exact integration of Poisson's equation using a similar method as in 2D. Using 2000 test particles we generate a (initially) consistent distribution in 6D phase space with the same bunch length (200 ns at 4σ) and synchrotron period (645 turns) as in the experiment.

The dependence on the working point is shown in Fig. 4 to give a similar behavior as in the experiment for $Q_x > 6.28$, but no loss for smaller tunes. For better comparison with the experiment we apply a Gaussian fit to the simulation data and determine the rms emittance from it, which puts the emphasis on the core emittance. Note that the relatively large emittance growth without accompanying loss reflects the large physical aperture in both experiment and simulation, if compared with the initial beam size. For an assumed 200 A octupole, instead, the simulation shows a continuous loss ($< 25\%$) due to the much reduced dynamical aperture.

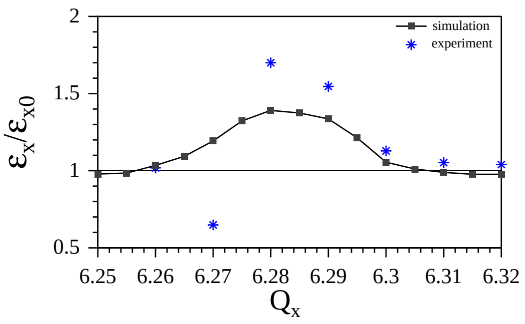

Figure 4: 3D simulation using analytical space charge (40 A octupole). Shown are simulated rms emittances (Gaussian fit) after $5 \cdot 10^5$ turns, and experimental values.

The rms emittance evolution of a typical case is shown in Fig. 5, which compares well with the measured data. The main difference is that the simulation rms emittance growth saturates, since particles with a small enough synchrotron amplitude (depending on Q_x) can never cross the resonance. We compare this result with a modification, where the growing emittance is used to update the horizontal rms size. As a result of this rms self-consistency space charge gets weaker, which allows more particles to cross the resonance. This enhances the growth and leads to even better agreement with the measurement.

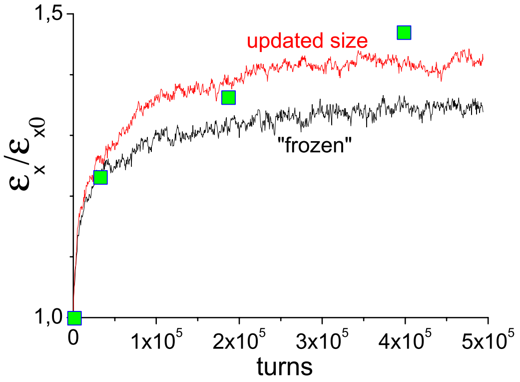

Figure 5: 3D simulation emittance evolution for $Q_x = 6.29$ comparing analytical “frozen” space charge with results obtained by using a continuously updated rms size (squares: measured values).

Our interpretation of the significant difference between 2D and 3D relies on synchrotron motion: in 2D particles can be on resonance for basically *one* value of the betatron amplitude, and if so they get easily de-tuned again with only small amplitude increase, due to the dominant space charge de-tuning; in 3D the synchrotron motion carries the particles from high to low space charge, thus a large number of particles is able to periodically cross the resonance at various betatron amplitudes. Eventually such particles are caught by the resonance, which implies that they are driven to larger transverse amplitude to compensate the enhanced space charge when moving back to the bunch center. As was shown in Ref. [4] such trapping may be followed by de-trapping after some time unless the particle hits the aperture. Therefore the resulting maximum halo increases for $Q_x \rightarrow 6.25$ (Fig. 6), which is the reason for the loss region in the experiment, where apparently further nonlinearities cause a smaller dynamic aperture than in the simulation and lead to extraction. Note that for $Q_x \rightarrow 6.32$, where the resonance loses its effect, the maximum halo size agrees with the initial beam edge of 3σ.

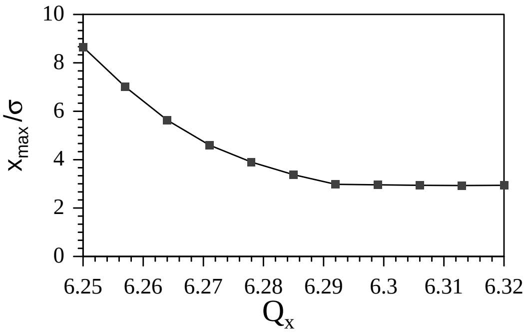

Figure 6: Halo radius (in units of initial σ).

4 CONCLUSION

The synchrotron oscillation has been shown to enhance significantly the response on the octupole. In the emittance growth regime quite good agreement is achieved with the measurements over half a million turns, which supports our 3D space charge model and interpretation. We predict the formation of a halo increasing in size for $Q_x \rightarrow 6.25$ and claim this is the source of the measured loss. Future measurements should consider weaker octupoles, where the predicted halo might be entirely inside the dynamic aperture. Cross-checks with fully self-consistent 3D simulation over some 10^4 turns are planned for the near future.

5 REFERENCES

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