BUNCH TRANSVERSE EMITTANCE INCREASES IN ELECTRON STORAGE RINGS

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Abstract

In this paper a theoretical framework to estimate the bunch transverse emittance grow up in electron storage rings due to short range transverse wakefield of the machine is established. The new equilibrium emittance equations are derived and applied to explain the experimentally obtained results in ATF damping ring.

INTRODUCTION

Required by the future e^+e^- linear colliders, damping rings are needed to provide the main linacs with extremely small transverse emittance beams. In an electron storage ring, it is observed that with the increasing bunch current not only a bunch suffers from bunch lengthening, increase in energy spread, but also transverse emittance growth. The usual explanation to the transverse emittance grow up is based on the intrabeam scattering theory [1][2][3] which has its origin from H. Bruck's idea [4]. Comparison of the emittance grow up between experimental results and those from intrabeam scattering theory shows, however, that in the vertical plane the agreement is not satisfactory [5][6]. In this paper we will draw attention to another important physical cause for the transverse emittance grow up in addition to the intrabeam scattering, i.e. the short range transverse wakefield of the machine.

EQUATION OF TRANSVERSE MOTION

The differential equation of the transverse motion of a bunch with zero transverse dimension is expressed as

$$\frac{d^2y(s,z)}{ds^2}+\frac{2}{\tau_y c}\frac{dy(s,z)}{ds}+k(s,z)^2y(s,z)$$
$$=\frac{1}{m_0c^2\gamma(s,z)}e^2N_eW_{\perp,y}(s,z)Y(s,z) \qquad (1)$$

where $y(s,z)$ is the particle's transverse deviation from the closed orbit, s is the longitudinal coordinate of the particle located at the center of a bunch, z denotes a particle's longitudinal position inside the bunch with respect to the bunch center, $k(s,z)$ describes the linear lattice focusing strength, $W_{\perp,y}(s,z)=\int_z^\infty \rho(z')\mathcal{W}_{\perp,y}(s,z'-z)dz'$, $\mathcal{W}_{\perp,y}(s,z)$ is the point charge wakefield, the bunch line charge density $\rho(z)$ is normalized as $\int_{-\infty}^{\infty}\rho(z')dz'=1$, c is the velocity of light, τ_y is the synchrotron radiation damping time in transverse y direction, m_0 is the rest mass of the electron, e is the electron charge, and $Y(s,z)$ is the deviation between particles and the geometric center of vacuum chamber. Due to synchrotron radiation effect, one can treat the particles in a bunch on the same footing by multiplying $\rho(z)$ on both sides of eq: 1 and make the integration from $-\infty$ to ∞ over z. Consequently, one gets

$$\frac{d^2y(s)}{ds^2}+\Gamma\frac{dy(s)}{ds}+k(s)^2y(s)=\Lambda \qquad (2)$$

where $\Gamma=\frac{2}{\tau_y c}$, $\Lambda=\frac{e^2N_ek_{\perp,y}(\sigma_z)Y(s)}{m_0c^2\gamma l_s}$, l_s is the circumference of the storage ring, $k_{\perp,y}(\sigma_z)=\int_0^{l_s}\left\{\int_{-\infty}^{\infty}\rho(z)W_{\perp,y}(s,z)\,dz\right\}ds$, and $\rho(z)=\frac{1}{\sqrt{2\pi}\sigma_z}e^{-\frac{z^2}{2\sigma_z^2}}$. $Y(s)$ is a random variable due to vacuum chamber misalignment error and close orbit distortion with $<Y(s)>=0$ ($<>$ denotes the average over s). Eq. 2 can be regarded as Langevin equation which governs the Brownian motion of a molecule.

To make an analogy between the movement of the transverse motion of an electron and that of a molecule, we define $P=\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}$, and regard $Y(s)P$ as the particle's "velocity" random increment ($\Delta\frac{dy}{ds}$) over the distance l_s. We assume that the random variable $Y(s)$ follows Gaussian distribution:

$$f(Y(s))=\frac{1}{\sqrt{2\pi}\sigma_Y}\exp\left(-\frac{Y(s)^2}{2\sigma_Y^2}\right) \qquad (3)$$

and the velocity (u) distribution of the molecule follows Maxwellian distribution:

$$g(u)=\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mu^2}{2kT}\right) \qquad (4)$$

where m is the molecule's mass, k is the Boltzmann constant, and T is the absolute temperature. The fact that the molecule's velocity follows Maxwellian distribution permits us to get the distribution function for Λl_s [7]:

$$\phi(\Lambda l_s)=\frac{1}{\sqrt{4\pi q l_s}}\exp\left(-\frac{\Lambda^2 l_s^2}{4ql_s}\right) \qquad (5)$$

where

$$q=\Gamma\frac{kT}{m} \qquad (6)$$

By comparing eq. 5 with eq. 3, one gets:

$$2\sigma_Y^2=\frac{4ql_s}{P^2} \qquad (7)$$

or

$$\frac{kT}{m}=\frac{\sigma_Y^2P^2}{2l_s\Gamma} \qquad (8)$$

Till now one can use all the analytical solutions concerning the random motion of a molecule governed by eq. 2 by a simple substitution described in eq. 8. Under the condition, $k^2(s) >> \frac{\Gamma^2}{4}$ (adiabatic condition), one gets [7]:

$$<y^2>=\frac{kT}{mk^2(s)}+\left(y_0^2-\frac{kT}{mk^2(s)}\right)$$
$$\times\left(\cos(k_1 s)+\frac{\Gamma}{2k_1}\sin(k_1 s)\right)^2\exp(-\Gamma s)$$
$$=\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2$$
$$+\left(y_0^2-\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2\right)$$
$$\times\left(\cos(k_1 s)+\frac{\Gamma}{2k_1}\sin(k_1 s)\right)^2\exp(-\Gamma s) \quad (9)$$

$$<y'^2>=\frac{kT}{m}+\frac{k(s)}{k_1^2}\left(y_0^2-\frac{kT}{mk^2(s)}\right)\sin^2(k_1 s)\exp(-\Gamma s)$$
$$=\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2$$
$$+\frac{k(s)}{k_1^2}\left(y_0^2-\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_y(\sigma_z)}{m_0c^2\gamma}\right)^2\right)$$
$$\times\sin^2(k_1 s)\exp(-\Gamma s) \quad (10)$$

$$<yy'>=\frac{k(s)^2}{k_1}$$
$$\times\left(\frac{kT}{mk(s)^2}-y_0^2\right)\left(\cos(k_1 s)+\frac{\Gamma}{2k_1}\sin(k_1 s)\right)\exp(-\Gamma s)$$
$$=\frac{k(s)^2}{k_1}\left(\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2-y_0^2\right)$$
$$\times\left(\cos(k_1 s)+\frac{\Gamma}{2k_1}\sin(k_1 s)\right)\exp(-\Gamma s) \quad (11)$$

where $k_1=\sqrt{k(s)^2-\frac{1}{4}\Gamma^2}$. The asymptotical values for $<y^2>$, $<y'^2>$, and $<yy'>$ as $s\to\infty$ are easily obtained:

$$<y^2>=\frac{kT}{mk^2(s)}=\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (12)$$

$$<y'^2>=k^2(s)<y^2>=\frac{\sigma_Y^2\tau_y}{4T_0}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (13)$$

$$<yy'>=0 \quad (14)$$

Inserting eqs. 12, 13, and 14 into the definitions of the r.m.s. emittance shown in eq. 15:

$$\epsilon_{w,y}=\left(<y^2><y'^2>-<yy'>^2\right)^{1/2} \quad (15)$$

one gets

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y}{4T_0k(s)}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (16)$$

or

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y<\beta_y(s)>}{4T_0}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (17)$$

where $<\beta_y(s)>$ is the average beta function of the machine in y plane. Before going on further, we have to remind the reader that at the beginning of this section it is assumed that the bunch has zero transverse dimension (the bunch is represented as a soft line), in reality, however, a bunch has finite transverse dimension. A particle inside the bunch can move like a molecule in a gas due to quantum effect of synchrotron radiation. In electron storage rings, the “banana” shape of the bunch cannot be sustained due to “mixing”, quite different from what happens in a linac and a hadron storage ring where there is no, or little, synchrotron radiations. Mathematically to take this fact into account, one can rewrite eq. 17 as follows

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y<\beta_y(s)>}{4T_0\mathcal{R}_{\epsilon,y}^3}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (18)$$

where $\mathcal{R}_{\epsilon,y}=\epsilon_{total,y}/\epsilon_{0,y}$, $\epsilon_{total,y}$ is the final emittance at a given bunch population N_e, $\epsilon_{0,y}$ is the emittance zero current, and the cubic functional dependence on $\mathcal{R}_{\epsilon,y}$ can be regarded as an Ansatz. Finally, we find the expression for the emittance of a bunch corresponding to a given bunch population

$$\epsilon_{total,y}=\epsilon_{0,y}+\epsilon_{w,y}$$
$$=\epsilon_{0,y}+\frac{\sigma_Y^2\tau_y<\beta_y(s)>}{4T_0}\left(\frac{e^2N_ek_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2 \quad (19)$$

If we distinguish now the horizontal plane denoted by the subscript x and the vertical plane denoted by the subscript y, one gets the two emittance equations

$$\mathcal{R}_{\epsilon,x}=\frac{\epsilon_{total,x}}{\epsilon_{0,x}}$$
$$=1+\frac{\sigma_X^2\tau_x<\beta_x(s)>}{4T_0\epsilon_{0,x}\mathcal{R}_{\epsilon,x}^3}\left(\frac{e^2N_ek_{\perp,x}(\sigma_{z0})}{m_0c^2\gamma\mathcal{R}_z^{\Theta}}\right)^2 \quad (20)$$

$$\mathcal{R}_{\epsilon,y}=\frac{\epsilon_{total,y}}{\epsilon_{0,y}}$$
$$=1+\frac{\sigma_Y^2\tau_y<\beta_y(s)>}{4T_0\epsilon_{0,y}\mathcal{R}_{\epsilon,y}^3}\left(\frac{e^2N_ek_{\perp,y}(\sigma_{z0})}{m_0c^2\gamma\mathcal{R}_z^{\Theta}}\right)^2 \quad (21)$$

where σ_{z0} is the bunch length of zero current, $\mathcal{R}_z=\sigma_z/\sigma_{z0}$, and $\Theta=0.7$, which corresponds to SPEAR scaling for transverse loss factor [8]. Since $\mathcal{R}_z$ is also a function of N_e, it is obvious that one can start to solve eqs. 20 and 21 only when $\mathcal{R}_z(N_e)$ has been solved from the bunch lengthening equation [9].

APPLICATION TO THE ANALYSIS OF ATF DAMPING RING EXPERIEMTAL RESULTS

ATF damping ring is a machine dedicated for the feasibility studies of future e^+e^- linear colliders [10]. In this section, by applying our theory established above and neglecting intrabeam scattering effects, we try to explain the ATF damping ring experimental results [6] with the following machine parameters: $E_0 = 1.3$ GeV, $< \beta_x >= 4.2$ m, $< \beta_y >= 4.6$ m, $\tau_x = 18.2$ ms, $\tau_y = 29.2$ ms, $\epsilon_{x0} = 1.1\times10^{-9}$ mrad, $\epsilon_{y0} = 5.8\times10^{-11}$ mrad, and the information about the bunch lengthening with respect to N_e can be obtained either from experimental results [11][12] or from analytical results [9]. Assuming $k_{\perp,x}(\sigma_{z0}) = k_{\perp,y}(\sigma_{z0})$=1020 V/pC/m, for $\sigma_X = 0.42$ mm and $\sigma_Y = 0.163$ mm, by using eqs. 20 and 21 one fits the experimentally measured emittance grow ups vs the bunch population as illustrated in Figs. 1 and 2, where the experimental results correspond to the values denoted in ref. [6] as “Wire scanner 2001/2/8”. It is seen clearly that both the horizontal and vertical emittances’ functional dependences on the bunch population fit well with the experimental results. We stress that $\sigma^2_{X,Y} = \sigma^2_{x,y,chamber} + \sigma^2_{x,y,co}$, where $\sigma_{x,y,chamber}$ are the vacuum chamber misalignment errors and $\sigma_{x,y,co}$ are the closed orbit distortion errors. It is obvious that to avoid excessive emittance grow ups, both the closed orbit distortions and the vacuum chamber misalignment errors should be under careful controlls with the same rigour.

To check further the validity of this theory one has to do more experiments by varying $\sigma_{X,Y}$ and to have more accurate values for $k_{\perp,x}(\sigma_{z0})$ and $k_{\perp,y}(\sigma_{z0})$.

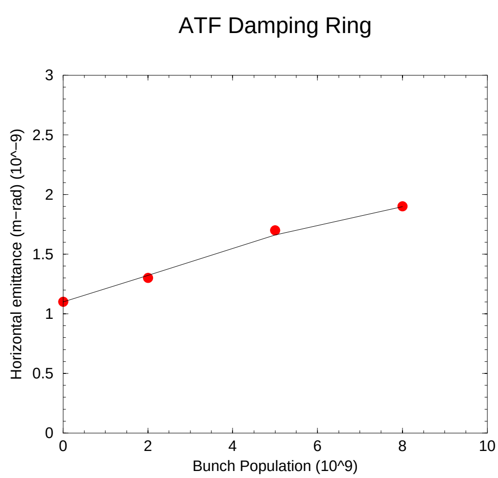

Figure 1: Horizontal emittance vs bunch population. The dots and solid line correspond to the experimental and theoretical values, respectively.

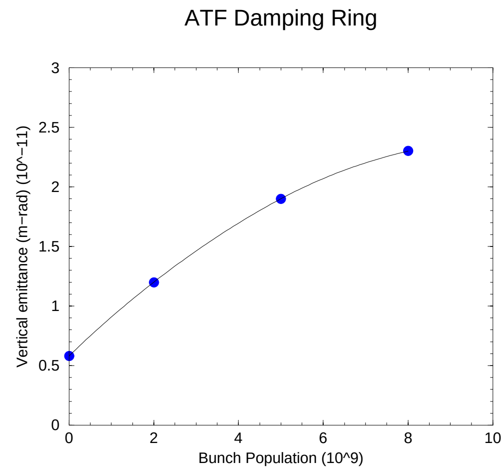

Figure 2: Vertical emittance vs bunch population. The dots and solid line correspond to the experimental and theoretical values, respectively.

CONCLUSION

In this paper we have established a theoretical framework to explain the bunch transverse emittance growth vs the bunch population in an electron storage ring. The new equilibrium emittance equations are given and applied to explain the experimental results from the ATF damping ring. More quantitative works need to be done in the future.

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