

FIRST EXPERIMENTAL TEST OF EMITTANCE MEASUREMENT USING THE QUADRUPOLE-MODE TRANSFER FUNCTION

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Abstract

We carried out the first experimental test of the emittance (thermal energy) measurement using the quadrupole-mode transfer function. We show that this method can be applied to measure the intrinsic thermal energy dynamical systems nondestructively. Since the QTF does not depend on the beam distribution, this method can be used to measure the rms emittance of different dynamical systems.

INTRODUCTION

Measurements of the thermal energy (or the rms emittance) are important in the study of the dynamical systems. Tools in the measurement of the rms beam emittance in high energy accelerators include the flying wire, ionization profile monitor, laser wire interference methods, synchrotron light monitor with CCD camera, etc. Most of these methods can cause beam dilution.

Recently, the quadrupole-mode transfer function has been proposed to measure the rms beam emittance of beams in storage rings [1]. The idea is to modulate the beam with an rf quadrupole field and detect the beam quadrupole-mode response function. From the beam quadrupole-mode transfer function, one can determine the rms emittance, betatron tune, or provide mismatch correction to the injected beams.

As the rf quadrupole field is adiabatically excited, particle motion is deformed according to the Hamiltonian contour without changing the phase space area (Liouville theorem). Since the Hamiltonian contour executes coherent motion under external harmonic modulation, the rms beam emittance can be more accurately measured by fast Fourier transform (FFT) method. Hardware for the measurement of QTF are the driver with rf quadrupole electromagnetic field, e.g. ferrite quadrupole magnet [2] or strip-line electrodes [3], and the quadrupole-mode monitors [4]. Even though the signal of the quadrupole moment is weak, the amplitude of the coherent beam shape oscillations can be enhanced by FFT analysis or a spectrum analyzer.

The aim of this quadrupole mode transfer function is intended to measure the transverse phase space area. However, the hardware for the transverse phase space measurement is not currently available in all accelerator laboratories. While the development of this measurement is going on, we study the feasibility of the quadrupole-mode transfer function measurement in the longitudinal phase space.

In fact, the longitudinal rms phase space area can be eas-

ily obtained from measuring the rms bunch length with a wall-gap monitor and then employing the well-known dynamics of the synchrotron motion to derive its conjugate phase space variable [5]. However, we will use the quadrupole-mode transfer function to measure the longitudinal rms phase space area in order to experimentally test the applicability of the method in the transverse phase space. This paper discusses the first experimental test of the longitudinal quadrupole transfer function for the measurement of the longitudinal rms emittance. Our experiment was carried out at the Taiwan Light Source, located in Hsinchu, Taiwan.

THE LONGITUDINAL QUADRUPOLE MODE TRANSFER FUNCTION

When the rf cavity voltage with angular frequency $h\omega_0$ is applied to charged particles in the accelerator, where h is the harmonic number and ω_0 is the angular revolution frequency of a synchronous particle, beam particles will execute stable synchrotron motion around the synchronous particle. Let ϕ and $\delta = \Delta p/p_0$ be the rf phase and the fractional off-momentum coordinates of a non-synchronous particle, and ϕ_s be the rf phase angle of a synchronous particle. The equations of motion are

$$\dot{\phi} = h\omega_0\eta\delta, \quad (1)$$

$$\dot{\delta} = \frac{\omega_0}{2\pi\beta^2 E} eV(\sin\phi - \sin\phi_s), \quad (2)$$

where the overdot is the derivative with respect to time t , η is the phase slip factor, V is the rf cavity voltage, β is the Lorentz velocity factor, and E is the beam energy. The small amplitude synchrotron tune is $Q_s = \sqrt{h|\eta \cos\phi_s| eV/2\pi\beta^2 E}$.

To excite the longitudinal quadrupole mode, the rf cavity voltage is sinusoidally modulated around twice the synchrotron frequency with modulation amplitude ratio b , i.e. $V = V_0(1 + b \sin\omega_m t)$ with $\omega_m \approx 2\omega_s$. The longitudinal quadrupole mode transfer function can be measured by using a spectrum analyzer with signal from a wall-gap monitor (or a beam position monitor).

In the presence of harmonic modulation near twice the synchrotron frequency, the synchrotron motion is coherently excited. In terms of the conjugate action-angle variables (J, ψ) , defined by

$$\phi = \phi - \phi_s = -\sqrt{2J} |\cos\phi_s|^{-1/4} \sin\psi, \quad (3)$$

$$\mathcal{P} = -\frac{h|\eta|}{\nu_s}\delta = \sqrt{2J}|\cos\phi_s|^{1/4}\cos\psi, \quad (4)$$

with $\nu_s = \sqrt{h|\eta|eV/2\pi\beta^2E}$ as the small amplitude synchrotron tune at $\phi_s = \pi$, the Hamiltonian can be approximated by [6]

$$H = \Delta J - \frac{1}{2}\alpha J^2 + GJ\cos(2\psi). \quad (5)$$

Here the resonance proximity parameter Δ , nonlinear detuning parameter α , and the resonance strength G are

$$\Delta = \left(\nu_s|\cos\phi_s|^{1/2} - \frac{\nu_m}{2}\right), \quad (6)$$

$$\alpha = \frac{\nu_s}{8}\left(1 + \frac{5}{3}\tan^2\phi_s\right), \quad (7)$$

$$G = \frac{1}{4}b\nu_s|\cos\phi_s|^{1/2}\left(1 + \frac{1}{3}\tan^2\phi_s\right), \quad (8)$$

where $\nu_m = \omega_m/\omega_0$ is the modulation tune. The Hamiltonian Eq. (5) closely resembles the quadrupole-mode Hamiltonian of the betatron phase space.

The parameters of the TLS are circumference $C = 120$ m; dipole bending radius $\rho = 3.495$ m; harmonic number $h = 200$; betatron and synchrotron tunes: $\nu_x = 7.22$, $\nu_z = 4.18$, $\nu_s = 0.010156$; phase slip factor $\eta_c \approx 6.1 \times 10^{-3}$; the cavity voltage $V_0 = 800$ kV, and $\phi_s \approx 166^\circ$. Electrons in synchrotrons radiate electromagnetic fields. An equilibrium beam emittance is attained from the effects of quantum fluctuation and synchrotron radiation damping. The damping time was $\tau_E = 3.1$ ms for the longitudinal phase space. The equilibrium distribution function obeying the Fokker-Planck equation is [7]

$$\rho(J, \psi) = \mathcal{N}e^{H(J, \psi)/E_{\text{th}}}, \quad (9)$$

where the normalization and the thermal energy \mathcal{N} and E_{th} are determined by

$$\int \rho(J, \psi) dJ d\psi = 1, \quad (10)$$

$$\sqrt{\langle\varphi^2\rangle\langle\mathcal{P}^2\rangle - \langle\varphi\mathcal{P}\rangle^2} = \epsilon_{\parallel}. \quad (11)$$

Here, ϵ_{\parallel} is the longitudinal rms emittance: [5]

$$\epsilon_{\parallel} = \left(\frac{h\eta}{\nu_s}\right)^2 |\cos\phi_s|^{-1/2} C_q \frac{\gamma^2}{J_E \rho}, \quad (12)$$

where $C_q = 3.84 \times 10^{-13}$ m, $J_E \approx 2$ is the damping partition number. The theoretical longitudinal rms emittance of the TLS is expected to be $\epsilon_{\parallel} \approx 8.243 \times 10^{-3}$, which will increase slightly resulting from synchrotron radiation in undulators and wigglers.

Now, we consider the coherent synchrotron mode of the beam. When a charged particle passes a wall-gap monitor, the image current can be expressed as

$$I_e(t) = e \sum_{\ell=-\infty}^{\infty} \delta(t - \hat{\tau} \cos(\omega_s t + \psi) - \ell T_0)$$

$$= \frac{e}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} j^{-m} J_m(n\omega_0 \hat{\tau}) e^{j[(n\omega_0 + m\omega_s)t + m\psi]}, \quad (13)$$

where e is the charge, $\hat{\tau}$ and ψ are the synchrotron amplitude and phase of the particle, T_0 is the revolution period, and J_m is the Bessel function of order m . The resulting spectra of the particle motion are classified into synchrotron modes, i.e. there are synchrotron sidebands around each orbital harmonic n . The amplitude of the m -th synchrotron sideband is proportional to the Bessel function J_m .

A bunch is made of particles with different synchrotron amplitudes and phases, the coherent synchrotron modes of the bunch can be obtained by averaging the synchrotron mode over the bunch distribution, i.e.

$$\begin{aligned} I(t) &= N_B \int I_e(t) \rho(\hat{\tau}, \psi) \hat{\tau} d\hat{\tau} d\psi \\ &= \sum_{n,m} A_{n,m} e^{i(n\omega_0 + m\omega_s)t}. \end{aligned} \quad (14)$$

The amplitude $A_{n,m}$ is called the coherent beam mode of the n -th revolution harmonic and m -th synchrotron sideband. When the beam distribution is only a function of $\hat{\tau}$, there exists only the revolution harmonics. Measuring all revolution harmonics, one can determine the beam distribution via inverse Hankel transformation [5]. Unfortunately, the detection system and the spectrum analyzer are limited by the bandwidth and intrinsic noises, it is difficult to carry out this type of analysis. Here, we demonstrate the method of QTF for the measurement of the rms beam emittance.

When the beam distribution is given by Eq. (9) with Hamiltonian given by Eq. (5), the amplitude of even order synchrotron sideband $m = 2\ell$ is

$$\begin{aligned} A_{n,2\ell} &= \frac{2\pi N_B \mathcal{N} e}{T_0} (-1)^\ell \int J_{2\ell}\left(\frac{n}{h} |\cos\phi_s|^{-1/4} \sqrt{2J}\right) \\ &I_\ell\left(\frac{G}{E_{\text{th}}} J\right) \exp\left\{\frac{\Delta J - \frac{1}{2}\alpha J^2}{E_{\text{th}}}\right\} dJ, \end{aligned} \quad (15)$$

where I_ℓ is the modified Bessel function of order ℓ . All odd order synchrotron sidebands vanish for the coherent quadrupole mode excitation.

Figure 1 shows a beam spectrum with center frequency at $n = h$ with a span of 130 kHz, a resolution bandwidth of 300 Hz, video bandwidth of 300 Hz, and a modulation frequency of $f_m = \omega_m/2\pi = 54$ kHz with the voltage modulation amplitude ratio at $b = 2.6\%$. Note that the spectrum shows the main rf peak of the revolution harmonic, and its coherent mode power at ± 54 kHz sidebands. In fact, the high intensity beam bunches in TLS suffer collective coupled bunch instabilities induced by the parasitic high order modes in the rf cavities. The resulting beam spectrum shows f_s , $2f_s$ and other noise harmonics. Choosing the rf voltage modulation frequency properly can effectively suppress the coupled bunch instability [6]. In this experiment,

we measure only the power spectra of $P_{h,0}$ and $P_{h,\pm 2}$ at $\omega = h\omega_0$ and $\omega = h\omega_0 \pm \omega_m$ respectively. Thus we can simultaneously measure the powers of the revolution harmonic $P_{h,0}$ and the quadrupole mode synchrotron sideband $P_{h,\pm 2}$ as a function of the modulation frequency ω_m . The quadrupole-mode transfer function (QTF), defined by

$$q_2(n, \omega_m, b) = P_{n,\pm 2}/P_{n,0}, \quad (16)$$

should be independent of the beam intensity.

At the maximum voltage modulation amplitude ratio of about 2.6%, the two-beamlet bifurcation of the synchrotron bucket occurs at about 51.5 kHz [6]. To avoid complication of hysteresis in our measurement resulting from nonlinear resonances [5], we vary the modulation frequency from 60 kHz downward till 50 kHz. The dependence of q_2 on the modulation frequency ω_m and modulation amplitude ratio b can be used to derive the rms emittance of the beam distribution.

Theoretically, the QTF is given by $q_2(n, \omega_m, b) = |A_{n,\pm 2}/A_{n,0}|^2$. Figure 2 shows a one-parameter fit to the experimentally data, where the modulation amplitude ratio varies from 0.26% to 2.6%. The solid lines show the fit with a longitudinal emittance of $\epsilon_{\parallel} = 0.18 \pm 0.01$ using the thermal distribution of Eq. (9). The fit is much larger than the theoretical rms emittance of $\epsilon_{\parallel 0} \approx 0.008$.

CONCLUSION

In conclusion, we have carried out experiments to test the applicability of using QTF to determine the longitudinal rms beam emittance. The derived rms beam emittance is much higher than the theoretical value. The possible reason for the enlargement may cause by the coupled bunch instability as shown in Figure 1. Figure 1 shows that there are synchrotron sidebands, i.e. the beam has been excited by parasitic modes. A new calculation including rigid dipole mode oscillations would be necessary to resolve the discrepancy between the derived emittance and the actual beam emittance. A single bunch experiment that can avoid the couple bunch mode excitation of the unwanted synchrotron sidebands would be very useful.

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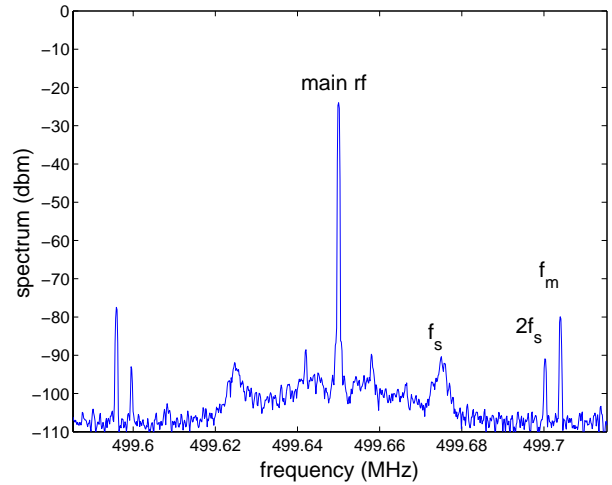


Figure 1: The beam spectrum obtained from a spectrum analyzer at the center frequency $hf_0 = 499.6649062$ MHz, span 130 kHz, resolution bandwidth 300 Hz, video bandwidth 300 Hz. The beam current is about 185 mA at the Taiwan Light Source. The rf voltage modulation frequency is $f_m = \omega_m/2\pi = 54$ kHz with modulation amplitude ratio $b = 2.6\%$. Besides the coherent mode frequencies at hf_0 and $hf_0 \pm f_m$, we also observe noise spectra at $2f_s$, $1f_s$, etc, arising from the parasitic modes in the rf cavities.

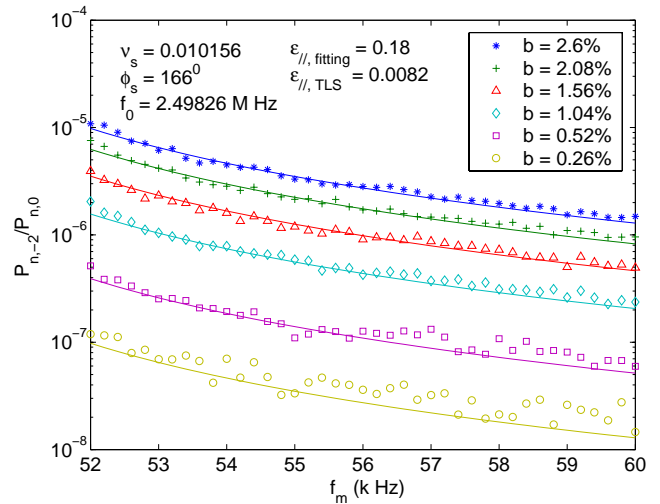


Figure 2: The QTF, $P_{n,-2}/P_{n,0}$, is shown as a function of modulation frequency f_m for different modulation amplitude ratio b from 2.6% to 0.26%. The symbols present the measured data and the solid lines are the fitting result of longitudinal rms emittance 0.18.

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