FLAT BEAM PRODUCTION IN LOW ENERGY INJECTORS

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Abstract

A source of ultra-fast synchrotron radiation pulses based on a recirculating superconducting linac is proposed at LBNL[1]. A flat beam will be produced in the low energy phase. High-brightness photocathode rf gun will produce electron beams in a solenoidal magnetic field. The electron beam will be transformed into flat beam with a large x/y emittance ratio by a skew-quadrupole-sequence adaptor[2]. A theoretical model is shown and simulations have been done with PARMELA. Space charge effect and possible solenoid setup are reported.

FLAT BEAM PRODUCTION

Beam Dynamics in Magnetic Field

For the production of flat beam, the cathode is immersed in solenoid magnetic field, hence, the beam is born with finite canonical angular momentum. Considering only the linear term of z component of solenoid field, B_z, we have equations of motion like:

$$x'' - Sy' - \frac{1}{2}S'y = 0 \tag{1}$$

$$y'' + Sx' + \frac{1}{2}S'x = 0 \tag{2}$$

where $S = \frac{eB_z}{P_z}$, P_z is the Z component of the particle momentum, prime indicates the derivative with respect to Z. Equations (1), (2) can be combined into a single complex equation:

$$\xi'' + iS\xi + \frac{iS'\xi}{2} = 0 \tag{3}$$

if we let:

$$\xi = x + iy$$

Equation (3) can be further simplified in Larmor frame

$$u'' - \phi'^2 u = 0 \tag{4}$$

by making a rotation transformation, letting:

$$u = \xi e^{i\phi}, \qquad \text{where} \quad \phi' = \frac{S}{2}$$

$\phi$is a non-periodic function of Z. There are two fundamental phase-amplitude form solutions exist for equation (4) [3]:

$$u = W(z)e^{i(\pm\psi(z))} \tag{5}$$

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If we subsitute this solutions into equation (4), we can get following relations:

$$\psi' = \frac{1}{W^2} \tag{6}$$

$$W'' + \phi'^2 W - \frac{1}{W^3} = 0 \tag{7}$$

For equation (4), the general solution can be written as:

$$u = AWe^{i(\psi+\delta_+)} + BWe^{i(-\psi+\delta_-)} \tag{8}$$

Where A, B, δ_+ and δ_- are constants decided by initial conditions. In phase space of u and u', we can get expressions for A as

$$\sqrt{2}Ae^{i(\psi+\delta_+ + \frac{\pi}{2})} = \frac{1}{\sqrt{2}}(u'W - uW' + \frac{iu}{W}) \tag{9}$$

Note, the modulus of the lhs of equation (9) is a constant, so will be the rhs. If we take the modulus of both sides and use the real and image part of u and u' to express the result, we will get:

$$\varepsilon_+ = \frac{1}{2}\varepsilon_r + \frac{1}{2}\varepsilon_i + L_u \tag{10}$$

where

$$\begin{aligned} \varepsilon_+ &= 2A^2 \\ \varepsilon_r &= \frac{u_r^2}{W^2} + (W'u_r - Wu_r')^2 \\ \varepsilon_i &= \frac{u_i^2}{W^2} + (W'u_i - Wu_i')^2 \\ L_u &= u_r u_i' - u_r' u_i \end{aligned}$$

similarly, from expression of B, we have

$$\varepsilon_- = \frac{1}{2}\varepsilon_r + \frac{1}{2}\varepsilon_i - L_u \tag{11}$$

where

$$\varepsilon_- = 2B^2$$

Here, we can introduce the Twiss parameter in Larmor frame,

$$\beta_l = W^2, \quad \alpha_l = -WW', \quad \gamma_l = \frac{1+\alpha_l^2}{\beta_l} \tag{12}$$

and get ε_r and ε_i in new expressions,

$$\varepsilon_r = \gamma_l u_r^2 + 2\alpha_l u_r u_r' + \beta_l u_r'^2 \tag{13}$$

$$\varepsilon_i = \gamma_l u_i^2 + 2\alpha_l u_i u_i' + \beta_l u_i'^2 \tag{14}$$

The real and image components of u and u' correspond to the movements in x and y dimensions in Larmor frame, respectively. When particles get out of the solenoid field region, the Larmor frame is same as lab frame, then ε_r becomes ε_x, and ε_i becomes ε_y. If we combine equation (10), (11), we can get following simple relations:

$$\varepsilon_+ + \varepsilon_- = \varepsilon_x + \varepsilon_y \quad (15)$$
$$\varepsilon_+ - \varepsilon_- = 2L_u \quad (16)$$

Note, ε_+ and ε_- are constants decided by initial conditions, no matter within or outside the solenoid field region. ε_x and ε_y are measured outside the solenoid field region, and they are regular emittances. Experimentally, we can get the value of ε_+ and ε_- by measuring the ε_x, ε_y and L_u.

Initial Conditions

So, What initial conditions decide ε_+ and ε_-? Let's simplify the problem by assuming uniform $\frac{B_Z}{P_Z}$ at the very first moment after electrons are emitted. Then we have:

$$\begin{aligned} \phi' &= constant \\ W &= constant \\ \psi &= \phi' Z \\ \alpha_l &= 0 \\ \beta_l &= \frac{1}{\psi'} = \frac{1}{\phi'} = \frac{2P_Z}{eB_Z} \end{aligned}$$

Then, in Lab frame, we have

$$\begin{aligned} x &= \sqrt{\frac{\varepsilon_+\beta_l}{2}} cos(\delta_+) + \sqrt{\frac{\varepsilon_-\beta_l}{2}} cos(-2\phi' Z + \delta_-) \\ x' &= \sqrt{\frac{2\varepsilon_-}{\beta_l}} sin(-2\phi' Z + \delta_-) \\ y &= \sqrt{\frac{\varepsilon_+\beta_l}{2}} sin(\delta_+) + \sqrt{\frac{\varepsilon_-\beta_l}{2}} sin(-2\phi' Z + \delta_-) \\ y' &= -\sqrt{\frac{2\varepsilon_-}{\beta_l}} cos(-2\phi' Z + \delta_-) \end{aligned}$$

In above equations, two terms on rhs involve ε_+, and these two terms describe the position of the center of the cyclotron motion. All other terms are ε_- related, they describe the cyclotron motion itself.

From above equations, we can get the expressions for ε_+ and ε_- in uniform $\frac{B_Z}{P_Z}$ region,

$$\varepsilon_+ = \frac{r_0^2 eB}{P_Z} \quad (17)$$
$$\varepsilon_- = \frac{P_Z v_\perp^2}{eB} \quad (18)$$

where $v_\perp^2 = x'^2 + y'^2$. r_0 is the distance from the center of the cyclotron motion to solenoid axis. Multiply both sides with P_Z, we get two normalized quantities,

$$\varepsilon_+ = r_0^2 eB \quad (19)$$
$$\varepsilon_- = \frac{P_\perp^2}{eB} \quad (20)$$

where $P_\perp$ is the transverse momentum. Hence, acceleration along solenoid axis is included. From here, we know that initial transverse distribution combined with B_z decide ε_+, and initial transverse momentum combined with B_z decide ε_-.

The Transformation

Since after particles leave the solenoid field region, ϕ won't change any more, we can combine it with another constant phase $\delta_\pm$:

$$\begin{aligned} \chi_+ &= \delta_+ - \phi \\ \chi_- &= \delta_- - \phi \end{aligned}$$

And in Lab frame, we can get the matrix form of solutions of equation (4)

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = V \begin{pmatrix} \sqrt{\varepsilon_+} cos\chi_+ \\ \sqrt{\varepsilon_+} sin\chi_+ \\ \sqrt{\varepsilon_-} cos\chi_- \\ \sqrt{\varepsilon_-} sin\chi_- \end{pmatrix}, \quad \text{or:} \quad \vec{X} = V\vec{a} \quad (21)$$

$$V = \begin{pmatrix} \sqrt{\beta_l} c & -\sqrt{\beta_l} s & \sqrt{\beta_l} c & \sqrt{\beta_l} s \\ \frac{-\alpha_l c - s}{\sqrt{\beta_l}} & \frac{\alpha_l s - c}{\sqrt{\beta_l}} & \frac{-\alpha_l c + s}{\sqrt{\beta_l}} & \frac{-\alpha_l s - c}{\sqrt{\beta_l}} \\ \sqrt{\beta_l} s & \sqrt{\beta_l} c & \sqrt{\beta_l} s & -\sqrt{\beta_l} c \\ -\frac{-\alpha_l s + c}{\sqrt{\beta_l}} & \frac{-\alpha_l c - s}{\sqrt{\beta_l}} & \frac{-\alpha_l s - c}{\sqrt{\beta_l}} & \frac{\alpha_l c + s}{\sqrt{\beta_l}}) \end{pmatrix} \quad (22)$$

where $c = cos\psi$, $s = sin\psi$. The transformation from round beam to flat beam is to block-diagonize matrix V by passing such a beam described by equation (21) through the adaptor. The quantites in $\vec{a}$ are constants during the transformation, and after transformation, ε_+ and ε_- are recognized as the emittances of x and y dimension, respectively. Because of the special initial conditions(the cathode is immersed in solenoid field), ε_+ is usually much larger than ε_-. Hence, we finally get a flat beam.

After the transformation, for the flat beam, we can use regular Twiss parameters to describe the beam. It can be derived that the Twiss parameters at the exit of the adaptor satisfy:

$$\beta_x = \beta_y \qquad \alpha_x = \alpha_y \quad (23)$$

and phase advance of the adaptor section satisfy:

$$\Delta\Phi_y = \Delta\Phi_x + \frac{\pi}{2} \quad (24)$$

Skew-quadrupole-triplet has been used as the adaptor in experiments to realize the transformation [4] for the capability of skew quadrupole to remove the canonical angular momentum. A0 group at FNAL has acheived flat beam production experimentally.

SIMULATIONS

We have done some simulations with PARMELA[5]. The space charge effect is not included in the above theoretical model, although it definitely contributes in experiments. We simulate A0 beamline at FNAL[4]. The bunch charge is about 1 nC, and the energy at the adaptor is about 15 MeV. In simulations, We let electron bunch travel until it arrive at the entrance of the adaptor. In this part 2D space charge effect is included in simulation. According to the particle distribution in 6D phase space, we can calculate $\varepsilon_{\pm}$ and search for the skew quadrupole gradients which can provide suitable transformation according to the above model. Then we set the skew quadrupole gradients and let the bunch resumne travlling. If we turn off space charge calculation, we do get a flat beam with $\varepsilon_{x,y}$ very close to $\varepsilon_{\pm}$, by "very close" I mean the descrepancy is less than 3%. But if we turn 3D space charge calculation, the final $\varepsilon_{x,y}$ of the flat beam is 20 to 30 percent higher, while we can fine-tune the skew quadrupole gradients to get the final $\varepsilon_{x,y}$ about 10% higher than $\varepsilon_{\pm}$.

FUTURE WORK

Simulations also show that, the transformation is very sensitive to the energy spread of the bunch. One possible way to reduce the energy spread is to reduce the longitudinal space charge force by increasing the bunch transverse size in strong space charge effect region. If we look at the solenoid field distribution of the typical solenoid setup for flat beam production, Fig. 1, we noticed that the solenoid provides a strong focusing at initial accelerating phase.

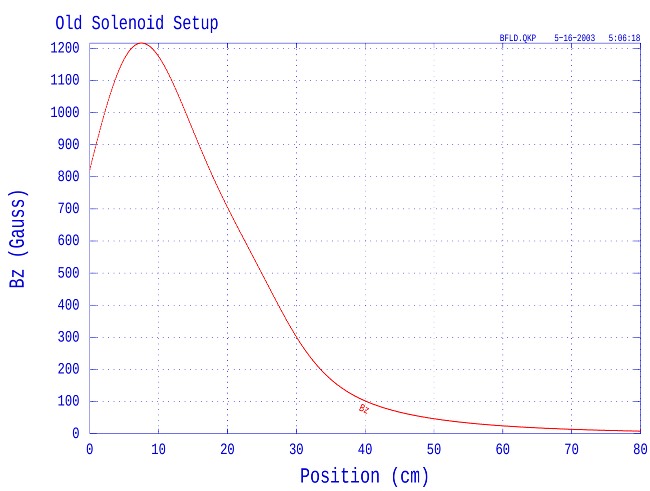

Figure 1: Bz from old solenoid setup

If we change the currents of the solenoid setup and get a new magnetic field distribution like in Fig. 2, the beam transverse size will be larger with weaker solenoid focusing. Simulations has shown obvious energy spread reduction for this new solenoid setup. Experiments need to be done to verify it. And possible side effect of the new solenoid setup need to be investigated.

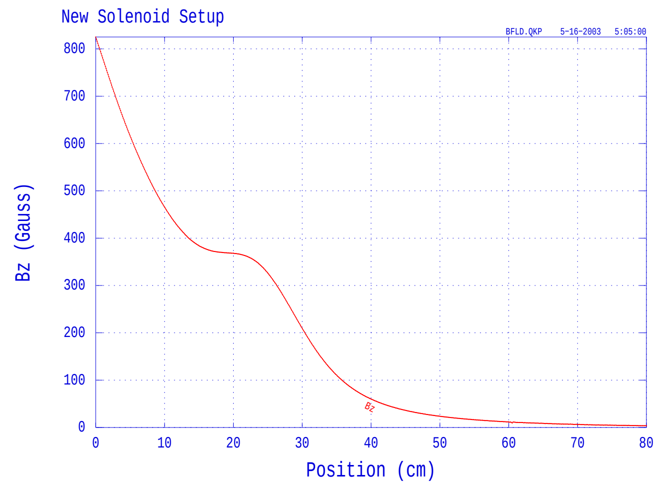

Figure 2: Bz from new solenoid setup

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