ANALYTICAL AND TIME-DOMAIN COMPUTATIONS OF SINGLE-BUNCH LOSS-FACTOR IN A PLANAR STRUCTURE

A. Smirnov, D. Yu, *DULY Research Inc., Rancho Palos Verdes, CA 90275*

Abstract

An analytical formulation is developed for a Gaussian bunch loss-factor in a periodic planar ("muffin-tin") structure. The short-range wakefield contribution is modeled with diffraction pattern expanded in a sum of quasi-eigenmodes of an equivalent open waveguide, which is excited by charge "image" fields induced on the iris edges. Comparisons with GdfidL time-domain multi-cell 3D simulations demonstrate a good accuracy of the model. Transverse wakes are computed numerically.

1 INTRODUCTION

Planar structures are of growing potential for application in linear colliders based on mm-wave structures [1]. Along with current projects based on state-of-the-art circular accelerating structures, planar accelerators may be the next evolutional step in the development of accelerator technology of multi-TeV linear colliders and compact accelerators. It would be based on modern microfabrication technologies [2] that have already been significantly advanced [3,4,5] towards mass-production of planar accelerating structures in the future.

Similar to conventional structures, short-range wakefields can be a potential source of instabilities, excessive bunch energy loss and phase-space distortions. However, unlike circular structures the short-range wakefields in planar structures are not yet studied comprehensively. Earlier we introduced extended models that allow describing the wakefields in the high-frequency domain [6]. The monopole short-range fields induced by a point charge in both periodic and single-cell, circular and rectangular structures were characterized analytically on the solid ground of Green-function and "image" field methods, diffraction and excitation theories of open cavities and waveguides [6,7,8]. Another semi-analytical approach employed in parallel [7] was based on matched field technique.

In this paper, wakefields induced by a Gaussian bunch are considered for a periodic planar structure (analytically) and corresponding for a multi-cell structure (numerically).

2 ANALYTICAL FORMULATION

Convenient form for practical calculation of bunch loss-factor per unit length k_{qL} in an arbitrary non-tapered slow-wave guide can be represented as follows:

$$k_{qL} = \sum_{s=0}^{s=s_1} \frac{\omega_s}{4} \frac{r_s(z,\vec{\rho}_\perp)\left|\tilde{\Phi}_s\right|^2}{Q_s\left|1 - v_{grs}/v\right|} + \frac{1}{\pi q^2} \int_{\omega_2}^{\omega_\infty\to\infty} \tilde{\eta}^2 r_\parallel(\omega,\vec{\rho}_\perp)d\omega, \quad (1)$$

where v is the bunch velocity, $\vec{\rho}_\perp$ is the bunch transverse displacement, ω_s are the modal synchronous frequencies at $v = v_{phs} = \omega_s/h'_s$, v_{grs} is the modal group velocity at this point of synchronism, $r_s(z,r_\perp) = E^0_{sz}(z,r_\perp)^2 / \left|dP_s/dz\right|$ is the longitudinal modal shunt impedance per unit length, ω_2 is the next modal frequency after ω_s at $s = s_1$, $\tilde{\eta} = \tilde{\eta}(\omega)$ is the Fourier-transform of the longitudinal space charge linear density $\eta(z)$, $\tilde{\Phi}_s = q^{-1}\int dz'\eta(z')\exp(-i\tilde{h}_s z')$ is the bunch modal formfactor, $\tilde{h}_s = h'_s - ih''_s/(v/v_{grs} - 1)$ is the complex dynamical wavenumber, $h''_s = \omega_s/2Q_s v_{grs}$ is the modal attenuation constant, and $r_\parallel(\omega,\vec{\rho}_\perp)$ is the real part of longitudinal impedance per unit length at higher frequencies $\omega \geq \omega_2$.

For a Gaussian bunch with rms length σ we have in (1):

$$\tilde{\Phi}_s = \exp\left(-(\tilde{h}_s\sigma)^2/2\right),\ \tilde{\eta}(\omega) = q\exp\left(-(\omega\sigma/v)^2/2\right). \quad (2)$$

The expression (1) consists of modal term, which describes usually low-frequency wakes having discrete spectrum, and short-range term written as integrated quasi-continuous wake impedance. The first (resonant) term now takes into account the group velocity effect correctly (see [9,10]). Note, HOM group velocity grows in (1) with frequency: $\max\left|v_{grs}\right| \xrightarrow[\omega_s\to\infty]{} c$, keeping $(r_s/Q_s)/\left|1 - v_{grs}/v\right|$ a finite value [11].

The smoothed wake resistance $r_\parallel$ is defined by spectral density of the point charge losses $dU/d\omega$. For the planar structure with period Λ it was found with systematic approach and diffraction model [7]:

$$r_\parallel(\omega,0) = \frac{\pi}{\Lambda q^2}\frac{dU}{d\omega} = \frac{Z_0 B}{4\pi^2\Lambda}\sum_{s=1,2}\alpha_s C_s, \quad (3)$$

where $1 << (kb)^2 << (\beta\gamma)^2$, $k = \omega/c$, $\beta = \sqrt{1-\gamma^{-2}}$, $Z_0 = 120\pi$ Ohms, $B = A + (1 + A^2)arctgA - \pi A^2/2$,

$$\alpha_{1,2} = \frac{2\pi^2}{\beta_\eta^2}\left\{\frac{m_{1,2}^2\left(M_x/\beta_\eta + 1\right)}{\left[\left(M_x/\beta_\eta+1\right)^2+1\right]^2} + \frac{n_{1,2}^2\left(M_y/\beta_\eta + 1\right)}{\left[\left(M_y/\beta_\eta+1\right)^2+1\right]^2}\right\},$$

$$\frac{C_1}{C_2} = \frac{-A + (1 - A^2)arctgA + \pi/2}{3A - (3 + A^2)arctgA^{-1} + \pi/2},\ C_1 + C_2 = 1,\ m_{1,2}=(2,1);$$

$n_{1,2}=(1,2)$; $\beta_\eta = 0.824$, $M_{x,y} = \sqrt{8\pi N_{x,y}}$, $N_{x,y} = \omega(d,b)^2/2\pi c(\Lambda - t)$ are the Fresnel numbers, t is the iris thickness, $2d$ is the horizontal dimension of the aperture (including side openings), $A=a/b$ is the aspect ratio, $2b$ is the vertical gap, and $2a$ is the horizontal dimension of each cavity. For high frequencies

$M_{x,y} >> \beta_\eta$ of short-range wakefields the wake resistance (3) varies as $\sim \omega^{-3/2}$. The practical option for the transition frequency ω_2 is based on the validity of Fresnel diffraction approximation ($b\omega_2 / c \geq 3$, see [8]). An additional criterion is smoothness of the function $k_{qL}(\sigma)$ (especially in the vicinity of the transition $\sigma \approx c/\omega_2$).

Note, cavity depth (i.e. maximum vertical dimension) is meaningful only for the trapped (discrete low-frequency) part of the full longitudinal coupling impedance (through the ω_s, r_s / Q_s and $v_{gr\,s}$), and does not have significant effect on the smoothed, short-range wake impedance (3) which is dominated by fields induced and diffracted only in the vicinity of the aperture [6]. A similar situation takes place with the dimension *d*.

3 NUMERICAL MODEL

The key problem in time-domain numerical simulation of quasi-periodic structures is the RAM (and CPU time) limitations. To make correct comparison between periodical and multi-cell models the minimum (or critical) number of cells to be included in simulations. It is defined by the dominant Fresnel number (see [12 and 13]):

$$N_{per} > N_{cr} = 2N_y . \qquad (4)$$

For a Gaussian bunch in the time domain this condition results in the same form as known for circular structures:

$$N_{cr} \approx b^2 \big/ \Lambda\sigma . \qquad (5)$$

Taking into account of Eq. (5), the total number of mesh elements scales as $\sim \sigma^{-4}$ as the bunch length decreases. An alternative numerical approach is eigenmode summation (see the first term in (1)) with modal characteristics calculated with 3D codes or matched field 3D models. However, to date there are still no eigenmode 3D finite-element solvers capable of computing the characteristics only for the *synchronous* modes (~ thousand of them for short bunches) using a single cell only, each mode having proper phase advance depending on its frequency. Note, such a code would have more affordable scaling factor $\sim \sigma^{-3}$ (similar to eigenmode problem for a single cell with periodical boundaries). The matched field model has, instead, a different problem: "missing zeros" and instability in solving the transcendent equation with matrix of big ranks [7] to find "proper" roots for HOMs in 3D structure. Nevertheless, the matched field model is very useful for the first few modes in (1) that make the computation of short-range wakefields very effective and completely analytical.

Time-domain simulations are performed here to verify analytical formulation (1-3) for a Gaussian, ultrarelativistic bunch of variable length using the GdfidL code with indirect algorithm [14]. The planar geometry considered here corresponds to the 37-cell 30 GHz 2π/3 section that was manufactured [5] and successfully tested [15]. A few cells of the model are depicted in Fig. 1. Parameters used in calculations are the following: *b*=1.8mm, Λ =3.332mm, *a*=3.344mm, *t*=0.7mm. We use here reduced side openings (horizontal dimension *d*=5.016mm) compared to the prototype [5,15] (*d*≈21mm) to reduce memory requirements. According to theory [7] it should not affect HOM losses significantly. Numerically we have only 0.08% difference in $\sigma = 62.5\mu m$ bunch loss-factor (and 0.04%, 0.09% for transverse factors) while *d* was reduced further by 27%.

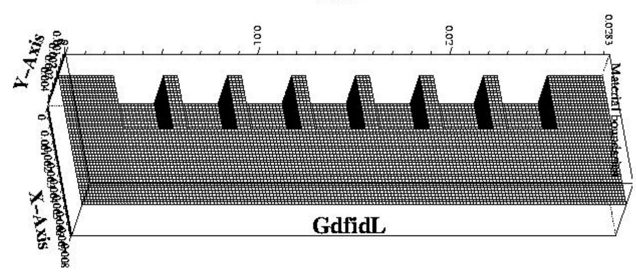

Figure 1: One-quarter of structure fragment model.

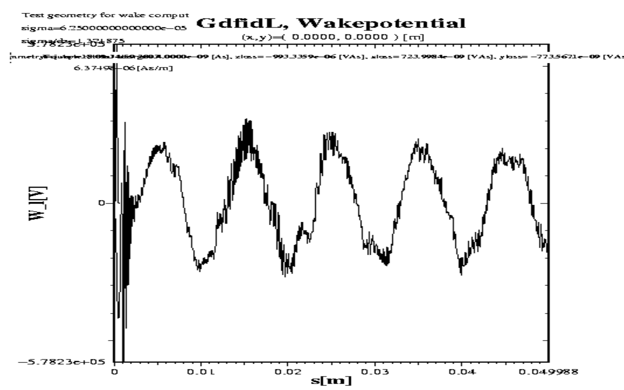

Figure 2: Longitudinal wake-function at $\sigma = 62.5\mu m$.

For long bunches ($\sigma > 200\mu m$), with a system RAM of <800MB it is possible to simulate the wakefields along the whole length (37cells). Unfortunately, limited memory implies reduced number of cells for shorter bunches. Figs. 2 and 3 demonstrate the wake potential plots in a 7-cell structure for a short bunch. The transverse wake-potentials are about the same for X and Y directions (1.15 and -1.17 V/nC·mm per cell for $\sigma = 62.5\mu m$), in agreement with the initial design and the "symmetric" transverse force concept [15]. Loss-factors calculated analytically and numerically are plotted versus bunch length in Fig. 4. The upper frequency in (1) is assumed $\omega_\infty \approx 3c/\sigma$, s_1=0, and the transition frequency chosen, ω_2=2π·42 GHz is one that immediately follows the fundamental one, ω_{s1}=2π·30 GHz. Direct integration is performed in (1) instead of analytical approximation with Gamma-function (see [16]), which introduces too high an inaccuracy in the model compared to the small difference between the original formulation (1) and time domain simulations (see Figs. 4). Growing discrepancy (Fig. 5) for bunch lengths <65μm indicates too few cells

($N_{cells} < N_{cr}$) as well as mesh lines per bunch length (σ / dz in Fig. 4).

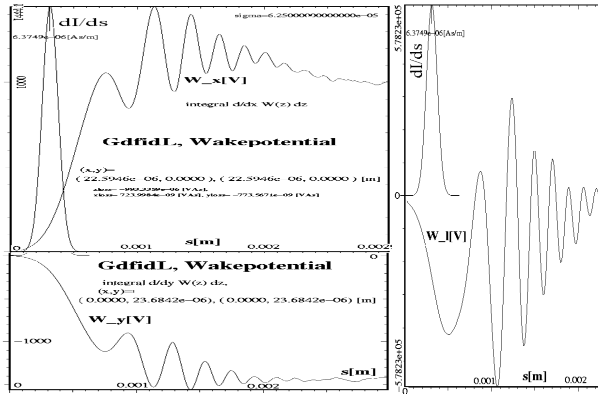

Figure 3: Short-range transverse (on the left) and longitudinal (on the right) wake-functions in 7-cell structure for $\sigma = 63\mu m$ and q=4nC.

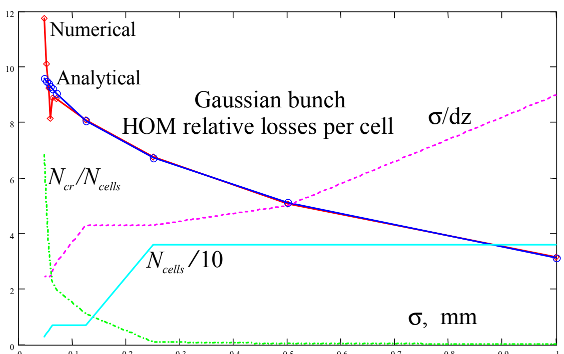

Figure 4: Monopole HOM energy losses related to fundamental mode loss per cell *vs* rms bunch length σ.

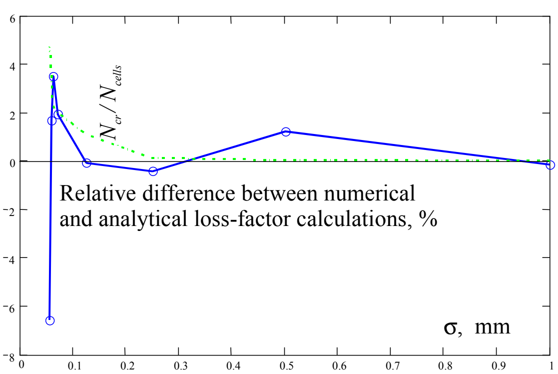

Figure 5: Relative difference [%] between numerical and analytical calculation of total energy losses per cell.

4 CONCLUSION

Comparison of analytical and time-domain modeling demonstrates very good agreement of bunch loss-factor calculations (~ a few percentages for ~200 fs bunch) and gives additional confirmation of the extended diffraction model.

Computations based on the approach of Eq. 1 can be completely analytical for conventional (e.g., cylindrical and planar) structures that makes it extremely fast and accurate. Practical benefits of this method can be extended to the asymmetric wakefields and some other types of structures.

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