SIMULATED GROWTH RATES FOR SINGLE-BUNCH INSTABILITIES DRIVEN BY A RESISTIVE IMPEDANCE

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Abstract

Vlasov simulations of instabilities driven by resistive impedance are used to determine growth rates of single-bunch instabilities. A method for measuring synchrotron tunes and growth rates from simulated synchrotron sidebands is described. Simulated growth rates are compared with Oide's calculation [K. Oide, Part. Accel. **51**, 43 (1995)].

INTRODUCTION

Vlasov-based and particle-tracking [1], and Perron-Frobenius [2] methods exist for the numerical calculation of single-bunch instabilities. These methods are capable of simulating the growth of modes arising from a few coupled synchrotron modes as well as instabilities in the microwave regime. Oide used a Vlasov- mode-based computer code to calculate growth rates for the first four synchrotron modes in the presence of a resistive impedance and in the absence of radiation damping [3]. He found that these rates are approximately proportional to the square of the beam current over a wide range of currents. This paper describes the time-domain simulation of growth of these instabilities and compares the simulated growth rates with Oide's calculations. These simulations are performed by integrating the Vlasov equation using the method of Warnock and Ellison [2] in a code written in Mathematica [4]. Nonlinear terms coming from potential-well distortion are simulated. Simulated quadrupole-mode growth rates agree very well with Oide's results while the dipole-mode growth rates are lower that Oide's results.

CALCULATION OF SIMULATED TUNES AND DAMPING RATES

A simulated bunch above threshold for instability that initially has the Haïssinski distribution will not, in principal, change with time. In computational practice, however, perturbations of the bunch distribution exist and these perturbations serve to seed the growth of unstable modes. In the model under consideration, the unstable modes Oide described [3] grow exponentially in time, each mode with its own growth rate. Each mode contributes a finite-width line to a synchrotron sideband reflecting the synchrotron tune of the mode and its growth rate.

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In this study, the tunes and damping rates of dipole and quadrupole modes are calculated from simulations in the linear regime by first Fourier transforming the simulated line density $\lambda(\phi, t)$ with respect to the phase coordinate ϕ at some revolution line n to obtain an approximation of a transformed pickup signal $\lambda_n(t)$.

$$\lambda_n(t) = \frac{1}{2\pi} \int d\phi \; \lambda(\phi; t) \; e^{-in\phi} \tag{1}$$

The expression is approximate to the degree that the line density does not to change significantly during the time the bunch traverses the pickup. This function is Fourier transformed with respect to t to obtain the spectral signal $\lambda_n(\omega)$, with ω the offset from the nth revolution line. It is assumed that that offset is small compared to ω_0. One then fits this function to resonances of multipole modes in the synchrotron sidebands. The resonances are each functions

$$\tilde{f}_j(\omega) = \left[1 - i\frac{\omega_j}{2\Gamma_j}\left(\frac{\omega}{\omega_j} - \frac{\omega_j}{\omega}\right)\right]^{-1}, \tag{2}$$

where ω_j and Γ_j are the resonant frequency and damping rate of the jth mode. These functions are the fourier transforms of the wake functions

$$f_j(t) = 2\Gamma_j e^{-\Gamma_j t}\left(\cos\overline{\omega}_j t - \frac{\Gamma_j}{\overline{\omega}_j}\sin\overline{\omega}_j t\right), \tag{3}$$

where $\overline{\omega_j} = \sqrt{\omega_j^2 - \Gamma_j^2}$. One then constructs a model $S(\omega)$ of the spectrum of the bunch as the superposition of N of these resonances and corrects for the finite duration of the simulations.

$$\begin{aligned} S(\omega) &= \sum_{j=1}^{N} a_j \tilde{f}_j(\omega)\left[1 - e^{i\omega t_{\max}}\left(\frac{f_j(t_{\max})}{2\Gamma_j}\right.\right. \\ &+ \left.\left. i\frac{\omega_j^2}{\omega\overline{\omega}_j} e^{-\Gamma_j t_{\max}} \sin\overline{\omega}_j t_{\max}\right)\right] \end{aligned} \tag{4}$$

In this expression, $t_{\max}$ is the duration of the simulation and the coefficients a_j are complex-valued weights. This differs significantly from a simple superposition of resonances (Eq. (2)) for a given mode if few e-folds growth of the mode are simulated, which is the case for the slow-growing dipole modes. The $4N$ real parameters embedded in Eq. (4) are all varied to fit the model to the simulated sidebands $\lambda_n(\omega)$ in a frequency range encompassing the synchrotron lines, i.e., the function

$$\int_{\min}^{\max} d\omega \, |\lambda_n(\omega) - S(\omega)|^2 \tag{5}$$

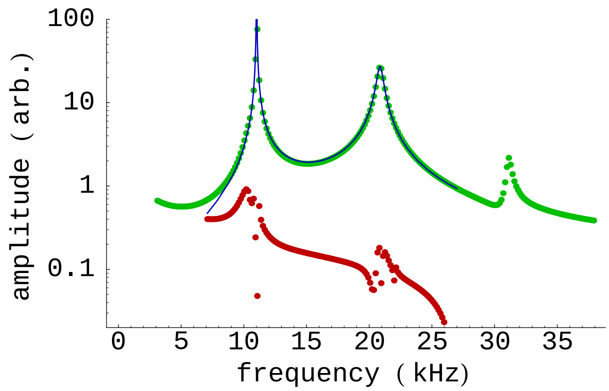

Figure 1: Example spectrum of a simulated bunch at 118 mA and corresponding fit as a function of frequency offset from a revolution line. The synchrotron frequency is 11 kHz and the revolution harmonic n is 55. The green points are of the simulated spectrum $\lambda(\omega)$, the blue trace is the fit function $S(\omega)$ covering the dipole and quadrupole lines, and red is the residual error. The sextupole line is also visible in the simulated spectrum.

is minimized. Figure 1 shows an example of a spectrum and its corresponding fit for $N = 2$.

SEEDING INSTABILITIES

A perturbation $\delta\Psi$ of the Haïssinski distribution $\Psi_{\text{Ha}}(\phi, p)$ [5], where ϕ and p are phase-space variables, serves to seed instability in the bunch. Using a seed ensures that mode signals are above the noise background in the synchrotron sidebands at the start of the simulation, a background that originates from an imperfectly calculated Haïssinski distribution. It was found that the quadrupole mode grows large more quickly than the dipole mode and nearly any seed results in strong quadrupole oscillations in the time there is significant growth of the dipole mode. So a perturbation that seeds the dipole mode preferentially was used. This allowed sufficient time that the growth rate of the dipole mode could be estimated before quadrupole-mode oscillations swamp the dipole mode.

The seed used has the form

$$\delta\Psi(\phi, p; t = 0) = \varsigma p \ \Psi_{\text{Ha}}(\phi, p), \tag{6}$$

where ς is the real constant chosen so that $\varsigma p_{\text{max}} \ll 1$, where p_{max} is the value of p at the edge of the grid (this ensures that $\delta\Psi$ is a small perturbation).

Oide and Warnock and Ellison discuss the 'trivial' dipole solution, which is the solution of the Vlasov equation where the Haïssinski solution translates in phase space in an orbit determined by the rf Hamiltonian. This translational solution exists only for hamiltonians harmonic in both the configuration and momentum variables. It exists because of the degeneracy of the frequencies (divided by the multipole orders) of the synchrotron modes and that the wake induced by the bunch has short range and tracks

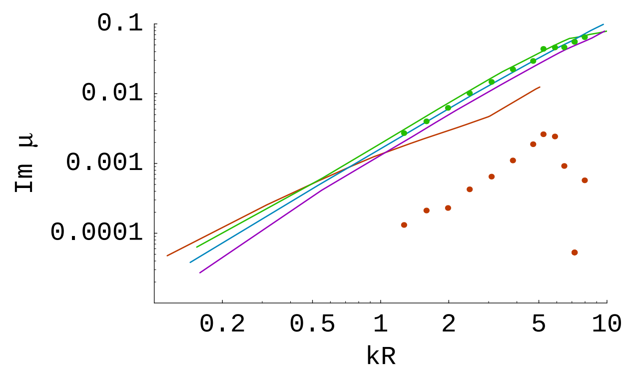

Figure 2: Simulated growth rates (dots) plotted on top of Oide's Fig. 2. Red represents dipole-mode growth rates while pale green represents quadrupole-mode growth rates. Sextupole and octupole modes are represented by aqua and violet traces without simulated data points.

the bunch [3, 2]. This translation is not a dipole mode in the sense of the dipole term of a multipole expansion of the distribution. So this solution is an inappropriate initial condition for seeding dipole oscillations: it results in undamped oscillations.

RESULTS

Oide expresses complex-valued synchrotron frequencies Ω normalized with respect to ω_s,

$$\mu = \Omega/\omega_s, \tag{7}$$

when referring to a coherent frequency of any synchrotron harmonic. The synchrotron tune is the real part of $\omega_s\mu$ and the damping rate Γ is

$$\Gamma = \omega_s \text{Im } \mu. \tag{8}$$

The beam intensity is represented by the dimensionless parameter product

$$kR \equiv eI_{\text{av}}R/\alpha\sigma_\epsilon^2 E_0, \tag{9}$$

where I_{av} is the average single-bunch beam current, α is the momentum compaction, σ_ϵ is the *fractional* natural energy spread of the ring, E_0 is the beam energy, and the ring impedance is the resistance R. While the product kR of Eq. (9) is the same as Oide's, there is a different convention for the dimension of R (a resistance) in Eq. (9) that changes the exact form of Eq. (9).

Simulations were performed at 12 beam currents with kR ranging from 1.27 to 7.95. Growth rates of the simulations are plotted together with Oide's results in Fig. 2. Figure 3 shows the corresponding synchrotron tunes.

There is good agreement between the quadrupole-mode growth rates predicted by Oide and my simulations. In these simulations, there are many e-folds growth of the mode and analysis of their synchrotron lines is very clean.

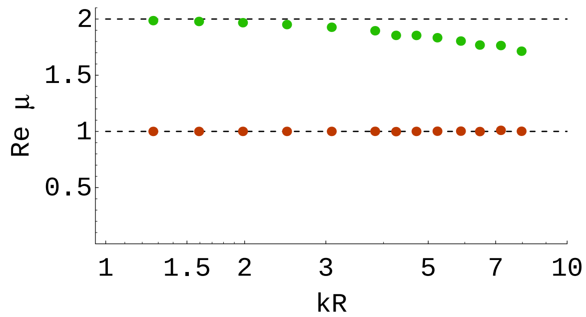

Figure 3: Simulated synchrotron tunes for the dipole (red) and quadrupole (green) modes.

A separate time-domain analysis of these growth rates gives the same results as the frequency-domain analysis described above.

Simulated dipole-mode growth rates are much lower than predicted by Oide, however. Calculations of this parameter using the method above were confirmed by inspection of time-domain plots of the intensity of dipole oscillations from a separate calculation based on the same simulations, which serves as a check of the method. Also by inspection of these plots, extraction of the three dipole-mode growth rates at the highest-current data points of Fig. 2 is compromised by the fast growth and intensity of the quadrupole-mode oscillations. The low growth rates of the dipole modes at these currents were not determined precisely due to the rapid growth of the quadrupole mode.

Errors in the simulation of growth rates result from, in part, the finite time step and the finite spacing of the grid in phase space. In most of these simulations, the time step is $\delta t = 2\pi/90\omega_s$ and the phase-space grid is 81×81. I then varied these parameters to check convergence. Figure 4 shows the dipole- and quadrupole-mode growth rates for a $kR = 5.87$ beam for varying number of grid points. The time step is 1.0 μs. Figure 5 shows the effect of varying the time step from 0.5 to 2.0 μs for 81×81 grids. Both plots show that the quadrupole-mode growth-rate predictions show little variation with these parameters except with the coarsest grids. So these calculations are robust. But the plot varying the time step shows that the dipole-mode growth-rate predictions have not seen enough exponential growth to accurately estimate the growth rate. At best, the calculations provide an upper limit that is a fraction of the quadrupole-mode rate.

There is still uncertainty regarding the appropriate seed for the dipole mode. If the 'trivial' dipole mode, as well as the 'true' dipole mode, is seeded, the growth rate appears artificially low until the 'true' dipole mode overpowers the 'trivial' one. Furthermore, one must see substantial exponential growth of the dipole mode before the quadrupole mode overpowers the dipole mode. A better estimate of the dipole coherent mode is necessary to better suppress the quadrupole mode and enhance the dipole mode. So the source of the discrepancy in the dipole-mode growth rates (compared with Oide's calculation) is not determined.

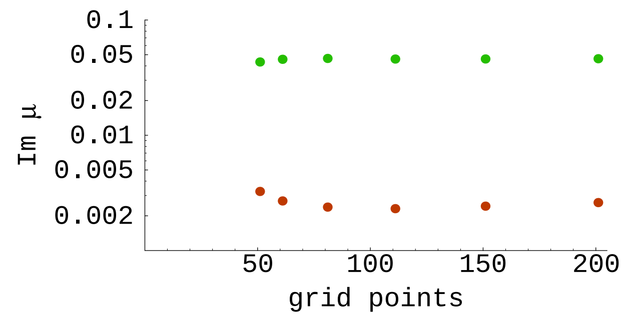

Figure 4: Variation of simulated dipole- and quadrupole-mode (red and green, respectively) growth rates with number of grid points. $kR = 5.87$ and the time step is 1.0 μs.

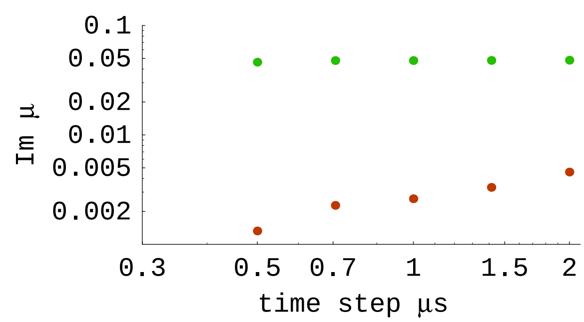

Figure 5: Variation of simulated dipole- and quadrupole-mode (red and green, respectively) growth rates with integration time step. $kR = 5.87$ and the number of grid points is 81×81.

CONCLUSION

Vlasov simulations of bunch instability driven by a resistive impedance were used to determine growth rates of the dipole and quadrupole modes. The method for determining these growth rates from simulated synchrotron sidebands was described. These simulations resulted in very good agreement with Oide's calculations [3] for the quadrupole mode but lower growth rates for the dipole mode.

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