# THE EFFECTS OF TEMPERATURE VARIATION ON ELECTRON BEAMS WITH RF VOLTAGE MODULATION

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#### Abstract

The correlation between the horizontal beam size vs. the cavity temperature, observed at the Taiwan Light Source (TLS), is explained by the combined effects of (1) cavity resonance frequency shift resulting from temperature change, (2) cavity voltage and synchronous phase angle change resulting from uncompensated beam loading in the low-level rf-feedback system, and (3) rf cavity voltage modulation for alleviating the coupled bunch instability. This experimental method can be used to evaluate the intrinsic resolution of the low-level rf-feedback system.

#### INTRODUCTION

At TLS, high brightness beam bunches in the storage ring have encountered longitudinal coupled-bunch instabilities (CBI), which have been suppressed by applying a sinusoidal rf voltage modulation [1]. The beam lifetime and beam stability have been substantially improved in routine operation. However, the beam bunches in the storage ring may become more sensitive to small perturbations to the rf cavities.

In a recent experiment at TLS, it was observed that a variation of 0.5 °C (1.5%) in the peak-to-peak rf cavity body temperature can induce 20 µm correlated peak-topeak variation in the horizontal beam size, measured by a synchrotron light monitor using a CCD camera from a dispersive location. This amounts to about 3% variation in the horizontal beam size with no associated vertical beam size variation. Thus, the horizontal beam size variation is related to the momentum spread of the beam. Since the single beam intensity is much less than the microwaveinstability threshold, the momentum spread should be independent of the rf cavity voltage. The observed horizontal beam size variation vs. rf cavity temperature is indeed puzzling because the rms momentum spread of electron beam depends only on the radiation damping and quantum fluctuation. This experiment sets a tight limit of the cavity cooling-water temperature control, which has attained the accuracy of  $\pm 0.1$  °C.

In this paper, we will analyze this phenomenon and show that it is related to the combined effects of cavity frequency shift, the change of effective accelerating voltage resulting from partially compensated low-level rffeedback system (LLRF), and the rf-voltage modulation used to combat the CBI. We will also demonstrate how to evaluate the intrinsic resolution of the low-level rffeedback system with this experimental method.

## **EXPERIMENTAL MEASUREMENTS**

A dedicated experiment was conducted to measure the sensitivity of electron beam size stability vs. the temperature of cavity cooling-water. The local feedback system of cavity cooling-water was adjusted so that a periodic variation of cavity temperature was possible. The horizontal beam size, measured by a synchrotron light monitor recorded by a CCD camera from a dispersive location, was observed to undergo periodic variation correlated with the cavity temperature as shown in Fig. 1. This phenomenon was observed while rf voltage modulation was applied to stabilize the CBI for electron beams. Table 1 lists relevant accelerator parameters for this experiment.

Table	1:	Operation	parameters	associated	with	the
archive	rchived data in Figure 1					

Parameter	Value	
rf frequency $f_{rf}$ (MHz)	499.648	
rf voltage $V_{\theta}$ (kV)	800	
modulation frequency $f_m$ (kHz)	49.663	
modulation amplitude $\varepsilon$	0.026	
synchrotron tune $v_s$	0.0100716	
synchronous phase $\phi_s$	169 <sup>°</sup>	
synchrotron frequency $f_s$ (kHz)	24.929	
momentum compaction $\alpha_c$	5.97×10 <sup>-3</sup>	
emittance (nm . rad)	19.5	
natural momentum spread $\sigma_{E}$	7.5×10 <sup>-4</sup>	



Figure 1: The temporal correlation between the horizontal beam size and the body temperature of rf cavities observed in the TLS storage ring. The data was recorded at a rate of 0.1 Hz.

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At TLS, the electron beam size can be measured by using a synchrotron light monitor with CCD camera, which is located at a beam line of the 3<sup>rd</sup> bending magnet of the three bend achromatic lattice where  $\beta_x = 0.5544$  m and the dispersion function is  $D_x = 0.15036$  m. At a dispersive location, the effective rms beam size is  $\sigma_{x0} = (\beta_x \varepsilon_x + D_x^2 \sigma_{\delta}^2)^{1/2}$ , where  $\varepsilon_x$  and  $\sigma_{\delta}$  are the rms emittance and fractional momentum-spread of the beam. Since  $\varepsilon_x$  and  $\sigma_{\delta}$  depend only on the synchrotron radiation damping and quantum fluctuation,  $\sigma_{x0}$  should be independent of the cavity temperature.

In the presence of rf voltage modulation at  $f_m \approx 2f_s$ , the invariant Hamiltonian tori coherently rotate at a frequency of  $f_m/2$ . The signal from the CCD camera of the synchrotron light monitor is a standard NTSC signal at 33 Hz. Let  $\hat{\delta}$  be the coherent bunch oscillation amplitude of the beam bunch. The effective beam size is

$$\sigma_x^2 = \sigma_{x0}^2 + \frac{1}{2}D_x\hat{\delta}^2 \tag{1}$$

The horizontal beam size under the normal operational conditions with voltage modulation was  $\sigma_x \approx 385 \ \mu\text{m}$ . For a fixed modulation frequency, the effective rms horizontal beam size will change if the synchrotron frequency is changed. We will review the effect of rf voltage modulation on particle motion and study mechanisms that can change  $\hat{\delta}$ .

## BEAM DYNAMICS WITH RF VOLTAGE MODULATION

When the rf voltage is modulated at a frequency near the second harmonic of the synchrotron frequency, the Hamiltonian tori may be divided into stable islands driven by parameteric resonances [2]. Those tori coherently rotate in the longitudinal phase space at exactly one half of the modulation frequency. Since the measured horizontal beam size is the time average of the quadrature of the betatron and momentum beam sizes, the increase in the energy spread of beam distribution may result in a larger measured horizontal beam size (see Fig. 2). At TLS, the rf cavity voltage is sinusoidally modulated with  $V_{rf} = V_0(1 + \varepsilon \sin(\omega_m t + \chi))$ , where  $\omega_m = 2\pi f_m$  is the modulation angular frequency,  $\varepsilon$  is the fractional rf voltage modulation amplitude, and  $\chi$  is an arbitrary phase factor. As the modulation frequency is near the 2<sup>nd</sup> order synchrotron sidebands, there exist solutions of stable fixed points (SFPs) [1] if  $v_m < v_{bif^+}$ 

$$J_{sfp} = \frac{24v_{s}|\cos\phi_{s}|^{1/2} - 12v_{m} + 2\varepsilon v_{s}|\cos\phi_{s}|^{1/2}(3 + \tan^{2}\phi_{s})}{v_{s}(3 + 5\tan^{2}\phi_{s})}$$
  
;  $J_{sfp} = 0$  if  $v_{m} < v_{bif} - or v_{m} > v_{bif} +$  (2)

where the bifurcation frequencies are

$$f_{bif\pm} = f_s \left[ 2 \pm \frac{\varepsilon}{2} (1 + \frac{1}{3} \tan^2 \phi_s) \right]$$
(3)

The peak fractional energy deviation  $\hat{\delta}$  of coherent

motion, associated with a Hamiltonian torus, is related to the action by



Figure 2: The images of synchrotron light monitor with (top) and without (bottom) rf voltage modulation.

Since the modulation frequency was in the region of  $v_{bif} < v_m < v_{bif^+}$  for the experimental data, the resulting bunch length in the longitudinal phase space would become larger [1]. The effective horizontal beam size measured at a location of nonzero dispersion function is given by Eq. (1). Using Eqs. (1), (2), and (4), we find the variation of horizontal beam size due to the change of energy spread as

$$\Delta \sigma_{\chi} = \frac{D_{\chi}^{2}}{4\sigma_{\chi}} \Delta(\hat{\delta}^{2})$$

$$= \frac{D_{\chi}^{2}}{2\sigma_{\chi}h^{2}\eta^{2}} \Delta(v_{s}^{2}|\cos\phi_{s}|^{1/2}J_{sfp})$$
(5)

Since the resonance frequency depends on the cavity size, the effective acceleration voltage may be changed through the cavity tuning angle through the partially compensated low-level rf-feedback system. The change of the horizontal beam size is

$$\Delta \sigma_X = \frac{D_X^2}{2\sigma_X h^2 \eta^2} (A - B \tan \phi_S) \frac{\Delta V_0}{V_0} \tag{6}$$

where

$$A = [24v_{s}^{2}|\cos\phi_{s}| - 6v_{m}v_{s}|\cos\phi_{s}|^{1/2} + 2\varepsilon v_{s}^{2}|\cos\phi_{s}|(3 + (7) + (7$$

## ANALYSIS OF BEAM SIZE VARIATION

The cavity detuning angle  $\psi$  is related to the resonance frequency  $\omega_r$  by

$$\psi = \tan^{-1} \left[ \frac{2Q_L(\omega - \omega_r)}{\omega_r} \right]$$
(10)

where  $Q_L$  is the loaded quality factor of cavities. We assume a uniform expansion/contraction of the cavity radius when the temperature was changed. With a small temperature variation  $\Delta T$ , the fractional deviation of resonance frequency for a cylindrical cavity is given by

$$\frac{\Delta\omega_r}{\omega_{r0}} = -\frac{\Delta b}{b} = -\alpha\Delta T \tag{11}$$

where  $\alpha$  is the coefficient of linear thermal expansion ( $\alpha$ =16.668×10<sup>-6</sup> per °C for copper). Considering a small variation of Eq. (10) with respect to the resonance frequency  $\omega_r$ , we arrive at

$$\Delta \psi \approx 2\alpha \ Q_L \cos^2 \psi \ \Delta T \tag{12}$$

For a steady state beam loading the expression of accelerating voltage in terms of complex phasor is [3]

$$\widetilde{V}_0 = \left[ I_g e^{j(\theta + \theta_g)} - I_i \right] R_{sh} \cos \psi \ e^{-j\psi} \tag{13}$$

where  $I_i$  is the rf beam image current,  $R_{sh}$  is the cavity shunt impedance. The fractional change of effective accelerating voltage due to the temperature change is given by

$$\frac{\Delta V_0}{V_0} = -2\alpha \, Q_L \sin \psi \cos \psi \, \Delta \psi \tag{14}$$

Using Eqs. (6) and (14), we find the relation between the change of horizontal beam size vs. the cavity temperature as

$$\Delta \sigma_x = -\frac{\alpha \, Q_L D_x^2 \sin \psi \cos \psi}{\sigma_x h^2 \eta^2} (A - B \tan \phi_S) \Delta T \qquad (15)$$

Here, the parameter *A* and *B* of Eqs. (7) and (8) are both positive, the cavity detuning angle  $\psi$  is also positive due to the Robinson stability, and  $tan\psi_s < 0$  for electron storage rings. Therefore, the deviation of horizontal beam size is of opposite sign to the temperature change. Based on Eq. (15), the calculated variation of horizontal beam size vs. cavity temperature variation is shown in Fig. 3(C), while the experimental data for temperature variation is reproduced in Fig. 3(A) and the measured rms horizontal beam size variation is shown in Fig. 3(B). Note that the calculated beam size variation is larger than the measured data, i.e. the effect of temperature variation has been partially compensated by the LLRF.

In fact, the LLRF is designed to compensate changes in cavity operational conditions so that the acceleration voltage would remain constant. The rf tuning angles are changed by the LLRF in response to temperature variation at 0.1 Hz. The top and middle plots of Fig. 4 show the data of cavity tuning angles recorded in response to the temperature variation for cavities 1 and 2 respectively. The required tuning angle derived from the observed data is shown in the bottom plot of Fig. 4. If the LLRF is perfect, we would not observe beam size variation.

According to the technical specification of TLS low level system [4], the accuracy of phase detector in the feedback system is  $\pm 1$  degree. One can use Eqs. (14) and (6) to estimate the uncompensated variation of horizontal beam size with rf voltage modulation. Using the worst estimate for the phase error of 2 degrees, we obtain 25  $\mu$ m peak-to-peak variation in the horizontal beam size. The peak-to-peak value of measured data is 20  $\mu$ m.



Figure 3: (A) the average temperature variation of rf cavities. (B) the measured variation of horizontal beam size. (C) the calculated variation of horizontal beam size arising from the change of rf resonance frequency. (D) the calculated variation of horizontal beam size due to the change of higher order parasitic loss factor.



Figure 4: Top and middle plots show the rf tuning angles for cavities 1 and 2 adjusted by the LLRF. The bottom plot shows the tuning angle derived from the observed horizontal data in Fig. 3(B).

Finally, we also have analysed the effect of higherorder parasitic-mode losses on the beam size variation and found its effect small. Because the beam size is very sensitive to small perturbations in the rf cavities in the presence of rf voltage modulations, this rf voltage modulation has applications in testing the intrinsic resolution of LLRF to various cavity perturbations.

#### REFERENCES

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