

## BEAM INSTABILITIES AT THE PLS STORAGE RING

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### Abstract

A simulation method to obtain beam tail distribution and beam lifetime in electron storage rings has been applied to the 2.5 GeV Pohang Light Source. The simulations were performed to obtain transverse and longitudinal tail distributions due to beam-residual gas bremsstrahlung, beam-residual gas scattering and intra-beam scattering at the Pohang Light Source. The beam lifetime that is obtained by the simulation is estimated to be around 19 hours in beam current of 180 mA. It is shown that the simulated beam lifetimes well agree with those of on normal operation.

### INTRODUCTION

The distribution of particles in a bunch can be divided into two regions: the core region at small amplitudes and the tail region at large amplitudes. The core distribution determines the brilliance in the synchrotron light source and the tail distribution may affect the beam lifetime. A simple and fast simulation method was proposed to obtain the beam tail due to rare random processes[1][2]. The simulation method investigates the beam tail distributions generated by rare and large-amplitude processes from the core distribution. In this paper, the same simulation method was applied to the 2.5 GeV Pohang Light Source (PLS). To see the effects of beam tails in the transverse and longitudinal distributions, we considered the cases of beam-residual gas scattering, intra-beam scattering and beam-residual gas bremsstrahlung. The beam lifetime at the PLS is also estimated in a simulation and the beam lifetimes obtained from the simulation show good agreements with those of on normal operation.

### DESCRIPTION OF THE SIMULATION METHOD

This section reviews the simulation method which follows from earlier work[1][2]. The simulation starts with  $n$  macroparticles that are given randomly with specified variances in six-dimensional phase space. Each macroparticle ( $i$ ) has a particle number ( $N_i$ ). Let  $p$  be the probability that an electron undergoes a random process during one turn. Once an electron in a macroparticle undergoes this process (the probability  $P$  is  $N_i p$ ), we create a new macroparticle ( $n+1$ )th. This new macroparticle has one particle ( $N_{i+1} = 1$ ) and the macroparticle which has undergone a random process now has a number of particles ( $N_i - 1$ ).

We assume that the variation in the random variable due to a random process is limited to a range between a minimum and a maximum value. To obtain the variation, first, calculate the probability ( $P$ ), and generate

one uniform random number ( $0 \leq x \leq 1$ ). If  $x < P$ , a random process occurs for the macroparticle. Second, generate a uniform random number ( $\theta_1$ ) in the interval between the minimum value ( $\theta_c$ ) and the maximum value ( $\theta_m$ ) and one uniform random number in the interval  $0 < y < (d\sigma(\theta)/d\theta)_{max}$ , and compare  $y$  and  $(d\sigma(\theta)/d\theta)_{\theta=\theta_1}$ . Here  $\theta$  is the scattering-angle random variable and  $(d\sigma(\theta)/d\theta)_{\theta=\theta'}$  is the cross section corresponding to  $\theta'$ . If  $y < (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$ , a random variation corresponding  $\theta_1$  is given to an electron. If  $y > (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$ , discard these  $\theta_1$  and  $y$ , and generate new  $\theta_1$  and  $y$  until the relation  $y < (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$  holds.

Each macroparticle in the simulation is tracked as follows:

1. *Input* We use the following normalized variables in tracking:

$$X = \frac{x}{\sigma_x^o}, P = \frac{\beta_x P_x}{\sigma_x^o}, Y = \frac{y}{\sigma_y^o} \quad (1)$$

$$Q = \frac{\beta_y P_y}{\sigma_y^o}, Z = \frac{z}{\sigma_z^o}, E = \frac{\epsilon'}{E_o \sigma_\epsilon^o} \quad (2)$$

Here, the  $\sigma_x^o$ ,  $\sigma_y^o$ ,  $\beta_x$  and  $\beta_y$  are nominal horizontal beam size, nominal vertical beam size, horizontal and vertical betatron functions, respectively.  $E_o$ ,  $\sigma_z^o$ ,  $\sigma_\epsilon^o$  and  $\epsilon'$  ( $= E - E_o$ ) are the nominal beam energy, nominal bunch length, relative energy spread and energy deviation due to a random process, respectively.

2. *Random process* When a transverse random process, such as beam-residual gas scattering, occurs, the momenta of a particle are varied by

$$P = P - \frac{\theta}{\sigma'_x}, Q = Q - \frac{\theta}{\sigma'_y}. \quad (3)$$

Here, the scattering angle ( $\theta$ ) is given by values between the minimum cutoff angle and the transverse aperture of the beam.  $\sigma'_x = \sigma_x^o/\beta_x$  and  $\sigma'_y = \sigma_y^o/\beta_y$ .

3. *Betatron oscillation*
4. *Synchrotron oscillation*
5. *Synchrotron radiation*

In the simulation program, it is assumed that new macroparticles do not undergo the random processes and are not tracked after they are produced by the random processes, although the original macroparticles are continuously tracked. CPU time at this simulation can be reduced by this method, and we can perform long-term runs.

## BEAM TAIL DISTRIBUTIONS BY A SIMULATION MODEL

It was shown that there is no great differences in their equilibrium states if we track over around 20000 macroparticles. We performed a weak-strong simulation with 30000 macroparticles in the phase spaces.

### Beam-Residual Gas Bremsstrahlung

The differential cross section for an energy loss due to bremsstrahlung between  $E$  and  $E + dE$  is given by[4]

$$d\sigma = 4\alpha r_e^2 Z(Z+1) \frac{du}{u} \frac{E'}{E_0} \left[ \left( \frac{E_0^2 + E'^2}{E_0 E'} - \frac{2}{3} \right) \log \frac{183}{Z^{1/3}} + \frac{1}{9} \right], \quad (4)$$

where  $Z$ ,  $\alpha$  and  $r_e$  denote the atomic number, the fine-structure constant and the classical electron radius, respectively. If we expand Eq.(5) by  $\frac{u}{E_0}$  and take first-order term, we obtain

$$d\sigma = 4\alpha r_e^2 Z(Z+1) \left( \frac{4}{3} \log \frac{183}{Z^{1/3}} + \frac{1}{9} \right) \frac{du}{u}. \quad (5)$$

We assume that one type of molecule uniformly exists in the ring.

### Beam-Residual Gas Scattering

The cross section of the elastic scattering with an atom is given by[4]

$$\frac{d\sigma}{d\Omega} = \left( \frac{2Zr_e}{\gamma} \right)^2 \frac{1}{(\theta^2 + \theta_{min}^2)^2}, \quad (6)$$

where  $\Omega$  is the solid angle,  $\theta$  the scattering angle,  $Z$  the atomic number,  $r_e$  the classical electron radius,  $\gamma$  the Lorentz factor and the screening of the atomic electrons is accounted by the angle  $\theta_{min}$ .

To obtain scattering angle, first, calculate the probability( $P$ ) that is scattered at higher angles  $\theta$  than minimum scattering angle  $\theta_a$ , and generate one uniform random number ( $0 \leq x \leq 1$ ) each turn to decide whether the scattering occurs or not. If  $x < P$ , the scattering angle is defined by

$$\theta = \theta_a / \sqrt{R}, \quad (7)$$

where  $R$  ( $0 < R < 1$ ) is the other uniform random number. On the other hand, beam-residual gas scattering causes the changes of momenta of a particle in the horizontal and the vertical directions. Then, third random number is used to define azimuthal angle  $\phi$  which is the angle between the horizontal and scattering planes. To obtain the changes of momenta due to the scattering in the normalized momenta, we have to multiply  $\theta_x = \theta \cos \phi$  and  $\theta_y = \theta \sin \phi$  with the value  $\beta_x/\sigma_x$ ,  $\beta_y/\sigma_y$ , respectively, taken at the position where the elastic scattering takes place. We use 4 m and 5.5 m as the average values of  $\beta_x$  and  $\beta_y$  in the ring, respectively.

### Intra-beam Scattering

The Touschek effect describes collision processes which lead to loss of both colliding particles. In reality, however, there are many other collisions with only small exchanges of momentum. Due to a scattering effect, two particles can transform their transverse momenta into longitudinal momenta. If the new longitudinal momenta of the two particles are outside the momentum acceptance, the particles will be lost. The differential cross section for Coulomb scattering of two particles with equal but opposite momenta in the non-relativistic approximation is given by the Möller formula[5]:

$$\frac{d\sigma}{d\Omega} = \frac{4r_o^2}{(v/c)^2} \left[ \frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta} \right], \quad (8)$$

where  $v$  is the relative velocity in the center of mass system and  $\theta$  is the scattering angle. Note that the scattering angle is measured in the laboratory system, while intra-beam scattering cross section is evaluated in the center of mass system. Accordingly, in order to obtain the scattering angle in the center of mass system, we have to perform a Lorentz transformation of the momentum and energy of a particle from laboratory system to the center of mass system. Then the new momenta are transformed back to the laboratory system.

Fig.1 shows the horizontal, vertical and longitudinal beam distributions due to beam-residual gas bremsstrahlung, beam-residual gas scattering and intra-beam scattering, respectively.

## BEAM LIFETIMES AS A FUNCTION OF APERTURES

We obtain the lifetime by using average  $\beta_{x,y}$  in stead of  $\beta(s)$  in the ring. The pressure-levels in total beam currents of 180 mA and 120 mA on normal operation at the PLS are  $0.6 \times 10^{-9}$  Torr and  $0.45 \times 10^{-9}$  Torr, respectively. The number of bunches on the operation is 400. Table 1 shows simulated beam lifetimes due to the bremsstrahlung as a function of the energy apertures. The lifetimes are obtained when the horizontal and the vertical apertures are set to  $100\sigma'_x$  and  $100\sigma'_y$ , respectively. Table 2 shows the simulated beam lifetimes that result from the beam-residual gas scattering and intra-beam scattering as a function of vertical aperture. The lifetimes are obtained when the horizontal and the energy apertures are set to  $100\sigma'_x$  and 1.5%, respectively.

### Comparison with operational beam lifetime

If we estimate the beam lifetime due to above considered three random processes, it gives the beam lifetime around 18.9 hours in the single bunch with beam current of 0.45 mA under of vacuum pressure of  $0.6 \times 10^{-9}$  Torr for the transverse apertures of  $100\sigma'_x$ ,  $100\sigma'_y$  and energy aperture of 1.5%. Fig.2 shows the beam lifetimes as a function of beam current on normal operation at the PLS which includes 400

bunches. It is shown that the beam lifetimes obtained from the simulation show good agreements with ones obtained on normal operation.

### DISCUSSION AND CONCLUSION

We have established the simulation method to obtain beam tail distributions due to the incoherent random processes. This simulation method provides a simple and fast means to obtain the tail distributions due to various random processes in the storage rings. Intra-beam scattering at the PLS more affects transverse tails than the beam-residual gas scattering. The tail distributions due to the beam-residual gas and intra-beam scatterings are obtained by considering relative large angle scatterings for the case of the PLS. These relative large angle scatterings only contribute a tail of large amplitude particles and do not affect the core of the beam distribution. Random processes influence the lifetimes as well as the tails of the beam distribution. The simulation study on the lifetime was performed as a function of the apertures. This simulation method for beam tails is also used to obtain the beam lifetime. This simulation showed good agreements with the operational beam lifetime.

### REFERENCES

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Table 1: Lifetimes due to the beam-residual gas bremsstrahlung

Energy aperture	1.2 %	1.5 %	1.8 %
Lifetime(h)	297.4	461	797.6

Table 2: Lifetimes due to the beam-residual gas scattering and intra-beam scattering.

<b>Vertical aperture</b>	<b>80<math>\sigma'_y</math></b>	<b>100<math>\sigma'_y</math></b>	<b>120<math>\sigma'_y</math></b>
Gas scattering(h)	319	487.4	731
Intra-beam scattering(h)	19.4	20	20.5

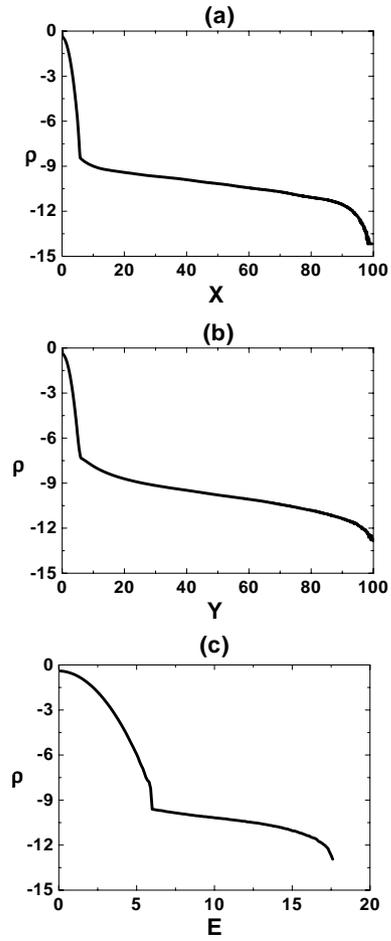


Figure 1: (a) Horizontal, (b) vertical and (c) longitudinal beam distributions due to the beam-residual gas bremsstrahlung, beam-residual gas scattering and intra-beam scattering after 560,000 turns. The horizontal axes are  $X$  and  $Y$ , the distance normalized by the nominal horizontal and vertical beam sizes and  $E$ , the energy deviation normalized by the relative energy spread. The vertical axes represent the distribution in  $X$ ,  $Y$  and  $E$  measured using the logarithmic scale.

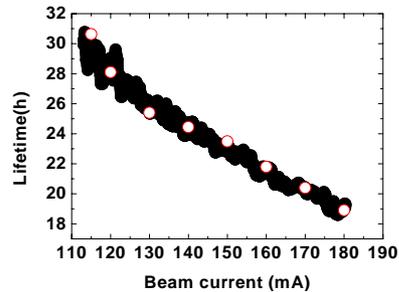


Figure 2: Circles show the simulated beam lifetimes that are obtained from the tracking of the beam-residual gas bremsstrahlung, beam-residual gas scattering and intra-beam scattering. Black line shows the beam lifetime verse beam current on normal operation.