

LONGITUDINAL MICROWAVE INSTABILITY IN LEPTON BUNCHES

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Abstract

The stability criterion for the longitudinal microwave instability in bunched lepton beams is derived using the mode-coupling formalism and taking into account the potential-well distortion. The new formula yields an intensity threshold which can be higher than the one given by the Keil-Schnell-Boussard approximation by a large factor. This result may explain why the classical instability threshold has been exceeded in some lepton machines.

1 INTRODUCTION

The longitudinal microwave instability for coasting beams is well understood [1-4]. It leads to a stability diagram, which is a graphical representation of the solution of the dispersion relation depicting curves of constant growth rates, and especially a threshold contour in the complex plane of the driving impedance. When the real part of the driving impedance is much greater than the modulus of the imaginary part, a simple approximation, known as the Keil-Schnell (or circle) stability criterion, may be used to estimate the threshold curve [3]. For bunched beams, it has been proposed by Boussard [5] to use the coasting-beam formalism with local values of bunch current and momentum spread. This approximation was expected to be valid in the case of instability rise-times shorter than the synchrotron period, and wavelengths of the driving wake field much shorter than the bunch length. This empirical rule is widely used for estimations of the tolerable impedances in the design of new accelerators. A first approach to explain this instability, without coasting-beam approximations, has been suggested by Sacherer through Longitudinal Mode-Coupling (LMC) [6]. The equivalence between LMC and microwave instabilities has been pointed out by Sacherer [6] and Laclare [7] in the case of broad-band driving resonator impedances, neglecting the Potential-Well Distortion (PWD). The complete theory describing the microwave instability for bunched beams is still under development [4,8].

It has been shown in Refs. [9,10], using the mode-coupling formalism for the case of proton bunch with a parabolic line density interacting with a broad-band resonator impedance, that a new stability criterion can be derived taking into account the PWD due to both space-

charge and resonator impedances. This new formula reveals in particular that it is better to operate the machine below transition (as already found in Ref. [11]). It also predicts a stability area below transition even in the presence of large space-charge impedances, without coasting-beam considerations of stability diagrams.

The case of a lepton bunch with a Gaussian amplitude density is discussed in this paper. Space charge is negligible in this case and the machine is operating above transition.

2 THEORY

Applying Sacherer's formula for LMC between modes m and $m+1$ yields the following intensity threshold condition [9]

$$I_b \leq \frac{3}{2} \times \frac{h \hat{V}_T B^3 |\cos \phi_s|}{F_1 \left| \frac{Z_l^{BB}}{p} \right|}, \quad (1)$$

with

$$F_1 = \frac{|m|}{|m|+1} \times \left| \frac{Z_l^{BB}}{p} \right|^{-1} \times \left\{ \left| j (Z_l^{eff})_{m,m+1} \times \frac{|m|+1}{\sqrt{|m|(|m|+2)}} + \frac{j}{2} \times \frac{|\cos \phi_s|}{\cos \phi_s} \right| \times \left[(Z_l^{eff})_{m,m} - \frac{(|m|+1)^2}{|m|(|m|+2)} \times (Z_l^{eff})_{m+1,m+1} \right] \right\}, \quad (2)$$

$$(Z_l^{eff})_{m,n} = \frac{\sum_{p=-\infty}^{p=+\infty} \frac{Z_l(\omega_p^l)}{p} h_{m,n}(\omega_p^l)}{\sum_{p=-\infty}^{p=+\infty} h_{m,m}(\omega_p^l)}. \quad (3)$$

Here, $I_b = N_b e f_0$ is the current in one bunch with N_b the number of protons in the bunch, e the elementary charge, and $f_0 = \Omega_0 / 2\pi$ the revolution frequency, h is the harmonic number, \hat{V}_T is the total (effective) peak voltage taking into account the PWD (the peak RF voltage is

\hat{V}_{RF}), $B = \tau_b f_0$ is the bunching factor with τ_b the total bunch length (in seconds) taking into account the PWD (the unperturbed total bunch length is τ_{b0}), ϕ_s is the RF phase of the synchronous particle ($\cos \phi_s < 0$ above transition) taking into account the PWD (the unperturbed synchronous phase is ϕ_{s0}), $|Z_l^{BB}/p|$ is the peak value of the Broad-Band (BB) resonator impedance, $m = \dots, -1, 0, 1, \dots$ is the longitudinal coherent bunch mode number, $j = \sqrt{-1}$ is the imaginary unit, Z_l is the longitudinal coupling impedance, $\omega_p^l = p \Omega_0 + m \omega_s$ with $-\infty \leq p \leq +\infty$, where $\omega_s = 2\pi f_s$ is the synchrotron angular frequency taking into account the PWD (the unperturbed synchrotron angular frequency is $\omega_{s0} = 2\pi f_{s0}$), and $h_{m,n}$ describes the cross-power densities of the m th and n th line-density modes. The broad-band resonator impedance is defined by

$$\frac{Z_l^{BB}(p)}{p} = \frac{\Omega_0}{\omega} R_s / \left[1 - j Q_r \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right) \right], \quad (4)$$

where R_s is the shunt impedance (in Ω), $Q_r = 1$ is the quality factor and $\omega_r = 2\pi f_r$ is the resonance angular frequency.

Considering a lepton bunch with Gaussian amplitude density, the following relations are obtained when PWD is taken into account [7]

$$g_0(\hat{\tau}) = \frac{8}{\pi \tau_b^2} e^{-8 \left(\frac{\hat{\tau}}{\tau_b} \right)^2}, \quad (5)$$

$$\phi_s = \phi_{s0} + \frac{2\pi I_b}{\hat{V}_{RF} \cos \phi_{s0}} \times \sum_{p=-\infty}^{p=+\infty} \text{Re}[Z_l(p)] \sigma_0(p), \quad (6)$$

$$\sigma_0(p) = \int_{\hat{\tau}=0}^{\hat{\tau}=+\infty} J_0(p \Omega_0 \hat{\tau}) g_0(\hat{\tau}) \hat{\tau} d\hat{\tau}, \quad (7)$$

$$\begin{aligned} \Delta &= \frac{\omega_{s1}^2 - \omega_{s0}^2}{\omega_{s0}^2} = -\frac{2\pi I_b}{h \hat{V}_{RF} \cos \phi_{s0}} \sum_{p=-\infty}^{p=+\infty} j \frac{Z_l(p)}{p} p^2 \sigma_0(p) \\ &= -\frac{6I_b}{\pi^2 h \hat{V}_{RF} \cos \phi_{s0} B^3} j \left[\frac{Z_l(p)}{p} \right]_{\text{Im,eff}}, \end{aligned} \quad (8)$$

$$\left[\frac{Z_l(p)}{p} \right]_{\text{Im,eff}} = \frac{\pi^3 B^3}{3} \sum_{k=-\infty}^{k=+\infty} \frac{Z_l(k)}{k} k^2 \sigma_0(k), \quad (9)$$

$$\omega_{s1}^2 = \omega_{s0}^2 \times \frac{\hat{V}_T}{\hat{V}_{RF}}, \quad (10)$$

$$\Delta_0 = \Delta \left(\frac{B}{B_0} \right)^3, \quad (11)$$

$$\left(\frac{B}{B_0} \right) \times \left| \frac{\cos \phi_{s0}}{\cos \phi_s} \right| = \left(\frac{B}{B_0} \right)^3 + \Delta_0, \quad (12)$$

where J_0 is the Bessel function of first kind and 0th order, and $g_0(\hat{\tau})$ is the stationary distribution of the synchrotron oscillation amplitude $\hat{\tau}$. The stability criterion of Eq. (1) can then be re-written

$$I_b \leq \frac{3}{2} \times \frac{h \hat{V}_{RF} B_0^3 |\cos \phi_{s0}|}{\left| \frac{Z_l^{BB}}{p} \right|} \times \frac{F_2}{F_1}, \quad (13)$$

where $F_2 = B/B_0$ is a factor found by equating the intensity at threshold of Eq. (13) to the one due to PWD (see Eq. (8)), which is given by

$$F_2 = \frac{B}{B_0} = \left\{ \left| \frac{\cos \phi_{s0}}{\cos \phi_s} \right| + \frac{9}{\pi^2 \frac{\cos \phi_{s0}}{|\cos \phi_{s0}|} F_1} \times \frac{j \left[\frac{Z_l^{BB}}{p} \right]_{\text{Im,eff}}}{\left| \frac{Z_l^{BB}}{p} \right|} \right\}^{1/2}. \quad (14)$$

Note that for sufficiently long bunches ($\tau_{b0} \gg 2/f_r$), the factor F_1 is independent of the bunch length and is given by $F_1 \approx 0.6$. In this case, the factor F_2 is given analytically by Eq. (14). This is not the case for shorter bunches, where Eq. (14) has to be solved numerically.

Neglecting the synchronous phase shift, the stability criterion for the longitudinal microwave instability can be written

$$I_{p0} \leq F \times I_{p0}^{KSB}, \quad (15)$$

with

$$I_{p0}^{KSB} = \frac{1}{\ln 2} \times \frac{(E/e) \alpha_p}{\left| \frac{Z_l^{BB}}{p} \right|} \times \left(\frac{\Delta p}{p_0} \right)_{\text{FWHH},0}^2, \quad (16)$$

where I_{p0}^{KSB} is the initial (low-intensity) peak intensity threshold from the Keil-Schnell-Boussard criterion for

Gaussian bunches [12], E is the beam energy, $\alpha_p = \gamma_{tr}^{-2}$ is the momentum compaction factor, and $(\Delta p / p_0)_{FWHH,0}$ is the initial (low-intensity) relative momentum spread (full width at half height). The factor F , which is given by

$$F = \frac{6}{\pi \sqrt{2\pi}} \times \frac{F_2}{F_1}, \quad (17)$$

is solved numerically and represented in Fig. 1.

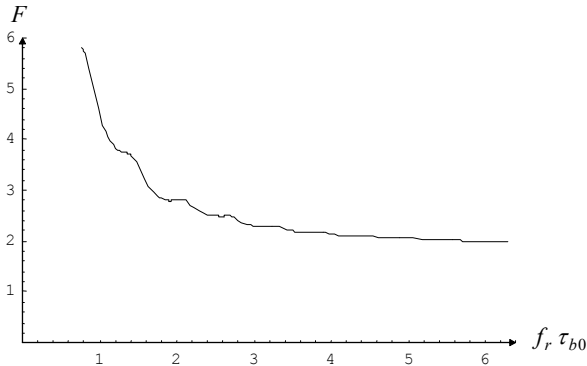


Figure 1: Plot of the factor F vs. $f_r \tau_{b0}$.

It is seen from Fig. 1 that for long (compared to the inverse of the resonance frequency of the impedance) bunches, the intensity threshold is ~ 2 times larger than from the Keil-Schnell-Boussard criterion. Furthermore, when the bunch length gets smaller, the threshold intensity increases, and if $f_r \tau_{b0} \leq 0.75$ there is no instability anymore (with this model...).

3 MEASUREMENTS

Microwave instability measurements made in the CERN SPS in 1993 for long proton bunches produced a value for the longitudinal impedance $|Z_l / p| \approx 20 \Omega$, with $Q_r \approx 1$ and $f_r \in [1.3-1.6 \text{ GHz}]$, using the Keil-Schnell-Boussard criterion [13]. To explain the absence of longitudinal microwave instability for short lepton bunches at high energies, i.e. when $\sigma_z \leq 5 \text{ cm}$, an effective longitudinal impedance 5 times lower than the long bunch value was needed, which was not understood.

Applying the new criterion for the case of the short bunch with $\sigma_z = 5 \text{ cm}$, i.e. $f_r \tau_{b0} \approx 1$, one sees that a factor of ~ 5 is predicted, in perfect agreement with the above (non) observations.

4 CONCLUSION

A new stability criterion for the longitudinal microwave instability of lepton bunches is given. For a sufficiently long bunch, the intensity threshold is found to be ~ 2 times larger than from the Keil-Schnell-Boussard

criterion. When the bunch length gets smaller, the threshold intensity increases. It is ~ 5 times larger than from the Keil-Schnell-Boussard criterion when $f_r \tau_{b0} \approx 1$, which is in perfect agreement with the (non) observations made in Ref. [13]. The new stability criterion may explain why the classical (Keil-Schnell-Boussard) instability threshold can be exceeded in lepton machines.

REFERENCES

- [1] J.L. Laclare, CERN Accelerator School, CERN 85-19, 1985.
- [2] V.K. Neil, A.M. Sessler, Rev. Sci. Instr. 6, 429, 1965.
- [3] E. Keil, W. Schnell, CERN Report ISR-TH-RF/69-48, 1969.
- [4] T. Roser, S.Y. Zhang, Workshop on Instabilities of High Intensity Hadron Beams in Rings, AIP Conference Proceedings 496, 412 p, Upton, New York, June/July 1999.
- [5] D. Boussard, CERN Report LabII/RF/Int./75-2, 1975.
- [6] F.J. Sacherer, "Bunch Lengthening and Microwave Instability", IEEE Trans. Nucl. Sci., Vol. NS-24, No. 3, June 1977.
- [7] J.L. Laclare, CERN Accelerator School, CERN 87-03, 1987.
- [8] E. Shaposhnikova, "Signatures of Microwave Instability", CERN-SL-99-008 HRF, 1999.
- [9] E. Métral, "Stability Criterion for the Longitudinal Mode-Coupling Instability in the Presence of Both Space-Charge and Resonator Impedances", CERN/PS 2001-063 (AE), October 2001.
- [10] E. Métral, "Stability Criteria for High-Intensity Single-Bunch Beams in Synchrotrons", CERN/PS 2002-022 (AE) or Proc. 8th EPAC, Paris, France, 3-7 June 2002.
- [11] K. Ng, "Potential-Well Distortion and Mode-Mixing Instability in Proton Machines", FERMILAB-FN-630, 1995.
- [12] A. Hofmann, "Single Beam Collective Phenomena-Longitudinal", Proc. First Course Int. School on Accelerators, Erice, 1976: Theoretical Aspects of the Behaviour of Beams in Accelerators and Storage Rings, CERN 77-13, p. 176, 1977.
- [13] T. Linnecar and E.N. Shaposhnikova, "Analysis of the Transverse Mode Coupling Instability of the Leptons in the SPS", CERN/SL/93-43 (RFS).