EFFECTS OF SPACE CHARGE AND NONLINEARITIES ON COLLECTIVE INSTABILITIES OF A LONG BUNCH *

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Abstract

We start with discussion of the effects of space charge and nonlinearities on the transverse microwave instability. A possibility of Landau damping with octupole correctors, using an example of the SNS ring, is explored. We also discuss the required strength of such nonlinearities for a bunched beam in the presence of the space-charge tune spread, as well as their effect on dynamic aperture and emittance dilution.

INTRODUCTION

Recently, the intensity limitation in the SNS associated with the transverse instability due to the extraction kickers was explored [1]. In this paper, we explore damping of the instability with the octupole correctors. Although some analytic theories on this subject exist, their application to a realistic situation may be limited. A complicated dynamics in a real machine can be studied in a more self-consistent way using computer simulations. In such an approach, many effects which influence dynamics of the beam can be accurately included. As a result of such studies for the SNS, we find a substantial damping of the instability even when the octupoles introduce very small tune spread, which is not expected based on the existing theories. A possible physical explanation is given, although ultimate conclusions are driven based on the simulations.

TRANSVERSE INSTABILITY IN THE SNS

The instability due to the extraction kickers in the SNS was studied using the UAL code [1]. It was found that even with the chromaticity of $\xi = -7$, the growth rate of the unstable harmonics below 10 MHz were relatively large ($\tau^{-1} \approx 4ms^{-1}$ for a 2 MW beam). This is despite the fact that such a chromaticity introduces the tune spread much bigger than the growth rate of the instability. An incomplete compensation happens because of the space charge tune shift. The onset of the instability with some halo growth was observed by the end of the accumulation in the SNS, which takes about $1\ ms$ [1]. Since that studies, the impedance of the extraction kickers was reduced by a factor of two, due to the aperture increase of the magnets [2]. As a result, the instability growth rate for the present impedance budget was reduced to just below $\tau^{-1} = 2\ ms^{-1}$. Here, we present studies of the instability damping using the old impedance of the SNS [1], which was chosen in order to observe a noticeable growth of the unstable harmonics by the end of the accumulation process.

THRESHOLDS AND SPACE CHARGE

For a long bunch in the SNS and a very slow synchrotron motion, a coasting beam model becomes a good approximation. The growth rate of the instability, due to the coupling impedance (in the absence of any damping mechanism), can be obtained from

$$\tau^{-1} = -Im(\Omega) = \frac{qcI_p}{4\pi E_0 \nu_0} Re(Z_\perp), \qquad (1)$$

where q is the charge of a proton, I_p is the peak current, $E_0 = \gamma mc^2$, ν_0 is the zero-current betatron tune, and Ω is the coherent dipole frequency.

To damp the growth rate with the frequency spread $\Delta\omega$, one should have $\Delta\omega > Im(\Omega)$. When $\Delta\omega$ comes from the momentum spread in the distribution, the stability condition is written, using Eq. 1, as [3]:

$$\mid Z_\perp \mid < F \frac{4E_0\nu_0\gamma\beta}{IR} \frac{\Delta p}{p} \Big[(n-\nu_0)\eta - \xi \Big], \qquad (2)$$

where R is the average machine radius, η is the slippage factor, $\xi = (\delta\nu)/(\Delta p/p)$ is the chromaticity, and F is the form factor which depends on the distribution. Since η is small in the SNS, the low n harmonics, which sample the peak of the extraction kicker impedance, can be effectively damped only by the chromatic term in Eq. 2. It was shown that one gets some chromatic damping even for the bunched beam in the SNS [1].

If there is also a frequency spread from the nonlinear elements, it contributes to the total spread $\Delta\omega$ required for damping. The stability condition in Eq. 2 does not take into account the effect of the space charge, which shifts the tunes of the incoherent particles even in the absence of the beam-pipe surroundings. A typical statement, which could be found in the literature, is that a tune spread due to the nonlinear elements, required for damping, should be comparable to the space-charge tune shift. Such a large spread may destroy the dynamic aperture. Here, we explore to what extent such guidelines are valid.

First, we review the role of the space charge in the stability of a collective dipole motion. Obviously, the fact that the particles inside the bunch have strong space-charge tune

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shift cannot influence the dipole oscillation of the beam, in the absence of the wall images. When one takes into account the effect of the images, both the incoherent frequencies and the dipole coherent frequency have the intensity-dependent shifts. In fact, the difference between the coherent dipole and incoherent self-field shifts is called the space-charge impedance. However, such an impedance should not be used in the stability condition directly. First, its incoherent part does not influence the coherent motion, and, more importantly, it has a pure imaginary contribution which is not directly responsible for the growth rate of the instability (see Eq. 1). On the other hand, the shift due to the imaginary space-charge impedance can influence the stability condition. Its effect will be different depending whether there exists a strong $Re(Z_\perp)$ contribution from other sources of the impedance. In the SNS, there is a large $Re(Z_\perp)$ from the extraction kickers. One can assume the stability diagram obtained from the dispersion relation without the space charge and then take into account the effect of space charge by introducing a shift along the imaginary axis. A more self-consistent approach is to derive the dispersion relation with the space charge [4]. With both approaches, one finds that, for the SNS case, the space charge has a destabilizing effect on the transverse instability, which was confirmed in simulations [1].

OCTUPOLES AND DAMPING

The octupole correctors in the SNS are intended for correction of the resonances, and thus produce only small tune spread of about 0.01. This is much smaller than the space-charge incoherent tune shift of 0.15 for a 2MW beam. However, the simulation studies with a full-intensity full-size beam showed that such weak octupoles may be effective in the damping process, which warranted further investigation. In addition, it was realized that, for the octupole spread to be effective, there should be a large amplitude particles within the beam. As a result, the spread becomes less effective because of the dynamic painting in the SNS, where the large size beam is reached only by the end of the accumulation. Such a finding forced us to perform a realistic study with a full 1060-turn injection process.

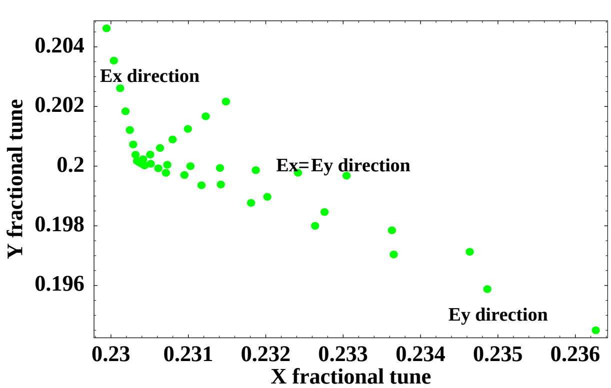

Figure 1: Tune spread by octupoles placed at large β_y.

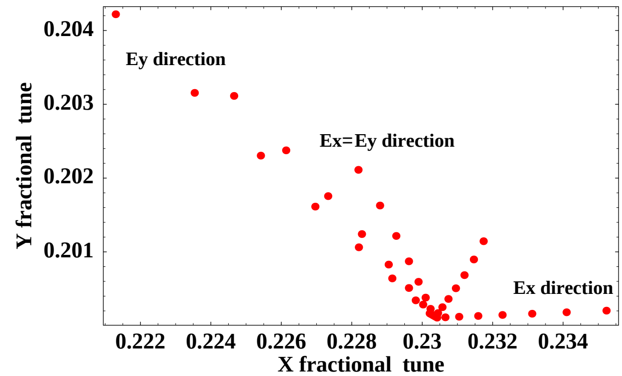

Figure 2: Tune spread by octupoles placed at large β_x.

EFFECTIVE SPREAD

Not surprisingly, one polarity of octupoles gave a stabilizing effect while the other did not, since a proper sign is necessary to introduce the incoherent spread in the direction where it overlaps the coherent spectrum. The damping effect from the octupoles was strongly influenced by their location because the octupole tune spread depends on the particle emittances and beta-functions. It is necessary to generate a spread which affects most of the particles in the transverse distribution. In the SNS, where we use the correlated painting, a substantial portion of the particle distribution has equal emittances. As a result, the most effective location and combination of the octupoles is the one which introduces the largest tune spread in the direction of $\epsilon_x = \epsilon_y$. We refer to such a spread as "effective".

For simplicity, in Figs. (1, 2, 4), only five values of $\alpha = 0, 0.25, 0.5, 0.75, 1.0$, which represent the correlation between the particle emittances, are shown. Here, the correlation is defined as $\alpha = \epsilon_x/(\epsilon_x + \epsilon_y)$, with $\alpha = 0$ corresponding to the particles in the y direction (indicated in Figs. as "Ey direction") with only ϵ_y emittances, and $\alpha = 1$ corresponding to the x direction (indicated in Figs. as "Ex direction"). Note that, due to the space-charge redistribution, the particles occupy all the emittances between the pure ϵ_x and ϵ_y motions. In each line, the dots correspond to eight different amplitudes of the particles within the beam. Only fractional tunes for the working point $(\nu_{0x}, \nu_{0y}) = (6.23, 6.20)$ are shown. The instability due to the extraction kicker impedance of the SNS occurs in the vertical direction, which requires (for damping) the incoherent spread to overlap the vertical coherent spectrum.

Figure 1 shows the tune spread due to a family of 4 octupoles located at large β_y. The strength of all correctors was taken equal with a negative polarity. The effective spread is not in the right direction, and no damping of collective oscillation is observed. A positive polarity gave spread in a proper direction but it was not sufficient. For the octupoles at the location of a large β_x and negative polarity, the effective spread (shown in Fig. 2) has a significant impact on the instability. Figure 3 shows the growth rates

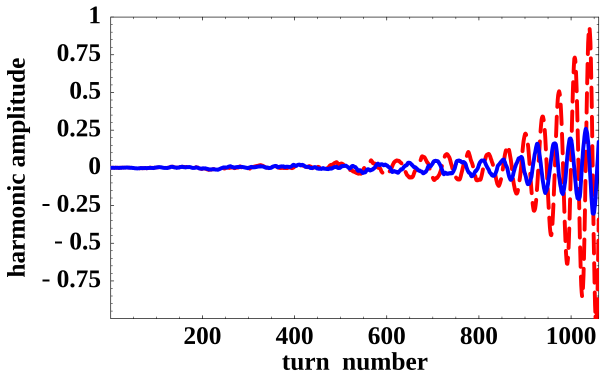

Figure 3: Time evolution for unstable harmonic: wrong sign of octupoles (red - dash line), right sign (blue - solid).

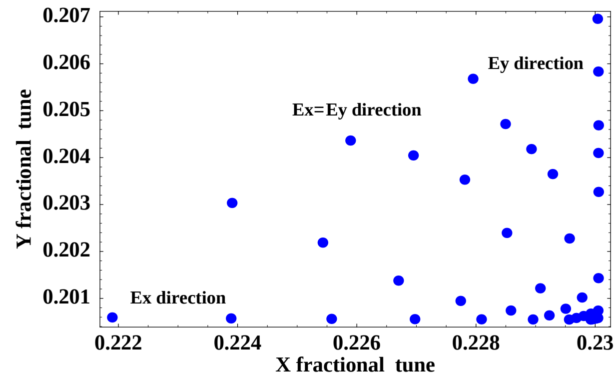

Figure 4: Tune spread with two families of octupoles.

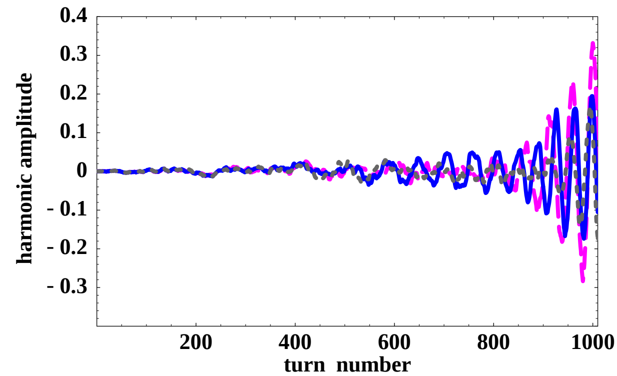

Figure 5: Time evolution for unstable harmonic: β_y family with positive sign (pink, long-dash), β_x family with negative sign (blue, solid), two families (black, short-dash).

of the unstable harmonic at 6 MHz. The red (dash) line indicates the growth rate for the octupoles with the incorrect polarity, corresponding to Fig. 1, while the blue (solid) line corresponds to the octupole spread of Fig. 2. In addition, Fig. 4 shows that a more effective tune spread can be obtained with the two families of octupoles. In this example, the octupoles at large β_y locations had positive polarity while the octupoles at large β_x had negative polarity. Both families were powered at a nominal current of 10 Amp. The corresponding damping of the unstable harmonic at 6 MHz is shown in Fig. 5. The damping rates were comparable to the case when only one family was used but powered at a maximum current of 17 Amp.

Note that simulations were done for the impedance which is a factor of two larger than the present impedance budget. For a presently expected impedance and the SNS base-line intensity of $N = 1.5 * 10^{14}$ (rather than $2 * 10^{14}$ used here), the growth rate of the instability was found to be only about $\tau^{-1} = 1\ ms^{-1}$, which was damped with the corrector settings corresponding to Fig. 4.

EFFECT ON DYNAMIC APERTURE

Although the strength of the octupoles was small, there is still a question whether they can effect the dynamic aperture. We suggest that, if the choice of correctors is done in a proper way, the effect on dynamic aperture may be minimized. For example, four correctors were placed one per superperiod of the SNS with the same phase advance between them. The strength and sign of all correctors in the family were the same so that only the systematic harmonics were driven. For the w.p. (6.23,6.20), the octupole resonances above the working point are driven by the $n = 25$ imperfection harmonic. As a result, we did not see emittance growth associated with these resonances when we used the correctors for Landau damping.

MECHANISMS OF DAMPING

We observed that the instability can be effected or even damped with the frequency spread from the octupoles being much smaller than the space-charge tune shift. To understand such effect one needs to take into account the tunes of the particles in the head and tail of the bunch correctly. These tunes are only weakly depressed by the space charge so that even small frequency spread in the right direction can influence the coherent rigid oscillation of the bunch. Also, the coherent shift due to the wall images for the head and tail of the bunch is different from the shift at the longitudinal bunch center which results in the effective spread along the bunch [1].

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