RF FOCUSING METHODS FOR HEAVY IONS IN LOW ENERGY ACCELERATORS*

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Abstract

A new type of axially symmetric RF field focusing (ARF) in a drift-tube accelerating structure is proposed and studied. Beam stability in both transverse and longitudinal directions is achieved by the variation of drift tube aperture and appropriate phasing. An analysis of 3D beam dynamics is performed using the smooth approximation technique. Beam dynamics in an APF structure and in a conventional RFQ are compared. An universal method is suggested to choose a voltage V and aperture in the APF structure designed for heavy-ions with charge-to-mass ratio $q/A \geq 1/66$ in the energy range above 20 keV/u. The proposed structure and analysing technique can be useful in the design of a post-accelerator for the Rare Isotope Accelerator Facility being designed in the U.S. [1].

INTRODUCTION

Transverse stability of beam motion in an accelerating structure can be ensured either by external focusing elements or by applying a special configuration of the accelerating RF field (RF focusing). The latter is especially effective for low energy ion linacs. Several types of RF focusing such as alternating phase focusing (APF) [2], radio frequency quadrupole (RFQ) [3] and undulator RF focusing (RFU) [4] have been proposed and investigated. In some applications the ARF focusing [5] may be preferable. In RFQ and ARF structures the beam is accelerated by a synchronous wave of RF field, in which case the acceleration gradient must be set to provide particle stability in both transverse and longitudinal directions.

BEAM DYNAMICS IN ARF LINAC

The ARF accelerating cavity can be designed as an interdigital structure. The longitudinal and transverse electric RF field components $E_{z,r}$ in a periodical resonant structure can be described by:

$$E_z = E_n \mathrm{I}_0(h_n r)\cos(\int h_n dz)\cos(\omega t),$$
$$E_r = E_n \mathrm{I}_1(h_n r)\sin(\int h_n dz)\cos(\omega t), \quad (1)$$

where $h_n = h_0 + 2\pi n / D$, $h_0 = \mu / D$, μ are the propagation factors and phase advance per period D of RF structure, n is the harmonic number, I_0, I_1 are the modified Bessel functions. A nonsynchronous harmonic can only focus the beam when its phase velocity $v_{ph,n} = \omega/h_n$ significantly differs from the average velocity of the particles v_b. The TF structure with a single nonsynchronous harmonic can be examind as a periodical sequence of axially symmetric electrostatic lenses. The synchronous s-th harmonic ($v_{ph,s} = v_b$) can be provided by modulation of the inner radius of the drift tubes. A configuration of the accelerating structure is characterized by the radii a_i of the drift tubes, the modulation factor m and structure period D, as is shown in Fig.1. Together with the electrode voltage $\pm V/2$, applied to neighbouring drift tubes, these parameters determine the acceleration and focusing fields.

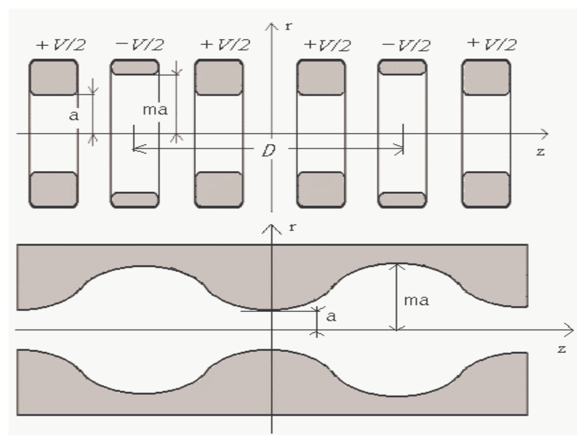

Figure 1: ARF and RFQ channels.

Let us consider particle acceleration in the two-wave approach when $\mu=\pi$, and a synchronous, s=0, and one nonsynchronous, n=1, harmonics are taken into account. In general, individual particles trajectories are complicated but can be represented as the sum of a slow variation $\bar{\boldsymbol{r}}$ and rapid oscillation $\tilde{\boldsymbol{r}}$. Accordingly, the beam momentum $\boldsymbol{p}$ can be represented as the sum of a slowly varying and a rapidly oscillating components, $\boldsymbol{p} = \bar{\boldsymbol{p}} + \tilde{\boldsymbol{p}}$. Following ref. [4] one can apply averaging over the rapid oscillations and obtain a time-averaged equation of motion for a non-relativistic ion:

$$\frac{d^2\boldsymbol{R}}{d\tau^2} = -\frac{d}{d\boldsymbol{R}}\tilde{\mathrm{U}}_{\mathrm{eff}}, \quad (2)$$

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where $\boldsymbol{R} = 2\pi\bar{\boldsymbol{r}}/\lambda\beta_s$ is a dimensionless coordinate and $\tau = \omega t$. $\tilde{U}_{eff}$ is the effective potential function determined by dimensionless field harmonics amplitudes of on the axis $e_{s,n} = eE_{s,n}\lambda/(2\pi mc^2\beta_s)$:

$$\tilde{U}_{eff} = -\frac{1}{2}e_0\left[\mathrm{I}_0(\rho)\sin(\psi_s+\chi) - \chi\cos(\psi_s)\right] + \\ + \frac{1}{16}\left\{5\left(\frac{e_1}{4}\right)^2 g(3\rho) + \left(\frac{e_0}{2}\right)^2 g(\rho)\right\}, \tag{3}$$

where $R = [\chi, \eta]$, $\chi = 2\pi(\bar{z} - \bar{z}_c)/\lambda\beta_s$, $\rho = 2\pi\bar{r}/\lambda\beta_s$, $g_{s,n}(\rho) = \mathrm{I}_0^{\,2}(\rho) + \mathrm{I}_1^{\,2}(\rho) - 1$; ψ, z_c, $\beta_s = \omega/\mathrm{h}_s\mathrm{c}$ are the phase, coordinate, and velocity of the synchronous particle.

The effective potential function can be rewritten as:

$$\tilde{\mathrm{U}}_{\mathrm{eff,a}} = -A_\mathrm{a}\frac{\mathrm{qV}}{8\mathrm{W_s}}\left[\mathrm{I}_0(\rho)\sin(\psi_\mathrm{s}+\chi) - \chi\cos(\psi_\mathrm{s})\right] + \\ + X_\mathrm{a}^2\left(\frac{\mathrm{qV}}{8\mathrm{W_s}}\right)^2\left\{\frac{45}{64}\mathrm{g}(3\rho) + \left(\frac{3A_\mathrm{a}}{4X_\mathrm{a}}\right)^2\mathrm{g}(\rho)\right\}, \tag{4}$$

where W_s is the energy of the synchronous particle, A_a and X_a determine acceleration and focusing gradients which are mutually coupled. If geometrical parameters *a*, *D*, *m* are known, A_a and X_a are defined as

$$A_a = \frac{2\left[\mathrm{I}_0(3m\rho_\mathrm{m}) - \mathrm{I}_0(3\rho_\mathrm{m})\right]}{\mathrm{I}_0(m\rho_\mathrm{m})\mathrm{I}_0(3\rho_\mathrm{m}) + 2\mathrm{I}_0(3m\rho_\mathrm{m})\mathrm{I}_0(\rho_\mathrm{m})}, \tag{5}$$

$$\mathrm{X}_a = \frac{1 - A_a\mathrm{I}_0(\rho_\mathrm{m})}{\mathrm{I}_0(3\rho_\mathrm{m})}, \qquad \rho_m = \frac{\pi a}{D}. \tag{6}$$

The effective potential function $U_{eff,a}$ describes the 3-D particle dynamics completely. In addition, it determines the system Hamiltonian

$$H = \frac{1}{2}\left(\frac{d\mathrm{R}}{d\tau}\right)^2 + U_{eff}. \tag{7}$$

Using the Hamiltonian (7) and analyzing the bunch form in the 4-dimensional phase space one can find the relationship between the longitudinal and transverse acceptances.

A necessary condition for simultaneous transverse and longitudinal focusing is the existence of a global minimum of $\tilde{U}_{eff}$. In this case the effective potential function is 3-D potential well in the beam frame. In the two-wave approach, the transverse stability condition (5) results in

$$\frac{qV}{mc^2}\left(\frac{\lambda}{2\pi a}\right)^2 > F_a = \frac{64}{135}\Phi_a(\rho)\frac{A_a}{X_a^2\rho_m^2}\sin(\psi_s+\chi), \tag{8}$$

where the form-factor Φ_a is

$$\Phi_a = \frac{\mathrm{I}_1(\rho)}{\mathrm{I}_1(3\rho)}\left[\mathrm{I}_0(3\rho) - \frac{\mathrm{I}_1(3\rho)}{6\rho}\right]^{-1} \tag{8a}$$

As it is seen in Eq. (8), the ARF efficiency depends significantly on the parameter X_a^2/A_a. Amplitudes of the RF field harmonics must be chosen to satisfy Eq. (8) along the structure. Eq. (8) also shows that for the considered range of beam energy the transverse focusing with an arbitrary synchronous phase can be achieved only if the amplitude of non-synchronous wave is larger than the amplitude of the synchronous one. The accelerating gradient is proportional to $A_\mathrm{a}(qV/8W_s)$. Thus the RF focusing effectiveness is limited by the acceleration gradient and sets restrictions on the parameter A_a. It also means that the ARF structure is effective for low energy beams. The ARF structure can be created by a special design of the focusing period consisting of two or more accelerating gaps.

BEAM DYNAMICS IN RFQ LINAC

The longitudinal and transverse electric RF field components E_v in an RFQ structure are described by:

$$E_z = \frac{h}{2}A_qV\ \ \mathrm{I}_0(\rho)\sin(\psi+\tau)\sin\tau, \quad h = 2\pi/\beta\lambda, \\ E_r = -hV\left[X_q\rho\cos(2\theta) + \frac{1}{2}A_q\mathrm{I}_1(\rho)cos(\psi+\tau)\right]\sin\tau, \tag{9}$$

where

$$A_q = \frac{m^2-1}{m^2\mathrm{I}_0(m\rho_\mathrm{m}) + \mathrm{I}_0(m\rho_\mathrm{m})}, \quad \mathrm{X}_q = \frac{1 - A_q\mathrm{I}_0(\rho_\mathrm{m})}{\rho_m^2}. \tag{10}$$

In this case, the effective potential function can be written as follows:

$$U_{eff,q} = -A_q\frac{qV}{8W_s}\left[\mathrm{I}_0(\rho)\sin(\psi_s+\chi) - \chi\cos(\psi_s)\right] + \\ + X_q^2\left(\frac{qV}{8W_s}\right)^2\left\{4\rho^2 + \left(\frac{A_q}{4X_q}\right)^2 g(\rho)\right\}. \tag{11}$$

The existence of the global minimum of the potential U_{eff} is a condition for simultaneous transverse and longitudinal stability. The oscillations will be stable if the condition

$$\frac{\mathrm{qV}}{\mathrm{mc}^2}\left(\frac{\lambda}{2\pi\mathrm{a}}\right)^2 > \mathrm{F_q} = \frac{1}{2}\Phi_\mathrm{q}(\rho)\frac{\mathrm{A_q}}{X_\mathrm{q}^2\rho_\mathrm{m}^2}\sin(\psi_\mathrm{s}+\chi) \tag{12}$$

is satisfied, where the form-factor $\Phi_q = \mathrm{I}_1(\rho)/\rho$. This inequality is very similar to the condition (8) and allows a detailed comparison of two methods of RF focusing.

COMPARISION OF ARF AND RFQ LINACS

The analysis of the 3D effective potential function allows us to find the conditions under which focusing and acceleration of the particles occur simultaneously. Universal curves which are described by the functions $\Phi_{a,q}$ in the right side of the inequalities (8), (12) allow to choose V, and λ/a at the given q/m, and a range of ion velocity change β_s.

In a RFQ linac the amplitude of rapid transverse oscillations $\tilde{r}$ is comparable with $\bar{r}$, and in an ARF linac, $\tilde{r} << \bar{r}$. As a consequence aperture of an RFQ linac a_q must be larger than for an ARF structure. The accelerating gradient, $A_{a,q}$, as function of a dimensionless aperture $\rho_m = a\omega/v$ for m=2 is shown in Fig. 2. In an ARF linac the acceleration gradient is larger than for a RFQ if $a_q \approx 2a_a$. The transverse focusing conditions (8) and (13) depend on the ratio A/X^2 as function of channel aperture ρ_m and form-factor $\Phi(\rho)$ The functions of $\Phi_{a,q}(\rho)$ for different ion beam radii are shown in Fig.2. The magnitude of $\Phi_a(\rho)$ is smaller than $\Phi_q(\rho)$ and drops as radius ρ increases. The latter means that the focusing condition (8) is easier to satisfy for non-axis particles in an ARF structure. For given V and λ the velocity range where the transverse stability conditions are satisfied is different for ARF and RFQ. For an RFQ structure one can find a minimal value of the normalized voltage $\tilde{V} = \frac{qV}{mc^2}\left(\frac{\lambda}{2\pi a}\right)^2$ (left part of the inequalities (12)) that provides transverse stability independently from the particle velocity. In an ARF structure transverse stability is achieved only for particle velocities above a lower limit of the voltage. The values of $\tilde{V}_{a,q}$ in Fig.3 were calculated for heavy ions with q/m=1/66, V=100 kV, λ= 25m, $\psi_s = 20^0$ and radius a=1 cm for RFQ and a=0.5cm for ARF. For these parameters in ARF structure beam stability is provided if $v > 1.8\omega a$.

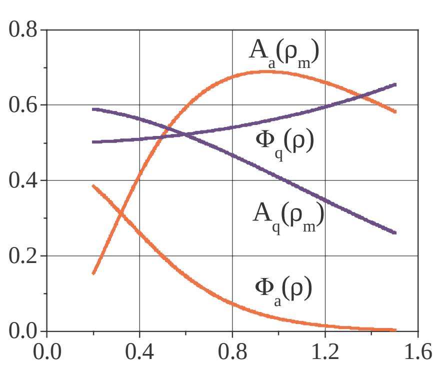

Figure 2: Accelerating gradients and form-factors as a function of unitless aperture, ρ_m, and radius, ρ.

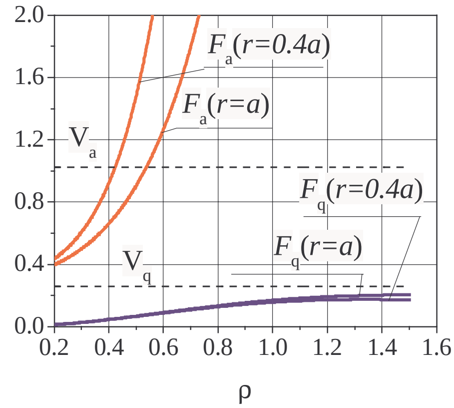

Figure 3: Transverse stability conditions.

CONCLUSION

New methods for axial-symmetric RF focusing and acceleration of heavy ions are studied and compared with the well-known RFQ structure. Universal curves are established to choose a voltage V and λ/a for a given q/m and ion velocity β_s in RFQ and ARF linacs. The acceleration gradient in ARF linac is larger than in the RFQ structure. Aperture of the RFQ linac must be larger due to large amplitudes of rapid transverse oscillations. Transverse stability in both ARF and RFQ structures depends on beam velocity range. For the same a/λ and V, the focusing properties of the ARF structure are extended to higher velocities. This result is especially interesting for the design of the RIA post accelerator [7] where ions with low charge-to-mass ratio q/m=1/66 are accelerated prior to injection into the superconducting linac.

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