OPTICS STUDIES FOR THE CERN PROTON SYNCHROTRON: LINEAR AND NONLINEAR MODELLING USING BEAM BASED MEASUREMENTS

R. Cappi, M. Giovannozzi, M. Martini, E. Métral, G. Métral, R. Steerenberg, CERN
A.-S. Müller, ISS, Forschungszentrum Karlsruhe, Germany

Abstract

The CERN Proton Synchrotron machine is built using combined function magnets. The control of the linear tune as well as the chromaticity in both planes is achieved by means of special coils added to the main magnets, namely two pole-face-windings and one figure-of-eight loop. As a result, the overall magnetic field configuration is rather complex not to mention the saturation effects induced at top-energy. For these reasons a linear model of the PS main magnet does not provide sufficient precision to model particle dynamics. On the other hand, a sophisticated optical model is the key element for the foreseen intensity upgrade and, in particular, for the novel extraction mode based on adiabatic capture of beam particles inside stable islands in transverse phase space. A solution was found by performing accurate measurement of the nonlinear tune as a function of both amplitude and momentum offset so to extract both linear and nonlinear properties of the lattice. In this paper the measurement results are presented and the derived optical model is discussed in detail.

PS LATTICE AND MAIN MAGNET

The PS lattice consists of ten super-periods each made of ten combined function magnets 4.4 m long, interlaced with eight 1.6 m and two 3.0 m drift spaces [1]. Every magnet is composed of two half-units, each made of five blocks with small gaps in between, with gradients of opposite sign ($|G| = 5.2 \text{ T/m}$), separated by a central junction. The control of the tunes and chromaticities is obtained by means of the three currents of the pole-face winding and figure-of-eight loop devices located on the magnet poles. As an example, the layout of a PS magnet unit in the extraction region is shown in Fig. 1. The latest PS magnetic field measurements using Hall probes were undertaken in 1992 [2] for different settings of the currents in the main coil, pole-face and figure-of-eight loop windings. The data of the vertical field, including measurements of the central field, the end and lateral stray fields, as well as the field in the junction between the two half-units, produced a discrete 2D field map [2].

The field measurements were carried out in a Cartesian co-ordinate frame (see Fig. 1 for its definition). In this reference system a regular mesh is defined and for each point in the mesh the value of B_y was measured in the median plane. The step size is 20 mm along the longitudinal z-axis and 10 mm along the radial x-axis. The mesh extends from -2.55 m to 2.73 m and from -70 mm to 310 mm in the longitudinal and radial directions respectively. As an example, the fitted 2D field map for the 26 GeV/c working point is shown in Fig. 2 (see Ref. [3] for more details). When the effect of the additional coils is taken into account,

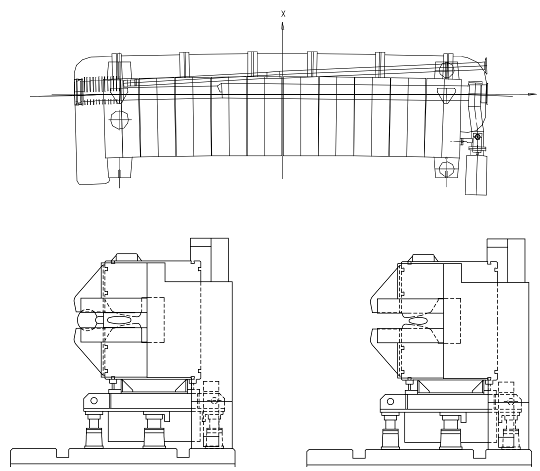

Figure 1: Layout of the PS magnet unit 16 (upper part). The vacuum pipes for both the circulating beam and the extracted one are visible. The cross sections of the entry face (open gap) and exit face (closed gap) are shown on the lower left and lower right respectively.

nonlinear field components have to be considered. Multipolar components may be derived from the measured field map to model the machine lattice. However, they hold only for the specific set of currents in the pole-face and figure-of-eight loop windings. Unfortunately, these values change according to the machine setting, thus a magnetic measurement is required for each new working point, which is not possible in practise.

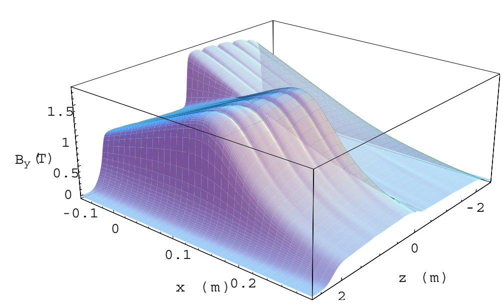

Figure 2: Polynomial field map fitted to the measured field values $B_y = B_y(x, z, 0)$ for the 26 GeV/c working point.

MEASUREMENT RESULTS

Measurement Technique

The technique used resembles that one used to derive the nonlinear field components using beam based measurements for the Super Proton Synchrotron (SPS) [4]. The tunes, both horizontal and vertical, are measured as functions of the momentum offset generated by a proper rf-perturbation. A polynomial fit of the measured curve is performed to extract numerical information on the different orders. Then, virtual nonlinear elements, represented by thin lens elements, are inserted in the machine lattice model and they are used as fit parameters to reproduce the fitted polynomial. This procedure is applied order-by-order, i.e. the quadrupole components are used to reproduce the constant term in the polynomial, the sextupole components the linear term and so on, up to the dodecapolar components. Due to the structure of the PS main magnet, the virtual nonlinear elements are located at both dipole ends, thus giving two free parameters per each element type. The computations on the PS model were performed by using the *MAD* program [5].

Injection Plateau - 2.14 GeV/c

The first case considered here, refers to the modelling of the injection flat-bottom. Although the tunes at injection are normally controlled by means of the low-energy quadrupoles, in recent years it was tried out to use the additional coils, i.e. pole-face windings and figure-of-eight loop, to improve the machine control, also including the chromaticity. Of course, this might lead to introducing unwanted nonlinear effects, particularly harmful for a high-intensity (hence large emittance) proton beam at injection. The measurement results shown in Fig. 3 show clearly a third-order effect on top of the expected linear behaviour. The coefficients of the best fitting polynomial are:

$$Q_x(\delta) = -3.92 \times 10^4\,\delta^3 - 96.94\,\delta^2 - 4.49\,\delta + 6.179$$
$$Q_y(\delta) = -2.64 \times 10^4\,\delta^3 - 146.26\,\delta^2 - 4.62\,\delta + 6.222$$

In the model, nonlinear elements have been added up to

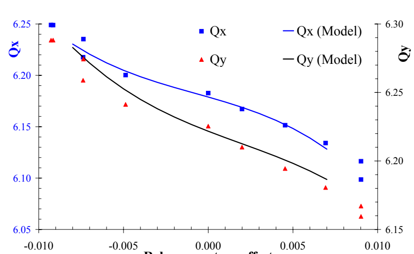

Figure 3: Tune vs. momentum offset: experimental data (markers) are compared with curves obtained using the lattice model with the fitted virtual thin lens multipoles.

decapoles only, as the agreement with measurement results is already quite good (see Fig. 3).

Intermediate Plateau - 14 GeV/c

High-intensity proton beams are normally extracted towards the SPS at 14 GeV/*c*. A detailed model of the PS machine for such energy is crucial for the proposed multi-turn extraction based on adiabatic capture inside stable islands of phase space using nonlinear magnets [6, 7] and also for the experimental measurements of this novel technique [8]. To this end, two main configurations were considered, i.e. standard working point (ST) and a special setting (SP) with reduced horizontal chromaticity (see Ref. [8] for more details). For each configuration, two sets of measurements were performed, namely with and without the sextupoles and octupoles necessary to create the stable islands for capturing and splitting the beam in the transverse phase space. Figure 4 refers to the normal working point and the results of the fitted PS model are directly compared with the measurements. A rather good agreement is visible for the horizontal tune curve, while a slightly worse situation (particularly for negative values of momentum offset) is visible for the vertical tune curve. The same considerations hold when the strong nonlinear elements are excited. It is worthwhile stressing that no fitting procedure was applied in this second case, i.e. the virtual kicks obtained without dedicated sextupoles and octupoles were kept unchanged when the special nonlinear elements were excited in the PS model. A somewhat different situation occurs

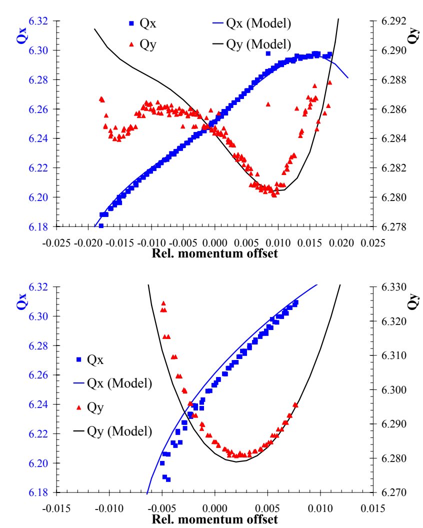

Figure 4: Tune vs. momentum offset: experimental data (markers) and fitted model (curves) without (upper) and with (lower) nonlinear elements used for the adiabatic capture tests. The standard working point is used.

when a special working point is used (see Fig. 5). The fitted model agrees quite well to measurement results without the dedicated nonlinear elements. However, the same virtual kicks do not give good enough results when comparing with measurements performed with dedicated sextupoles and octupoles. Two considerations might explain this observation. Firstly, the fitting procedure produces only an *effective model*: the details are not exactly reproduced, but the average behaviour is well described. Secondly, the nonlinear effects in the dynamics break the superposition principle. Therefore, in some cases, the final result of combining a good fitted model (computed without dedicated nonlinear elements) with the effect of dedicated sextupoles and octupoles might not be in good agreement with experimental results. The fitted polynomials for the two configurations, including dedicated sextupoles and octupoles are

$$Q_x^{\mathrm{ST}}(\delta) = -1.96 \times 10^7\,\delta^4 + 1.97 \times 10^5\,\delta^3 - 362.15\,\delta^2 - 6.79\,\delta + 6.255$$

$$Q_y^{\mathrm{ST}}(\delta) = 3.60 \times 10^6\,\delta - 4.30 \times 10^4\,\delta^3 + 688.01\,\delta^2 - 2.88\,\delta + 6.285$$

$$Q_x^{\mathrm{SP}}(\delta) = -1.40 \times 10^4\,\delta^3 - 249.90\,\delta^2 + 1.26\,\delta + 6.243$$

$$Q_y^{\mathrm{SP}}(\delta) = 1.76 \times 10^4\,\delta^3 + 599.03\,\delta^2 + 5.43\,\delta + 6.282$$

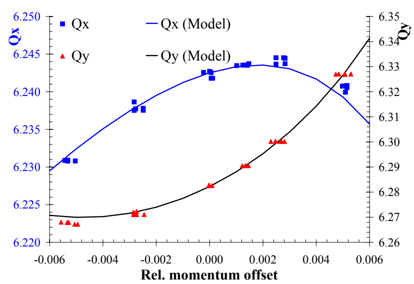

Figure 5: Tune vs. momentum offset: experimental data (markers) and fitted model (curves) with nonlinear elements used for the adiabatic capture tests. The special working point with low horizontal chromaticity is used.

Top Plateau - 26 GeV/c

Finally, the model of the PS machine at top energy (26 GeV/c) is considered. This is particularly interesting as the main PS magnet shows signs of saturation effects at high-energy that could enhance the nonlinear components introduced by the special coils used to control the working point. The measurement results are shown in Fig. 6, where the good agreement with the fitted model is clearly visible. The best fit polynomials for the measured tune curves are:

$$Q_x(\delta) = 1.72 \times 10^6\,\delta^4 - 1.97 \times 10^4\,\delta^3 - 317.10\,\delta^2 + 2.27\,\delta + 6.239$$

$$Q_y(\delta) = 4.57 \times 10^5\,\delta^4 + 9.03 \times 10^2\,\delta^3 - 63.67\delta^2 + 0.27\,\delta + 6.266$$

In this case magnetic field components up to dodecapole have been used in the model. The summary of the values of the fitted multipolar components is presented in Table 1, the expansion of the magnetic field in multipoles being:

$$B_y + iB_x = B_0\,\rho_0 \sum_{i=1}^{M} [K_s + iJ_n]\,\frac{(x+iy)^n}{n!} - B_0$$

$$K_n = \ell\,\frac{1}{B_0\rho_0}\,\frac{\partial^n B_y}{\partial x^n}, \quad J_n = \ell\,\frac{1}{B_0\rho_0}\,\frac{\partial^n B_x}{\partial x^n},$$

where ρ_0 is the nominal bending radius and ℓ the length of the magnetic element.

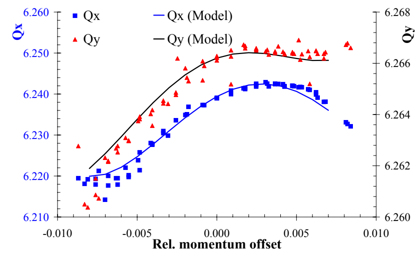

Figure 6: Tune vs. momentum offset: experimental data (markers) are compared with curves obtained using the lattice model with the fitted virtual thin lens multipoles.

GeV/c		K_1 (10^{-4})	K_2	K_3	K_4	K_5
2.14	F	-10.3	0.013	0.026	-105.53	–
	D	10.2	-0.0014	-0.17	150.79	–
14	F	-0.96	0.04	0.07	-1.79	-478.81
(ST)	D	1.29	-0.03	-0.06	0.11	462.27
14	F	-2.51	0.06	0.02	1.05	–
(SP)	D	2.21	-0.05	0.01	-2.04	–
26	F	-3.29	0.04	-0.63	-36.21	5013.04
	D	3.50	-0.03	0.69	39.89	-7239.44

Table 1: Summary of virtual kick values in units of m^{-n} (F, D stands for the half-unit type of the PS magnet).

CONCLUSIONS AND OUTLOOK

A model of the PS machine derived from measurements of the tune as a function of the momentum offset was successfully computed for various situations, i.e. injection energy, intermediate extraction energy and top-energy, including different configurations of dedicated nonlinear elements. The outcome of these studies will be beneficial for many other topics under investigation, such as the nonlinear resonance benchmarking experiment [9], the measurements [8] as well as the simulations concerning the adiabatic trapping inside stable islands of phase space [7].

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