ACTION AND PHASE ANALYSIS TO DETERMINE SEXTUPOLE ERRORS IN RHIC AND THE SPS *

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Abstract

Success in the application of the action and phase analysis to find linear errors at RHIC Interaction Regions [1] has encouraged the creation of a technique based on the action and phase analysis to find non linear errors.

In this paper we show the first attempt to measure the sextupole components at RHIC interaction regions using the action and phase method. Experiments done by intentionally activating sextupoles in RHIC and in SPS [2] will also be analyzed with this method.

First results have given values for the sextupole errors that at least have the same order of magnitude as the values found by an alternate technique during the RHIC 2001 run [3].

INTRODUCTION

Under ideal conditions, the action J and phase φ of betatron oscillations of a particle should remain constant all around the ring. Magnetic errors in the different elements of the ring can lead to a change of these two constants of motion. These changes are used to determine the location of such errors and their strengths.

Action and phase associated with particle orbits at particular position in the ring are obtained from pairs of adjacent Beam Position Monitor (BPM) measurements. BPM measurements are converted into action and phase by inverting the equations:

$$\begin{aligned} x_1 &= \sqrt{2J\beta_1}\sin(\psi_1-\varphi) \\ &= \\ x_2 &= \sqrt{2J\beta_2}\sin(\psi_2-\varphi) \end{aligned} \qquad (1)$$

where, x_1 and x_2 correspond to any two adjacent BPM measurements, β_1, β_2, ψ_1 and ψ_2 are their corresponding beta functions and phase advances.

Eq. 1 is applied to all adjacent BPM measurements in the ring to obtain functions of action and phase with respect to s, the azimuthal location.

During the RHIC 2000 run, studies of action and phase indicated significant coupling errors at the RHIC IRs. A method based on first-turn orbit measurements and action and phase analysis was developed to find the magnitude of the coupling errors and to perform the corresponding correction [1].

The positive results obtained from the previous studies stimulate the development of a general method able to determine no only skew quadrupole errors but also gradient errors and non linear errors. This method was used during the RHIC 2001 run to confirm the skew error measurements done with orbits taken in the RHIC 2000 run (see [4]). The action and phase analysis was then used to measure integrated gradient errors giving very precise results (see [5]) . The accuracy of the method to determine skew quadrupole errors and gradient errors as well was also demonstrated with a series of experiments performed during the RHIC 2001 run (see [5]). This paper covers experiments performed during the RHIC 2001 proton run and the experiments performed during the RHIC 2003 dAu run to determine sextupole errors with the action and phase analysis method. Results obtained with SPS orbits with sextupoles intentionally introduced in the accelerator are also presented.

DETERMINATION OF ERRORS FROM THE ACTION AND PHASE ANALYSIS

The magnitude of the magnetic kick that particles experience due to the presence of an optical error located at some arbitrary position s_0 is given by:

$$\Delta x'(s_0) = \sqrt{\frac{\left(J_x^L + J_x^R - 2\sqrt{J_x^L J_x^R}\cos(\psi_x^L-\psi_x^R)\right)}{\beta_x(s_0)}} \qquad (2)$$

where J_x^L, J_x^R, ψ_x^L and ψ_x^R correspond to the action and phases for $s < s_0$ (superindice L) and $s > s_0$ (superindice R) respectively. There is an equivalent expresion for $\Delta y'(s_0)$.

On the other hand, $\Delta x'(s_0)$ and $\Delta y'(s_0)$ can also be expressed as function of A_1 and B_1 , the skew quadrupole and gradient errors present at s_0, and all other non linear components like A_2 and B_2, the skew and normal sextupole errors. Such expression is given by:

$$\begin{aligned} \Delta x' &= (A_1 y_0 - B_1 x_0 \\ &\quad +2A_2 x_0 y_0 + B_2(-x_0^2+y_0^2)+...) \\ \Delta y' &= (A_1 x_0 + B_1 y_0 \\ &\quad +2B_2 x_0 y_0 + A_2(x_0^2+y_0^2)+...) \end{aligned} \qquad (3)$$

where x_0 and y_0 are the horizontal and vertical positions of the beam at s_0. The expansion shown in Eq. 3 is valid only for an error localized in a single point or in good aproximation for a single magnet. When magnet structures like the RHIC triplets or the RHIC interaction regions are responsible for the now so called integrated magnetic kick $\Delta x'$ the

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new expressions are [5]:

$$\Delta x' = A_1^{eq} y_0 - B_1^a x_0 + 2A_2^a x_0 y_0 - B_2^a x_0^2 + B_2^b y_0^2 + ... \quad (4)$$

$$\Delta y' = A_1^{eq} x_0 + B_1^b y_0 + 2B_2^b x_0 y_0 + A_2^a x_0^2 - A_2^b y_0^2 + ...$$

where the superindices employed in the coefficients point to the fact that except for the equivalent skew error, A_1^{eq}, all the other coefficients are not longer symmetric and hence they have to be splitted in two, one with superindice a and one with superindice b. It is possible to evaluate the different multipoles components in Eq. 3 if a set of measurements of the delta kicks versus the beam position at s_0 are available. The procedure to obtain such measurements is basically to record orbits with significant betatron oscillations (usually produced by adjusting a dipole corrector to strengths several times bigger than its normal setting); create the so called difference orbits by subtracting the baseline from the orbits created with the different settings of the dipole corrector; apply Eq. 1 to the difference orbits to obtain action and phases before and after s_0, and finally apply Eq. 2 to obtain $\Delta x'$ and $\Delta y'$ with an equivalent equation. The beam position (x_0, y_0) at s_0 it is usually approximated with the nearest beam position monitor.

NON LINEAR ANALYSIS OF RHIC 2001 PROTON EXPERIMENTS

During the RHIC 2001 proton run, difference orbits were taken to study non linearities at one of the interaction regions of RHIC with the action and phase method. The orbits were taken by changing the strength of a horizontal and a vertical dipole correctors 10 times which allows to have 10 points in the graphs of magnetic kick versus beam position. A fitting of the graphs (see the graphs obtained with the horizontal dipole corrector in Fig. 1) obtained in both cases give the coefficients defined in Fig. 1. Those coefficients are related with the multipole errors by linear formulas (see [5]) that were used to obtain Table 1.

Table 1: Equivalent multipole errors obtained from the fits of Fig. 1 and its corresponding figure in the vertical plane.

A_1^{eq}	0.122 ± 0.003
B_1^x	0.386 ± 0.001
B_1^y	-0.142 ± 0.002
A_2^{ya}	0.0121 ± 0.0003
A_2^x	-0.0012 ± 0.0011
B_2^y	0.0061 ± 0.0011
B_2^{xa}	0.0037 ± 0.0025

Table 1 indicates that the linear components can be precisely extracted from data and also sextupolar components can be extracted but not with the same precision as linear errors can be determined. Even though the precision of the sextupole errors determined in this experiment is not completely satisfactory, these errors are still comparable with the corresponding values found by an alternate method used during the RHIC 2001 run [3].

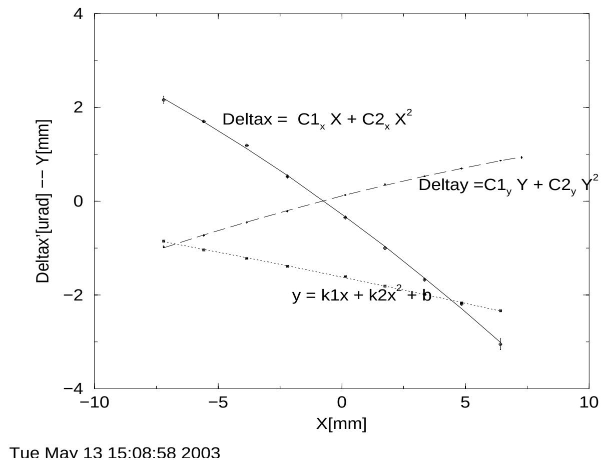

Figure 1: Graphs of magnetic kick vs beam position extracted from orbits obtained by changing the strength of a horizontal dipole corrector in RHIC. Even though the linear errors dominated these curves, nonlinear behavior is also present and it is possible to determine such nonlinearities from polynomial fits.

CALIBRATION OF THE TECHNIQUE TO FIND NON LINEAR ERRORS

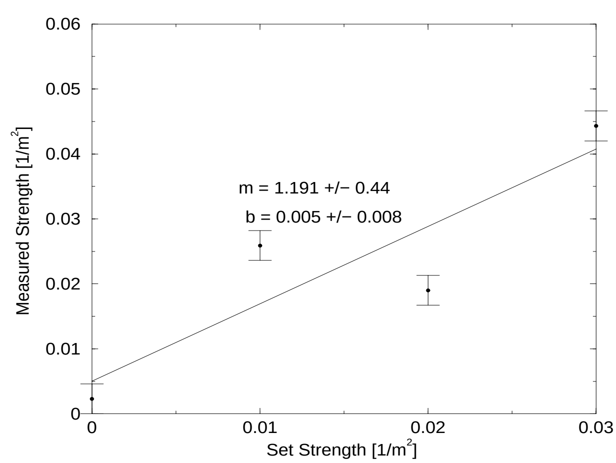

Figure 2: Sextupole calibration curve obtained with difference orbits collected during the RHIC 2003 dAu run.

Experiments to calibrate the action and phase method to determine sextupole errors were done during the RHIC 2003 dAu run. The experiment is basically to set a sextupole corrector to some known strength and then take a series of orbits with different strengths of a particular dipole corrector, first in the horizontal plane and then in the vertical plane. The experiment is then repeated for other 3 different sextupole corrector strengths. From every series of orbits it is possible to measured a sextupole component with the method described earlier and a calibration curve like the one shown in Fig. 2 can be obtained. The errors shown are propagated errors derived from the estimated errors of the graphs of magnetic kick versus position from which the calibration curve was obtained. The calibration curve follows the expected trend but the propagated errors seem to be very small when compared with the general deviation of the points from the model. There are evidence that errors associated with the magnetic kicks from which the sextupole were extracted were underestimated. Indeed the quadratic fits done to the curves of magnetic kick versus the beam position give values for χ^2 equal to 2.3, an indication of too small uncertainties. Apart from this problem the general deviation of the data points is still significant and more experimentation will be needed to reduce this deviation. The uncertainties associated with RHIC 2001 proton experiments magnetic kicks were 4 times smaller than the ones found in the RHIC 2003 dAu experiments. This might be due to some temporary condition of the machine but also might be related with the particle used. If this is the case, then it will be convenient to repeat this experiment with protons. Another factor that will reduce the errors is increasing the number of points used to determine each sextupole strength. Due to the time limitations only 4 points per sextupole strength were used in the RHIC 2003 dAu experiment. Increasing the amplitude of the betatron oscillation will definitively help to resolve the strength with better precision but the feasibility of increasing the amplitude beyond the maximum amplitud used of about 10 mm must be carefully examined.

ACTION AND PHASE ANALYSIS WITH SPS ORBITS

Orbits taken originally in the SPS to study resonance driving terms [2] were also analized with the action and phase method. The graphs of phase (see Fig. 3) obtained by inverting Eq. 1 show regular behavior of the phase with jumps at some places. Most of these places exactly correspond to the places were strong sextupole were on during the data taking of orbits at the SPS. The jumps are more or less clear depending on the turn that is being analyzed.

The graphs of phase also have a tilt (the phase graphs are expected to be horizontal lines with jumps at the places where the errors are located) that probably is due to the fact that the model used for the analysis and the machine model were tuned slightly different. The next step in the analysis of the SPS orbits with the action and phase method is to numerically determine the magnitude of the sextupole strengths and compare them with the set sextupole strengths in the control room.

CONCLUSIONS

Very precise measurements of linear components were obtained in the first experiment presented in this article (RHIC 2001 proton run) and the feasibility of extracting non linear errors has been demonstrated with the same data.

Data collected during the RHIC 2003 dAu run has allowed to test the calibration of the action and phase analysis to determine sextupole components. The calibration curve obtained is in agreement with the expected curve but more experimental data will be neccesary to improve the precision of the measurements.

Application of the action and phase analysis in turn by turn orbits of the SPS has allow to identify clearly the places where sextupoles were intentionally turn on. Future analysis will also give the strengths of such sextupoles.

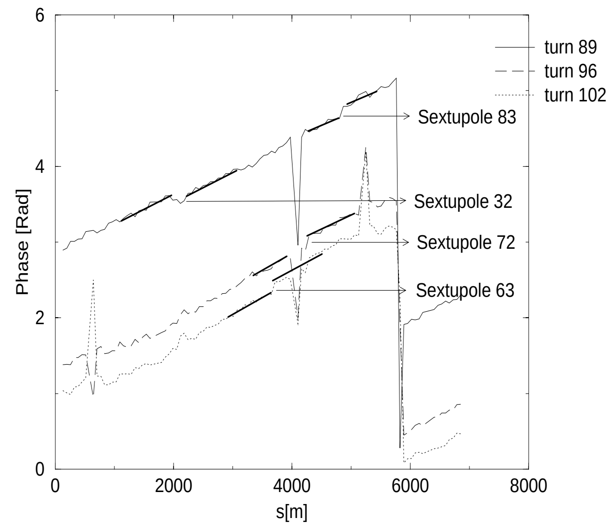

Figure 3: Phase analysis of SPS orbits. The sextupoles that were introduced intentionally in the accelerator can be clearly identified by the jumps in phase.

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