SELF-CONSISTENT, UNBIASED EXCLUSION METHODS OF EMITTANCE ANALYSIS

Martin P. Stockli, R. F. Welton, SNS, ORNL, Oak Ridge, TN 37830, USA*
R. Keller, SNS, LBNL, Berkeley, CA 94720, USA*

Abstract

We present a self-consistent method for analyzing measured emittance data that yields unbiased estimates for the rms emittance as well as its associated uncertainty. The self-consistent, unbiased elliptical exclusion analysis, SCUBEEx, uses an exclusion ellipse to determine the bias from the data outside the ellipse, before calculating the emittance from the bias-subtracted data within the ellipse. Variations of the ellipse size, shape, and orientation allow for objectively estimating the bias and the rms emittance.

INTRODUCTION

The emittance of a particle beam is the six-dimensional distribution of all position coordinates along the three configuration space directions and their associated velocity coordinates. Projecting it into the two-dimensional planes, {x-x'}, {y-y'}, and {z-z'}, respectively, reduces the emittance into three subsets. The description can be further reduced to the fractional area emittance, describing the area in either two-dimensional subset occupied by a fraction of the particle beam, which obviously is a function of the fraction value.

The description of the typical spread of the distribution with one single value was accomplished with the introduction of the root-mean-square emittance [1]. Based on the quantity of particles $c(x,x')$ passing through the position coordinate x with a velocity component x', the rms emittance is defined as

$$\varepsilon = \sqrt{\left\langle x'^2 \right\rangle \left\langle x^2 \right\rangle - \left\langle xx' \right\rangle^2} \qquad \text{with}$$

$$\left\langle x^2 \right\rangle = \frac{\sum_{all} x^2 c(x,x')}{\sum_{all} c(x,x')} \ , \quad \left\langle x'^2 \right\rangle = \frac{\sum_{all} x'^2 c(x,x')}{\sum_{all} c(x,x')} \ ,$$

$$\text{and} \quad \left\langle xx' \right\rangle = \frac{\sum_{all} xx' c(x,x')}{\sum_{all} c(x,x')} \ .$$

The emittance often is given normalized to the particle velocity v and the speed of light c: $\varepsilon_{norm} = \varepsilon \cdot v \cdot (c^2 - v^2)^{-1/2}$, a factor of 0.0118 for our 65 kV H⁻ beam.

Usually all coordinates are measured from the center of the particle distribution, which is accomplished by translating x and x' so that their first moments become zero. These translations minimize the rms emittance. The orientation and aspect ratio of the rms emittance ellipse are described by the Twiss parameters, namely,

$$\alpha = -\frac{\left\langle xx' \right\rangle}{\varepsilon} \ , \quad \beta = \frac{\left\langle x^2 \right\rangle}{\varepsilon} \ , \text{and} \quad \gamma = \frac{\left\langle x'^2 \right\rangle}{\varepsilon} \ .$$

All these terms are well defined and can be evaluated without any problems as long as $c(x,x')$ is well defined, for example in simulation data. Problems can arise when evaluating measured emittance data, because they contain noise and typically a small bias. These problems are normally avoided by excluding most of the background data through a threshold or through exclusion boundaries, although those methods can bias the results [2]. We present a method that systematically minimizes the net contributions from the background and self-consistently estimates the rms emittance and its uncertainty [3].

EMITTANCE MEASUREMENTS

Emittance measurements are in most cases double-slit experiments where the first slit samples a small fraction of the particle beam. The sampled beam spreads out before it is intercepted by the second slit to determine the associated transverse velocity distributions. When both slits are centered on the beam they record only a small fraction of the total particle flux, typically 1%. This fraction can drop to the 10^{-4} range when measuring the beam wings outside the beam core. Going further away from the beam core, one may find even smaller particle fluxes, the halo, until the flux gradually fades away and one measures pure background.

We describe the pure background with two components: the noise and the bias. The noise describes the local, quasi-random variations with a zero average. The bias describes the mean value, which is constant, at least locally. In principle the bias might vary gradually as a function of the position and velocity coordinates. However, we restrict this paper to uniform biases, as the presented background data do not exhibit any significant position or velocity dependence.

In the absence of an actual particle flux and a bias, one observes pure background noise, characterized by about the same number of small positive and small negative signals. We define positive as having the same polarity as the output when measuring the beam core, while negative refers to the opposite polarity. A positive bias can be recognized by a dominance of positive signals, whereas a negative bias by a dominance of negative signals.

Figure 1 shows measured emittance data as a density plot versus position x and velocity component x'. The large signals measured from the beam core occupy an
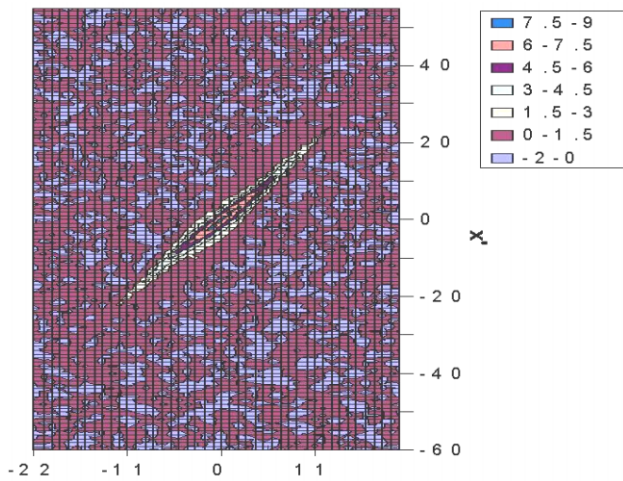
527

Figure 1: Emittance data from an expanding beam.

ellipse-like area with a diagonal orientation indicating an expanding beam. This colored structure is surrounded by a narrow zone of small, exclusively positive signals indicated in purple. Only further away from the beam core appear negative signals indicated in blue. Roughly 2/3 of the plotted area is consistent with pure background noise.

Applying the previously given formulas to all data yields α=-4.11, β=2.27, and γ= 7.88. For the rms emittance, one obtains ϵ=17.2 mm·mrad, which is the product of the two half axes of the rms emittance ellipse. Use of the half-axis product (HAP) with the dimension of mm·mrad is consistent with the emittance definition. We avoid the confusing π written as a part of the unit when it is supposed to be the multiplier for calculating the area.

THRESHOLD ANALYSIS

Applying a threshold commonly means that all values of a distribution above the threshold remain unchanged while all values below the threshold are set to zero before summing over all data.

Figure 2 shows the rms emittance estimated by applying a threshold to the data shown in figure 1 as a function of the threshold value. These thresholds, like all other values related to the measured signals, are quoted in percent of the maximum measured particle flux.

Some analysis codes exclude the negative signals from the emittance evaluations because they try to avoid having to deal with the "unphysical" reversed polarity. This is

equivalent to setting a threshold at 0%, which overestimates the rms emittance as 203.4 mm·mrad. This grossly inflated value is caused by large contributions from the positive noise signals found at large x and x' values.

To reduce such inflated values, most involved analysts increase the threshold to exclude all background data, often to a point where the emittance estimate no longer changes with small threshold changes. Figure 2, for example, shows the slope to change sevenfold at +6%, resulting in an rms emittance estimate of 12.2 mm·mrad.

There is, however, no absolute need to exclude the negative signals. On the contrary, their contributions compensate the contributions from the positive noise signals as one can see in Figure 2. When the threshold is set to −8%, which includes all data, the rms emittance is estimated at 17.2 mm·mrad, as previously found.

However, adding only a 0.005% bias to the data shown in Figure 1 increases the unthresholded rms emittance estimate to 23.9 mm·mrad, while subtracting a 0.005% bias reduces it to 4.6 mm·mrad, a factor of 5 difference caused by a small bias change of 0.01%. This severe sensitivity is caused by the large amount of background data in our example, especially those with large x and x' values. It causes rms emittance estimates from unthresholded data to be unreliable because small biases are not uncommon, and often unnoticed.

ELLIPTICAL EXCLUSION ANALYSIS

The reliability of rms emittance estimates can be improved by excluding pure background data, especially those located far from the core of the beam. The most reliable estimates are obtained when the exclusion boundary surrounds the data tightly without excluding any real signal. This task is difficult because some of the real signals are normally hidden in the noise. However, the absence of any significant net signal can be ascertained if one can vary the size of the exclusion area over a significant range without significantly changing the rms emittance estimate. Ellipses are best suited to conform tightly to typical emittance data. To demonstrate the elliptical exclusion analysis, we select as ellipse parameters the Twiss parameters α and β calculated from the data in Figure 1 after thresholding them at 10% to exclude all background signals.
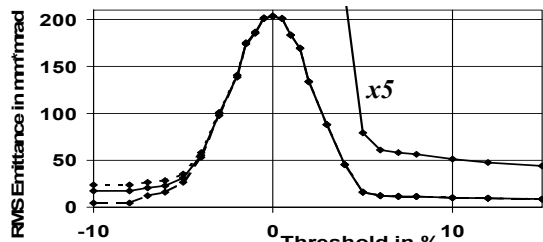

Figure 2: RMS emittance as a function of threshold applied to the data from figure 1 (solid line). The dotted and dashed lines show the emittance of the same data after adding to (dotted line) or subtracting from (dashed line) all data a bias of 0.005%.
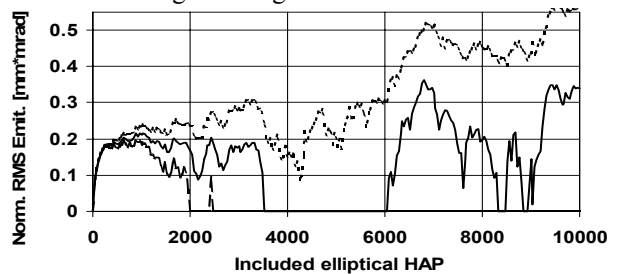

Figure 3: Normalized rms emittance estimates as a function of the exclusion ellipse HAP for the data of figure 1 (solid line), and after adding to (dotted line) or subtracting from (dashed line) all data a 0.05% bias.

The rms emittance estimates in Figure 3 vary wildly for large exclusion ellipses due to the same reasons why unthresholded estimates are unreliable. Shrinking the exclusion ellipse reduces these fluctuations until the solid line in Figure 3 forms a plateau below 2000 mm·mrad. Below 250 mm·mrad, the estimates start to fall off because one starts excluding data from real particle flux. The estimates, however, are sensitive to small biases as one can see from the two curves representing a +0.05% bias (dotted) and a –0.05% bias (dashed), indicating the need for a thorough bias analysis.

SELF-CONSISTENT BIAS ESTIMATION

Figure 4 shows the average of the signals measured outside the exclusion ellipse. Large ellipses yield large fluctuations because of the granularity of the few data found outside, e.g., only 11% of the data are outside 10,000 mm·mrad. More reliable bias estimates can be obtained for ellipses smaller than 3000 mm·mrad excluding more than 2/3 of the data.
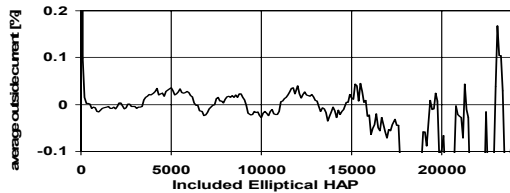

Figure 4: Average current signal outside the exclusion ellipse as a function of its HAP.

SELF-CONSISTENT UNBIASED ELLIPTICAL EXCLUSION ANALYSIS

The self-consistently determined bias needs to be subtracted from the raw data to counteract the sensitivity observed in the elliptical exclusion analysis. The top of Figure 5 shows the average of the current signals outside exclusion ellipses. The averages are fairly constant between 250 and 3000 mm·mrad, consistent with a bias of -0.008% ±0.01%.

The bottom of Figure 5 shows the rms emittance evaluated from the data within the ellipse after subtracting
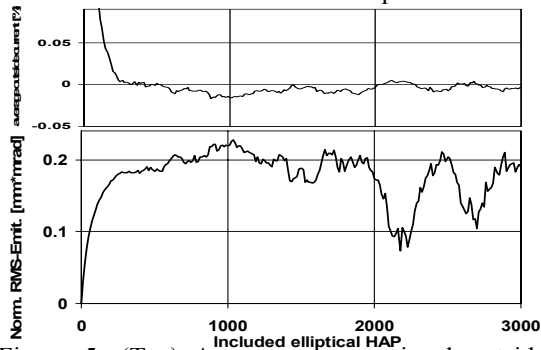

Figure 5: (Top) Average current signal outside and (bottom) normalized rms emittance estimated from the data inside the exclusion ellipse after subtracting the corresponding average outside current as a function of the exclusion ellipse HAP.
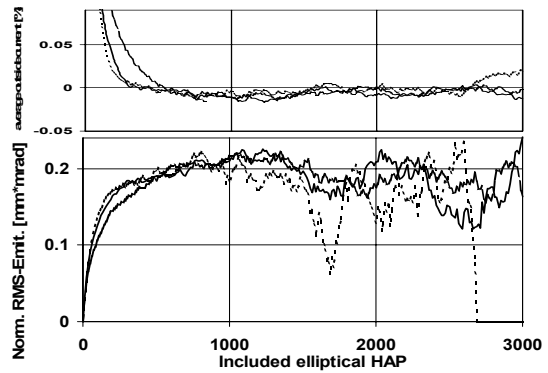

Figure 6: (Top) Average outside current and (bottom) normalized rms emittance of the bias subtracted data within the ellipse for three different exclusion ellipse sets.

the average current signal found outside the ellipse. The fluctuations caused by the noise clearly penetrate into the plateau. Even so, between 250 and 1400 mm·mrad the normalized rms emittance stays within 0.20 ±0.02 mm·mrad, consistent with a plateau.

Figure 6 is identical to Figure 5 except that the aspect ratio and orientation of the exclusion ellipse was determined from the data in Figure 1 after thresholding them at 5% (dotted), 20% (solid), and 90% (dashed). The figure shows that the dashed line requires a larger ellipse before the estimates reach the plateau, caused by a slightly different orientation of the ellipse determined from the 10% most intense measured signals. A threshold of 5% includes some noise leading to a less excentric ellipse and thus compresses the scale of Figure 5. However, Figure 6 shows that all evaluations are consistent with a bias of 0.008 ±0.01 and a normalized rms emittance of 0.20 ±0.02 mm·mrad.

CONCLUSIONS

With the robustness demonstrated we estimate with confidence the normalized rms emittance in our example at 0.20 ±0.02 mm·mrad. By chance, this is very close to the unnormalized 17 mm·mrad found earlier as the unthresholded estimate. It is, however, significantly larger than the estimate established through thresholding, which shows that thresholding is likely to exclude real signals from the wings and halo of the particle flux distribution.

REFERENCES

[1] C. LeJeune and J. Aubert, in "Applied Charged Particle Optics", edited by A. Septier (Academic Press, New York, 1980) p. 159.

[2] M.P. Stockli, R.F. Welton, R. Keller, A.P. Letchford, R.W. Thomae, and J.W.G. Thomason, in "Production and Neutralization of Negative Ions and Beams", edited by M.P. Stockli (AIP, New York, 2002) p 135.

[3] R.F. Welton, M.P. Stockli, R. Keller, R.W. Thomae, J.W.G. Thomason, J. Sherman, and J. Alessi, in "Production and Neutralization of Negative Ions and Beams", edited by M.P. Stockli (AIP, New York, 2002) p 160.