PERIOD LENGTH OPTIMIZATION FOR THE LNLS UNDULATOR

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Abstract

The aim of the present work is to find the optimum magnetic period length for the LNLS elliptically polarizing undulator (EPU). An objective function for the radiation which takes into consideration both intensity and polarization over the energy range desired by LNLS users is proposed and used in the optimization calculation. Results of such calculation are presented and discussed after a review of EPUs and their radiation characteristics.

ELLIPTICALLY POLARIZING UNDULATORS

APPLE-II Undulator

Planar undulators can only produce on-axis vertical fields that accelerate electrons that in turn radiate light with fixed linear polarization. If a variably and/or elliptically-polarized radiation is sought then the simple design of two parallel Halbach cassettes has to be abandoned. Many designs have been proposed in order to realize elliptical fields. A particularly optimized undulator that presents horizontal field with high amplitude was proposed by S.Sasaki[1]. Sasaki devised an EPU, named APPLE-II in the literature, which is composed of four parallel Halbach cassettes, one in each of the four quadrants in the transverse $x - z$ plane. Sasaki's undulator can be thought of as a combina-

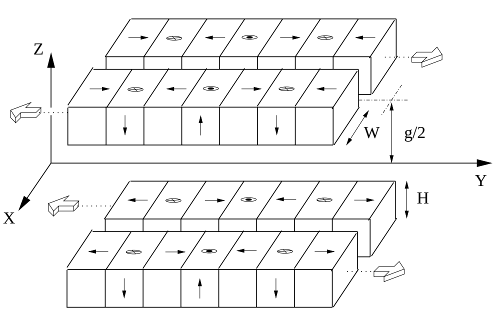

Figure 1: APPLE-2 undulator with its four Halbach cassettes. Diagonally opposed cassettes are fixed relative to each other but move longitudinally with respect to the other two cassettes. In the picture, the undulator is shown in the phase $\phi = \pi/2$

tion of two other undulators, each made of two diagonally-opposed cassettes. These two undulators create on-axis fields with the same profile, both the horizontal and vertical components. But while the vertical field components from the undulators are in phase, the horizontal field components are 180-degree out of phase. In this way, by shifting longitudinally the undulators with respect to each other, it is possible to create elliptical magnetic fields[1]:

$$\begin{aligned} B_x &= B_x^{max}(g/\lambda)\cos\phi\sin qy = B_{x0}\sin qy, \\ B_z &= B_z^{max}(g/\lambda)\sin\phi\cos qy = B_{z0}\cos qy, \end{aligned} \quad (1)$$

where $\phi = \pi\delta l/\lambda$ and $q = 2\pi/\lambda$. The shift between cassettes δl is relative to an undulator configuration in which the vertical field is zero. By symmetry the longitudinal component B_y vanishes for a period undulator. The maximum amplitudes B_x^{max} and B_z^{max} of the field components depend on undulator parameters: gap g between upper and lower cassettes, period length λ, transverse dimensions and remanent field of the permanent magnets (PMs). The plot in Fig.[2] shows this dependence on the magnetic gap. Note from the plot that, since the ratio B_x^{max}/B_z^{max} varies with g/λ, whenever the gap is changed in order to tune the energy peaks of radiation, the relative cassette phase ϕ has to be adjusted as well, if a specific field ellipticity is to be kept.

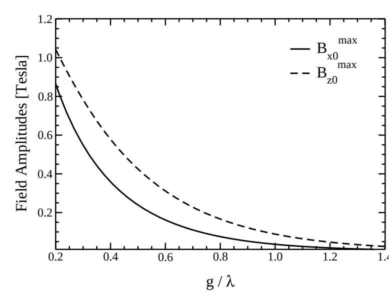

Figure 2: Vertical and Horizontal maximum field amplitudes as a function of the gap for an APPLE-II undulator with λ = 50 mm. Permanent magnets have transversal dimensions W = 40 mm, H = 40 mm and remanent magnetization B_r = 1.2 T. Separation between front and back cassettes is 1 mm.

UNDULATOR RADIATION OF A ZERO EMITTANCE E-BEAM

As electrons traverse the undulator they experience the magnetic field of Eq.[1] and, for an arbitrary EPU phase ϕ, they start moving in a helical trajectory. The equations of motion can be expanded in the parameter $1/\gamma$ and solved analytically up to second order. Such calculated trajectory can then be used to evaluate the radiation characteristics of the EPU[2]. In the Radiation Regime, the photon flux

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expression is given by

$$\frac{dN}{dt} = n_0 \gamma^2 N_p^2 \sum_{k=1,3,...} \left(\frac{kK}{1+K^2/2}\right)^2 g_k^2(\omega)|\mathbf{F}_k|^2. \quad (2)$$

This flux is proportional to both the number of undulator periods N_p and the e-beam energy γ squared. This holds true because the approximative assumption of small solid angle Ω_0 for the observation window was used ($\Omega_0\gamma^2 \ll \pi$). In the expression for the flux, $n_0 = (I/e)\alpha_f(\Delta\omega/\omega)\Omega_0$ is a quantity with dimension of particle flux that is proportional do the electron flux, fine-structure constant, frequency integration window and solid angle of the observation window. The complex vector $\mathbf{F}_k = \sin\phi_0(J_n - J_p)(i\mathbf{x}) + \cos\phi_0(J_n + J_p)\mathbf{z}$ contains contributions from both horizontal and vertical oscillating electrical field components. $J_n = J_{(k-1)/2}(u)$ and $J_p = J_{(k+1)/2}(u)$ are integer-order bessel functions of the first kind evaluated at $u = kK^2\cos 2\phi_0/(4+2K^2)$, where K is the undulator parameter. For an arbitrary EPU phase, $\cos^2\phi_0 \equiv B_{x0}^2/(B_{x0}^2+B_{z0}^2)$ is neither zero nor one and the two contributions for $\mathbf{F}_k$ lead to elliptically polarized radiation on axis. When the EPU cassette phase is set in a way that the amplitudes B_{x0} and B_{z0} of horizontal and vertical field components are equal, the argument u of the bessel functions vanishes and only the first harmonic $k = 1$ contributes to the radiation. All higher harmonics are absent when the EPU operates with circularly polarized light.

The complex vector $\mathbf{F}_k$ can be expanded in any vector basis. In particular, using the basis $\sqrt{2}\mathbf{e}_+ = \mathbf{x} + i\mathbf{z}$ and $\mathbf{e}_- = \mathbf{e}_+^*$, it is possible to decompose the expression $|\mathbf{F}_k|^2$ in Eq.[2] into right-handed and left-handed circularly polarized radiation contributions, dN^+/dt and dN^-/dt respectively. For magnetic dichroism experiments, where pure circularly polarized light is needed, a figure of merit can be defined as the normalized different between left and right circularly polarized radiation components: $p_k \equiv (dN^+/dt - dN^-/dt)/(dN^+/dt + dN^-/dt)$. This expression can be explicitly evaluated from $\mathbf{F}_k$:

$$p_k = \frac{\sin 2\phi_0(J_n^2 - J_p^2)}{\sin^2\phi_0(J_n - J_p)^2 + \cos^2\phi_0(J_n + J_p)^2} \quad (3)$$

OBJECTIVE FUNCTION

There are two polarization cases of radiation that users may be interested in: horizontal and circular. These two cases can be realized with the EPU that is going to be built at LNLS by setting its phase ϕ appropriately. Energy ranges which can be covered with radiation harmonics depend on the choice of period length λ. For each energy and polarization state there is a unique λ that maximizes photon flux. Moreover, users of the undulator radiation are interested in a broad energy range, from absorption lines of elements Carbon (286 eV), Nitrogen (400 eV) and Oxygen (525 eV) (linear light polarization) to L-edge transitions in metals like Manganese ($\sim$ 640 eV), Iron ($\sim$ 710 eV), Cobalt ($\sim$ 780 eV) and Nickel ($\sim$ 860 eV), for circular magnetic dichroism experiments. As a consequence, the choice of period length is not obvious and a compromise between radiation output for different energies has to be achieved. The most generic method that can take into account this compromise is one in which a weighting number is assigned to each energy-polarization state. An objective function then is defined as the summation of the radiation flux at each energy multiplied by its corresponding weighting number. The optimized period length comes out of maximizing this function.

$$F(\lambda) = \int d\epsilon \left\{ W_H(\epsilon)\frac{dN_H}{dt} + W_C(\epsilon)\frac{dN_C}{dt} \right\} \quad (4)$$

It is important to realize that the condition of 100% circularly polarized light for higher energies can not be attained since the first order harmonic is the only term that exists. In order to cover the L-edge transition lines of metals the ideal unit polarization ratio has to be abandoned and elliptically polarized light from the third and fifth harmonics has to be used instead. This consideration implies that, for the case of circular radiation, the quantity that enters the objective function has to deal with the compromise between polarization ratio and photon flux. For such quantity the product of the flux and the polarization ratio can be chosen, as it appears in Eq.[4]. The weighting functions picked for both polarization cases are plotted in Fig.[3].

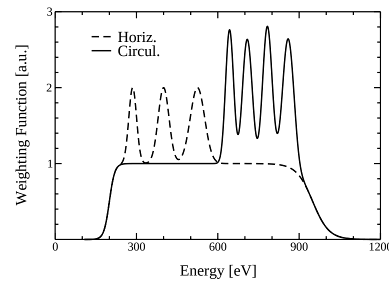

Figure 3: Weighting functions for horizontally (dash lines) and for circularly (solid lines) polarized light cases.

OPTIMIZATION OF OBJECTIVE FUNCTION

The algorithm for evaluating the integral in Eq.[4] consists in calculating maximum flux for each of the two polarization cases, over all energies, for λ values. The calculation for the horizontal case is simple: for a given λ and energy ϵ, check for which harmonic bands this energy lies within its limits, calculate the photon flux for those bands, pick the one with higher value and add its contribution to the integral over energy. The case of elliptically polarized radiation is more elaborate because for each λ and photon energy the optimum EPU cassette phase that maximizes the figure if merit has to be calculated. Given λ and ϵ, the optimum phase is calculated for each harmonic. The figure of merit for each band is evaluated and the biggest one is chosen and used in Eq.[4]. Table[1] lists all parameters used in the optimization just described and Fig.[4] shows the result of the optimization calculation. The contributions from the horizontal and circular cases to the objective function are plotted separately, as well as the objective function itself. The optimum period length for the undulator turned out to be 48 mm. Fig.[5] shows the flux for the horizontally polarized radiation case with optimum λ. Three bands corresponding to the first, third and fifth harmonics can be identifies in the plot. Since the vertical magnetic field for the minimum undulator gap is such that $K > 2$, there are no gaps between the bands. In Fig.[6] the on-axis radiation flux and polarization ratio are plotted as functions of photon energy for the elliptical case. For energies covered by the first harmonic it is apparent from the plot that the polarization ratio is one and therefore the radiation is 100% circularly polarized. This is shown in the plot for energies above 183 eV. Energies below this value can not be attained with perfectly circular polarization because the maximum K value with minimum gap is not high enough. As for the third and fifth harmonic bands, the undulator phase that maximizes the figure of merit does not correspond to a circular polarization state and it is energy-dependent over the whole energy interval of the bands.

Table 1: Parameters used in the calculation for the period length optimization

EPU	PM width	W = 40 mm
	PM height	H = 40 mm
	Min. gap	g_{min} = 23 mm
	Length	L = 2900 mm
	PM magnetization	B_r = 1.2 Tesla
E-beam	Energy	E = 1.37 GeV
	Emittance	ϵ = 0
Observation Window	Opening angle	θ = 0.2 mrad

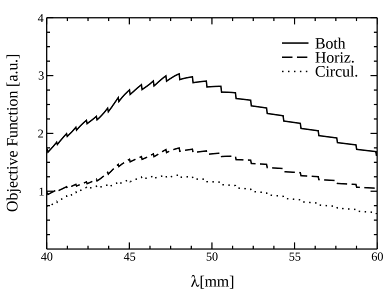

Figure 4: Weighting functions for horizontally (dashed lines) and for circularly (solid lines) polarized light cases.

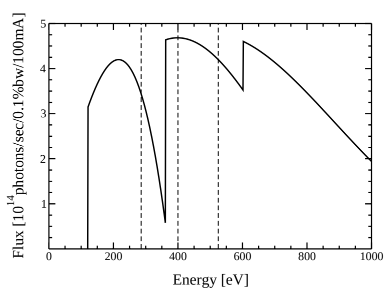

Figure 5: The three vertical dash lines correspond to transition energies of Carbon (286 eV), Nitrogen (400 eV) and Oxygen (525 eV).

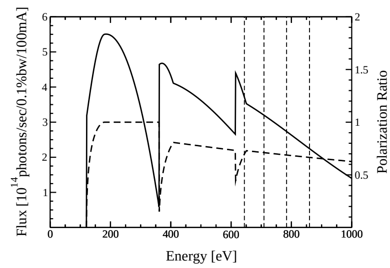

Figure 6: The solid curve is the radiation flux and the dash curve is the polarization ratio. Vertical lines correspond, in energy order respectively, to transitions in metals: Mn, Fe, Co, Ni.

FINAL REMARKS AND ACKNOWLEDGEMENTS

We have optimized the undulator period length by maximizing an objective function that takes into account the energy range and the polarization states of radiation that are relevant for the beamline users. The optimum value was 48mm. It was assumed that the solid angle related to the observation window is much smaller than the opening angle of the radiation. This assumption overestimates the photon flux, specially for higher harmonics and higher energies. A proper convolution integral of the radiation over the observation window has to be implemented for a more accurate calculation of the photon flux. On the same footing the beam emittance of the storage ring should be considered. The lower energy limits of the harmonic bands are expressed in terms of the maximum field amplitudes for minimum undulator gap. In this work the field amplitudes were calculated assuming a unit permeability for the permanent magnet blocks. For typical NdFeB magnets this permeability is slight above one. Demagnetization effects can lead to somewhat smaller field amplitudes and consequently they can change the lower limits of the harmonic bands. Finally I would like to thank Pedro F. Tavares, G. Tosin and A.R.B. de Castro for fruitful discussions on the subject of e-beam dynamics and radiation characteristics of undulators.

REFERENCES

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